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Elise Koch

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THE RECORD
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June, 1925

ADDRESS BY THE PRESIDENT, PERCY H. EVANS

The President of the American Institute of Actuaries is expected to open each meeting with an address. This is a serious responsibility, but it is considerably lightened by the fact that he does not have to submit his remarks in advance to the Committee on Papers, and as the presidential addresses are not discussed, he avoids the embarrassment of having his mistakes gently but firmly pointed out from the floor.

Our program presents an unusually large number of original papers, besides discussion of three valuable contributions from the Joint Meeting last November and informal discussion on three interesting topics. Because of this very full program, we shall have no time to waste, and I wish to assure you that if I had consulted my own preferences instead of established precedent, I would not take your time with an address. However, in submission to the force of precedent, I have prepared an address on a subject which I hope will be of interest to you.

Before proceeding with my subject, please allow me at this time to express my gratitude for the willing help and co-operation I have had during the past year from all of the officers, the Board of Governors, the various committees, and the membership of the Institute. I do not wish to single out particular officers or committees; each and every one has earned my personal gratitude as well as that of the Institute; but I do think it is proper that you should realize and appreciate that a very heavy burden falls on the Secretary, the Editor, and the Examination Committee. Their work has

been done promptly and efficiently. They are entitled to special consideration, and one of the ways in which we can give evidence of our appreciation is by returning prompt answers to their communication in connection with the matters on which they may have occasion to address the membership.

Happiness has been well defined as the "feeling of continuous progress in the pursuit of a desirable object." There is serious danger in satisfaction with results; nevertheless, I think we have good reason for being pleased with the progress of the American Institute of Actuaries in the realization of its functions. Without going into other details, I think its progress in all functions is well indicated by its growth in its educational activity as reflected in the examinations. In 1921 there were 153 papers written with 89 passes; in 1922, 179 papers with 93 passes; in 1923, 298 papers with 164 passes; in 1924, 357 papers with 178 passes; and for the examinations last April, 455 papers with 231 passes.

Now with some embarrassment in appropriating precious minutes from our printed program, I will pass to the address I have written.

THE TREND OF MORTALITY

There are few subjects that excite popular interest more readily than the span of human life. The least significant table printed in life insurance ratebooks is the "Expectation of Life," yet the layman who examines one of these manuals will always stop at that page and read the tabulated "expectation" at his own age. This very natural and human interest in the remaining period of vital existence is responsible for the success of propaganda based on statements of alleged statistical results which are often absurd exaggerations. I have somewhere seen a statement made with the appearance of an established fact that in consequence of some social activity or other the span of life has within a very short period been increased twelve years. It is not at all uncommon to read statements, based on an increase in the average after-lifetime at birth, made as if the increase applied at the average age.

Between the popular interest in vital statistics and the special interest of biometricians there are many other interests. The trend of mortality in the general population should be of particular interest to life insurance actuaries from its bearing on the cost of insurance. *The Record of the American Institute* and the *Transactions of*

the Actuarial Society contain studies of the trend of interest rates which, if not particularly conclusive, are at least most interesting. What I have to say is in no sense a study of future mortality rates but merely a suggestion that the subject is quite as important and worthy of study as the trend of interest rates. Such studies might at least serve to correct erroneous popular notions of what is happening to the span of life in consequence of the various forces at work.

Probably the actuary of any company who has placed on his desk a digest of the results of successive mortality investigations covering a long period of time will find reason for being impressed by the changes in the mortality rates. Table I is a digest such as I have in mind, covering a number of successive issue groups:

TABLE I

Issues	Medico-Actuarial Table			American Men Table			
	First 5 Years	After 5 Years	Aggregate	First 5 Years	After 5 Years	Aggregate	Exposed during First Policy Year
1885–1890	95.7	97.7	97.4	.....	.....	.....	$ 202,544,600
1891–1895	86.4	96.0	94.1	.....	.....	.....	236,709,500
1896–1900	73.2	91.2	86.6	.....	.....	.....	288,665,000
1901–1905	76.3	80.9	79.0	84.2	76.4	79.2	389,019,000
1906–1910	68.0	78.0	70.9	75.3	74.3	74.9	562,830,400
1885–1905	80.5	92.3	89.3	89.0	87.3	87.7	$1,116,938,100
1885–1910	76.3	91.4	86.5	84.3	86.5	85.8	1,679,768,500
1906–1915	Excluding "flu" and war losses			78.0	79.5	78.8	$1,161,495,400
1906–1915				71.8	68.0	69.8	.....

It is scarcely necessary to mention the fact that life insurance experience can give no exact value to forces operating on population vitality, not only because the lives are selected, but because the selection has been more or less different at different times. A medically selected group does not cease to be select at the end of five or any other number of years. However, the exclusion of the early policy years does minimize the effects of selection and uncover, to some extent, the effect, if any, of improvements in medical, sanitary, and hygienic science, or rather the popular knowledge and employment of such improvements in everyday life.

Table II, shows for three large groups the mortality experience by attained ages after the fifth policy year by the American Men (ultimate) Table.

The exclusion of death losses of $3,709,800 from the 1906–15 groups, covering all the deaths during 1918, 1919, and 1920 from influenza, pneumonia, and military or naval causes, resulted in a reduction of the rate from 79.5 to 68 per cent. This ratio is doubtless considerably less than the result would have been if the pandemic and the war had not occurred. It is certain that the ratio for this group would have been higher than the experience during 1924, and possibly as high as 72 per cent.

The variations in selection at different times, whether resulting from company policy or efficiency, cannot be compensated. In the

TABLE II

Attained Ages	Issues, 1885–1905, Exp. to Ann., 1915	Issues, 1906–15, Exp. toAnn., 1920 (Ex. Flu and War)		Issues, 1899–1919, Exp. during 1924
20–24	62.3	132.1	65.9	60
25–29	98.4	118.4	60.8	64
30–34	91.7	107.7	61.4	59
35–39	88.9	93.2	68.4	92
40–44	86.9	76.3	64.3	52
45–49	85.2	75.2	69.7	73
50–54	87.8	76.9	71.4	55
55–59	86.1	73.5	70.5	68
60–64	84.0	65.8	61.9	76
65–69	92.0	76.4	68.9	74
70–74	89.8	113.0	108.7	88
75–79	90.6	.....	.....	83
80	73.0	.....	.....	90
Total	87.3	79.5	68.0	69

selection of these particular groups there was little if any change in company policy during the period covered, but there was, without doubt, considerable improvement in efficiency. Improvement in the system of inspections may be mentioned. Also the group issued 1906–15 had the benefit of blood-pressure reports beginning in a limited way in November, 1906, until at the end of 1912 some 90 per cent of the applications contained a blood-pressure reading.

However interesting these results may be at any time, and especially when American companies are giving so much attention to insurance without medical examination, it seems clear that the figures do little more than suggest that during the past forty years there has been an improvement in general mortality rates which has supplemented the medical selection of life insurance risks. Of course

the answer to the question, certainly the best answer and probably the only possible answer, is to be found in the work of our own Dr. J. W. Glover in constructing the United States Life tables from the census returns. These tables show the following expectations:

TABLE III

United States Life Tables

Complete Expectation

Age	1890	1901	1910	1920
	Mass. Males	O.R.S. White Males	O.R.S. White Males	O.R.S. White Males
2	51.57	55.54	56.88	58.62
7	50.60	53.03	53.85	55.34
12	46.82	48.85	49.56	51.02
17	42.82	44.57	45.18	46.71
22	39.30	40.71	41.13	42.69
27	36.02	37.06	37.21	38.77
32	32.72	33.43	33.33	34.93
37	29.38	29.87	29.61	31.12
42	26.03	26.33	25.99	27.32
47	22.69	22.82	22.46	23.56
52	19.44	19.40	19.02	19.91
57	16.42	16.16	15.77	16.51
62	13.56	13.17	12.85	13.38
67	10.80	10.48	10.25	10.56
72	8.42	8.11	7.95	8.17
77	6.41	6.09	6.04	6.17
82	4.76	4.54	4.56	4.53
87	3.27	3.39	3.49	3.21
92	2.19	2.54	2.70	2.10

Considerable discussion ensued when it was found that the census of 1910 showed a decrease in the expectation at the older ages where degenerative diseases make their appearance. Life insurance results had not shown evidence of increase in such ailments. However, when the results of the 1920 census became available, the fear that modern conditions of life were causing an increase in such diseases was quieted. The results of the 1920 census are of course very satisfactory, but it seems to me that we are unduly optimistic if we expect the next census returns to show a marked increase over 1920, because I believe the 1920 expectations were favorably influenced by the influenza epidemic. The census report says that the lack of an improvement in mortality rates in adult years was due to the influenza epidemics of 1919 and 1920. Life insurance

experience indicates that the peak of the pandemic occurred in the last quarter of 1918. The United States Life expectations were based on the death of 1919 and 1920. It is not unreasonable to expect that the lives which survived the worst of the influenza in 1918 might show an increased expectation, though of course not so great as if the disease had not been prevalent from January to April, 1919, and during February and March, 1920.

As I have said, the trend of mortality appears to be quite as important as the trend of interest rates to the actuaries of life insurance companies. The attempt of Messrs. Elderton and Oakley, in *The Mortality of Annuitants, 1900-1920*, to forecast the rates likely

TABLE IV

Age	70 Per Cent A.M. (Ult.)	Dublin
20	50.53	49.82
30	41.90	41.06
40	33.16	32.49
50	24.74	24.08
60	17.22	16.01
70	11.10	9.17
80	6.62	5.29
90	3.70	3.03
100	1.72	1.85

to prevail in future has a special importance in the case of non-participating annuities, but such studies are also of importance to life insurance. The vital statistics of Massachusetts indicate an increase in the expectation at birth from forty years in 1855 to fifty-five years in 1920. Dr. Dublin has constructed a hypothetical table of limiting expectations which he believes to be reasonable, based on the United States Life Table of 1910. I am unable to see anything unreasonable in his assumptions. He increases the expectation at birth to sixty-five years. At age twenty his figure is 49.82 years as compared with 45.93 by the American Men (ultimate) Table, with 16.01 at age sixty as compared with 14.29. I have already given a life insurance experience showing mortality about 70 per cent of the American Men Table. Modifying the American Men Table on that basis results in expectations that are in excess of those under Dublin's Hypothetical Table (Table IV).

Since the foregoing was written, Mr. C. D. Rutherford has

contributed to the *Transactions of the Actuarial Society*, "A Note on Mortality Variations" in which he offers the conclusion that the ultimate mortality to be expected at the present times does not exceed 75 per cent of the A.M. (5) Table.

In a discourse of this character, where no attempt is being made to prove anything, more or less unlimited speculation may be pardonable as to the possible origins of future improvement in mortality. This is an age of popular education in life-conservation. If anything like the improvement reflected by our figures is humanly possible, we should be able to realize a considerable part of it within the next fifty years. Those who are interested in anti-tuberculosis campaigns are optimistic because the death-rate from that disease has declined to about one-half the rate experienced in 1900. Surely this is a most impressive fact, but the apparent gain may not be real, and if real it may not be permanent. When the forces of nature are repressed at one point by human interference, they have a tendency to boil over at another point. It is well, therefore, that we do not think too highly, and expect too much, of our powers of interference; neither would it be wise to retire from the field and do nothing at all about these conditions on the ground that you cannot beat Nature at her very ancient job of selection. If the anti-tuberculosis people wish to believe that they have worked a miracle, why object? If the proponents of the doctrines popularized by Mr. Wiggam in *The Fruit of the Family Tree* prefer to believe that Nature brought about the change as a part of her scheme of natural selection, why dispute it? As life insurance men, we are more concerned with the proximate trend of mortality than in arguments as to whether racial freedom from tuberculosis will eventually come because the anti-tuberculosis society has killed all the germs, or because Nature has killed all the people who are obnoxious to the germs. As actuaries, we are strictly practical, and we are in favor of any activity that can add a year or any fraction of a year to the expectation of life among sufferers from this disease.

A certain eminent biologist with a talent for publicity, who finds no ground for believing that the decline in the tuberculosis death-rate has been affected one way or the other by any social activity, has also told the public something pleasant to hear about alcohol and longevity. Based on a study of 1,259 male lives, he concludes that moderate drinking is not associated with a shortened expecta-

tion as compared with total abstention. Indeed, the moderate use of alcoholics is, it appears, rather beneficial if this data is accepted, because the moderates show a year longer average total lifetime. Since I can see a number of things in life that seem to me more important than living a year or two more or less, I am inclined to agree that the question cannot be disposed of by considering only the undoubted evils of alcohol in excess as, alas, we continue to know them years after the country has gone dry by enactments made and duly printed in the statute-books. In an article in the *American Mercury* of February, 1924, Dr. Raymond Pearl said the man on the street

> has been told that the experience of life insurance companies has proved that the use of even the smallest amount definitely shortens life. The deductions of the actuary have a great reputation for deadly precision among persons who know nothing about their basis. This reputation is probably somewhat higher than the real merits of the case would warrant. Certainly in the matter of present interest what the insurance companies actually know about the effect of alcohol upon mortality can by no possibility be held to justify the conclusions which the public have drawn.

What we require is the cold, scientific truth about the effect of alcohol on longevity regardless of the beliefs and emotions of any person or group of persons. It is quite true that conclusions have been drawn from published life insurance experience and used as prohibition propaganda that were not logical consequences of the facts. It may be true that some individuals who happened to be actuaries have found in the experience more support for their personal opinions than the available facts scientifically justified. Even actuaries are human. But it is not true that in the latest and most extensively published investigation of insurance results from the angle of liquor habits, so far as they could be defined, there was one word to support Dr. Pearl's unnecessary remark about actuaries as a class. The experience referred to was published as a "Note on Mortality by Habits Representation." The paper contained the following caution:

> The outstanding consideration is the instability of this habit in consequence of which a classification based on representations made at the time of application can tell us very little of the influence of a given alcohol consumption on longevity. The general reader must therefore be warned that nothing better than broad inferences concerning the effects of alcohol can be drawn from data based on original applications where the subse-

quent histories have not been traced and periodical reclassifications made on some significant basis.

Let me mention here the results of this investigation of mortality under 286,392 policies, issued 1885–1900, according to what the applicants claimed their liquor habit to be in making application. The group covered 30,069 deaths for $86,691,000 of insurance. The "total abstainers" showed a mortality of 84.3 per cent; "moderate users," 97.2 per cent; "regular beer-drinkers, 111.3 per cent; and "regular spirit-drinkers," 128.9 per cent of the Medico-Actuarial Table of Mortality. A later count, covering the issues of 1901–8, showed the following percentages of the American Men Table for the several groups in the same order, i.e., 69.6 per cent, 77.6 per cent, 91.2 per cent, and 127.3 per cent.

Why do applicants for life insurance who claim to be abstainers at the time of medical examination persistently show the most favorable mortality? Of course we do not suppose they all told the exact truth. We may also assume that many of them took to drink later on. But it seems as if one of two things must be said about this "abstainer" group: Either there is some benefit to longevity in abstinence or else we must believe there was just enough, and no more, moderate tippling going on in the "abstainer" class to give the whole group the alleged benefits of moderate doses of alcohol while the other three groups, including the so-called "moderate users," were in point of fact using alcohol to excess. This whole question of "moderate use" is still unanswered. Mr. Emory McClintock said in 1905: "Statistics have never yet shown one atom of information on that subject and it is doubtful if they ever can. All that is possible is to argue about it."

Professor Ross says that when the biologists found they could not agree about the transmission of acquired characteristics without more facts, they called a truce and got busy cutting off the tails of successive generations of mice. It may be that the present question is insoluble. Nevertheless, I believe that there is, in the absence of proof to the contrary, a rather sound basis for belief that the frequent creation of abnormal somatic conditions by the application of alcohol in "moderate" doses is probably not conducive to longevity. Such limited use of alcoholics may be psychologically helpful in meeting the "storm and stress" of life, but that is an entirely different question.

In one of his books William James quoted an eminent physiologist to the effect that if all rivers and springs ran alcohol instead of water, either all men would now hate it or our nerves would have been selected so as to drink it with impunity. In spite of the extent to which the odor of alcohol was abroad in the land up to the Volstead era, it is fairly evident that neither of these results came to pass. All men do not hate it, and even the "Wets" agree that in spite of a certain tolerance shown by peoples long habituated to alcohol, excessive indulgence increases mortality. Let us assume that the present generation of alcoholics dies out and the quantity of alcohol consumed is reduced to a minimum. Surely this will affect the trend of mortality until Nature finds some new and equally effective way of killing off the unfit. After all, alcohol is a relatively inefficient instrument of natural selection because, among other reasons, it operates too slowly. My guess is that as the insurance-buying class gradually forgets to think so much about alcohol, the mortality rates among moderate and free users will tend toward and perhaps below the rates heretofore exhibited by applicants claiming to be abstainers. We may then find it necessary to revise our build classifications and possibly also our notions of average blood pressure which appears to be related in some manner to overweight. Dr. J. W. Fisher pointed out in 1915 that persons weighing 20 per cent or more in excess of the average for height and age show an average pressure about 4 millimeters higher than the general average. The Metropolitan Life Insurance Company, in a study of hypertension correlations, reached the conclusion that of the five characteristics studied overweight alone seemed to have a definite relationship to high blood pressure.

It appears to be the general opinion among actuaries that during the next twenty-five years interest rates will tend downward. There is also a possibility, at least, that during the same period we may, in the case of lives selected with that end in view, also realize some further decrease in mortality rates in consequence of improved living conditions, research, and education. Whether or not conservation of life-activities will make any permanent change in the curve of racial evolution as it is read geologic ages hence; whether such activities tend to conserve and increase desirable human characteristics or to perpetuate the undesirable; these seem to me questions for the biologist rather than the actuary who has a very definite rela-

tion to a group of living persons with whom contracts have been made involving life-contingencies.

PRESENTATION OF DIPLOMAS

(The President made the following remarks at the time of presenting diplomas to new Fellows of the Institute:)

The Board of Governors, by virtue of authority delegated to it by the American Institute of Actuaries, hereby confers on each of you the degree of Fellow of the Institute. This certificate or diploma is granted only to those who have obtained Fellowship by examination. It is therefore evidence that you have successfully passed the very arduous tests of your knowledge of pure and actuarial mathematics and also, under the wise rule of the Institute, of your familiarity with the practical requirements of your profession. As personalities you will give different values to the several parts of your acquirements. I venture to suggest to you that in the practice of your profession your human contacts will for the most part be with men who do not have actuarial training, and for that reason among some others you should not fail to appreciate the practical side of your work or what may be called "common sense of actuarial science."

Now this certificate means something more than technical proficiency. The Board of Governors means it to be also a certificate of character. They would not have granted it to you in spite of your success in passing twelve examinations if they were not satisfied that those who employ your professional services will always receive from you an honest opinion, free from self-seeking or fear of consequences, on any question which it may be your duty to consider. In accepting this certificate of proficiency and character, you create a moral obligation that your conduct both professional and private shall be an honor to your profession and to the American Institute of Actuaries which by this parchment has given you the support of its honorable name. I am sure that you will live up to this obligation, and because of that sureness I feel it a great honor as President of the Institute to exercise the privilege of transmitting these certificates to your several hands. The best wishes of the membership of the Institute go with you in your future progress upward.

ON LIDSTONE'S DEMONSTRATION OF THE OSCULATORY INTERPOLATION FORMULA

By J. F. REILLY

In a note at the end of my paper, "Certain Generalizations of Osculatory Interpolation," which appeared in *The Record*, XIII (1924), 4–22, reference was made to Lidstone's "Alternative Demonstration of the Formula for Osculatory Interpolation."

I there pointed out that the generalized Lidstone method did not always produce the same results as the generalized Sprague-King method, and cited one case where the two formulas could not agree since they were of different degrees.

In the present paper Lidstone's method has been generalized, and the results contrasted with those previously obtained by the Sprague-King method.

STATEMENT OF THE PROBLEM

The polynomial of degree $2h$ with given values y_x corresponding to $x=0, 1, 2, \ldots, 2h$ may be written

$$y_x=u_x=y_0+x\,\Delta y_0+\frac{x(x-1)}{2!}\,\Delta^2 y_0 + \ldots + \frac{x(x-1)\ldots(x-2h+1)}{(2h)!}\,\Delta^{2h}y_0 . \quad (a)$$

Also the polynomial of degree $2h$ with given values y_x corresponding to $x=1, 2, 3, \ldots, 2h+1$ may be written

$$y_x=v_x=u_x+\frac{(x-1)(x-2)\ldots(x-2h)}{(2h)!}\,\Delta^{2h+1}y_0 . \quad (b)$$

Lidstone's method of deriving an osculatory interpolation formula when generalized requires the determination of a polynomial ψ_x such that the curve

$$y_x=u_x+\psi_x \quad (c)$$

shall pass through the points (h, y_h), and $(h+1, y_{h+1})$, and shall have contact of order k with curve (a) at the point (h, y_h), and with curve (b) at the point $(h+1, y_{h+1})$. Curve (c) will have contact of order k with curves (a) and (b) if the first k derivatives of y with respect to x obtained from (a) are the same as those obtained from (c) when $x=h$, and if the first k derivatives of y with respect to x obtained from (b) are the same as those obtained from (c) when $x=h+1$.

THE REQUIRED CURVE

From (a) $y_h=u_h$, and $y_h^{(m)}=u_h^{(m)}$, $m=1, 2, \ldots, k$, where $y_h^{(m)}$ is the value of the mth derivative of y_x when $x=h$.

From (b) $y_{h+1}=v_{h+1}$, and $y_{h+1}^{(m)}=v_{h+1}^{(m)}$, $m=1, 2, \ldots, k$.

From (c) $y_h=u_h+\psi_h$, and $y_h^{(m)}=u_h^{(m)}+\psi_h^{(m)}$, $m=1, 2, \ldots, k$; and $y_{h+1}=u_{h+1}+\psi_{h+1}$, and $y_{h+1}^{(m)}=u_{h+1}^{(m)}+\psi_{h+1}^{(m)}$, $m=1, 2, \ldots, k$.

Imposing the conditions that curve (c) shall pass through the point (h, y_h) and have contact of order k with curve (a) at this point, we obtain

$$\psi_h=\psi_h'=\psi_h''=\ldots=\psi_h^{(k)}=0.$$

Hence ψ_x will contain $(x-\mathrm{h})^{k+1}$ as a factor.

Again, imposing the conditions that curve (c) shall pass through the point $(h+1, y_{h+1})$ and have contact of order k with curve (b) at this point, we obtain

$$\psi_{h+1}=0 \text{ (since } u_{h+1}=v_{h+1}),$$

$$\text{and } v_{h+1}^{(m)}=u_{h+1}^{(m)}+\psi_{h+1}^{(m)},\ m=1, 2, \ldots, k.$$

Hence ψ_x will contain $(x-h-1)$ as a factor and must satisfy k additional conditions.
We may thus write

$$\psi_x=(x-h-1)[a_{k+1}(x-h)^{k+1}+a_{k+2}(x-h)^{k+2}+\ldots+a_{2k}(x-h)^{2k}] \quad (d)$$

where the coefficients $a_{k+1}, a_{k+2}, \ldots, a_{2k}$ are to be determined to satisfy these k additional conditions. When the a's are thus determined, $u_x+\psi_x$ will be the osculatory interpolation formula.

THE EQUATIONS FOR THE DETERMINATION OF THE a'S

From curve (b) we may write

$$v_x - u_x = \frac{(x-1)(x-2)\ \ldots\ .\ (x-2h)}{(2h)!}\,\Delta^{2h+1}y_0\ ,$$

or adopting the summation notation of the paper referred to above,

$$v_x - u_x = \frac{1}{(2h)!}\sum_{2h}(x-1)(x-2)\ \ldots\ .\ (x-2h)\,\Delta^{2h+1}y_0\ .$$

Differentiating m times with respect to x, and then setting $x=h+1$, we obtain

$$v_{h+1}^{(m)} - u_{h+1}^{(m)} = \frac{m!}{(2h)!}\sum_{2h} h(h-1)\ \ldots\ .\ (-h+m+1)\,\Delta^{2h+1}y_0\ ,$$

$$m = 1, 2, \ldots\ .\ , k\ .$$

Again, differentiating (d) m times by the aid of Leibnitz' theorem for the derivative of a product, we obtain

$$\psi_x^{(m)} = (x-h-1)\sum_{r=1}^{r=k} a_{k+r}\frac{(k+r)!}{(k+r-m)!}(x-h)^{k+r-m} + m\sum_{r=1}^{r=k} a_{k+r}\frac{(k+r)!}{(k+r-m+1)!}(x-h)^{k+r-m+1}\ ,$$

which, when $x=h+1$, becomes

$$\psi_{h+1}^{(m)} = m\sum_{r=1}^{r=k} a_{k+r}\frac{(k+r)!}{(k+r-m+1)!}\ ,\ m = 1, 2, \ldots\ .\ , k\ .$$

The k additional conditions which were of the form

$$v_{h+1}^{(m)} = u_{h+1}^{(m)} + \psi_{h+1}^{(m)},\ m = 1, 2, \ldots\ .\ , k,$$

may now be written

$$m\sum_{r=1}^{r=k} a_{k+r}\frac{(k+r)!}{(k+r-m+1)!} = \frac{m!}{(2h)!}\sum_{2h} h(h-1)\ \ldots\ (-h+m+1)\Delta^{2h+1}y_0,$$

$$m = 1, 2, \ldots\ .\ , k\ ,$$

or dividing by m,

$$\sum_{r=1}^{r=k} a_{k+r}\frac{(k+r)!}{(k+r-m+1)!}=\frac{(m-1)!}{(2h)!}\sum_{2h} h(h-1)\ldots(-h+m+1)\Delta^{2h+1}y_0\,,$$

$$m=1,\ 2,\ \ldots\ldots,\ k\,.$$

These k linear equations will determine the k quantities a_{k+1}, a_{k+2}, ... , a_{2k}, and will complete the determination of the function ψ_x.

THE SOLUTION OF THE EQUATIONS

Using the symbol β_{m-1} to represent

$$\frac{(m-1)!}{(2h)!}\sum_{2h} h(h-1)\ldots\ldots(-h+m+1)\,\Delta^{2h+1}y_0,\ m=1,\ 2,\ \ldots\ldots,\ k\,,$$

the system of linear equations becomes

$$\frac{(k+1)!}{(k+1)!}a_{k+1}+\frac{(k+2)!}{(k+2)!}a_{k+2}+\ldots\ldots+\frac{(2k)!}{(2k)!}a_{2k}\quad=\beta_0$$

$$\frac{(k+1)!}{k!}a_{k+1}+\frac{(k+2)!}{(k+1)!}a_{k+2}+\ldots\ldots+\frac{(2k)!}{(2k-1)!}a_{2k}=\beta_1$$

. . . . . .

$$\frac{(k+1)!}{2!}a_{k+1}+\frac{(k+2)!}{3!}a_{k+2}+\ldots\ldots+\frac{(2k)!}{(k+1)!}a_{2k}\ =\beta_{k-1}\,.$$

This system of equations is similar to that solved in the paper referred to above, and we may therefore omit the details of solution, giving only the results. The denominator determinant is equivalent to $\zeta_2^1(2k,\ 2k-1,\ \ldots\ldots,\ k+1)$, and this difference-product reduces to $1!\ 2!\ 3!\ \ldots\ldots\ (k-1)!$ which we represent by $(k-1)!!$

The numerator determinant for a_{k+r}, $r=1,\ 2,\ \ldots\ldots,\ k$, may be expanded in terms of the elements of the rth column and their cofactors, resulting in

$$\beta_0 K_{r,\,0}+\beta_1 K_{r,\,1}+\ldots\ldots+\beta_{k-1}K_{r,\,k-1},$$

where $K_{r,i}$ is the cofactor of β_i.

The value of $K_{r,0}$ is $(-1)^{r+1}\frac{(2k)!\,(k-2)!!}{k(k+r)\,(k-r)!\,(r-1)!}$,

and that of $K_{r,k-1}$ is $(-1)^{r+k}\frac{(k-1)!!}{(k-r)!\,(r-1)!}$,

while in general

$$K_{r,i}=(-1)^{r+i+1}\sum_{s=0}^{s=k-i-1}\mu_{i+s,2}\times{}_rp_{k-i-s-1}\times\frac{(k-1)!!}{(k-r)!\,(r-1)!},$$

where $\mu_{i+s,2}$ has the same meaning as in the paper referred to above, and ${}_rp_{k-i-s-1}$ is the sum of the products $k-i-s-1$ at a time of $2k,\ 2k-1,\ \ldots,\ k+r+1,\ k+r-1,\ \ldots,\ k+1$, and $p_0=1$.

Then since $a_{k+r}=\frac{\beta_0 K_{r,0}+\beta_1 K_{r,1}+\ \ldots\ +\beta_{k-1}K_{r,k-1}}{(k-1)!!}$,

and the K's have been determined, the coefficients in (d) are all known, and ψ_x is completely determined.

VALUES AND PROPERTIES OF THE FUNCTION $\mu_{i+s,2}$

To facilitate the finding of special cases from the general formulas, a table of the values of $\mu_{i+s,2}$ is added (Table I), together with certain properties of this system of numbers. These properties were found from a study of the numbers themselves, and enable us to extend the table to larger values of both i and s. It should, of course, be possible to obtain these properties also from the definition of the function $\mu_{i+s,2}$ itself.

The values of $\mu_{i+s,2}$ have the following properties:

(1) $\mu_{1+s,2}=(-1)^s$

(2) $\mu_{i+1,2}$ has second differences constantly equal to -1

$\mu_{i+2,2}$ has fourth differences constantly equal to 1.3

$\mu_{i+3,\,2}$ has sixth differences constantly equal to $-1.3.5$

$\mu_{i+4,\,2}$ has eighth differences constantly equal to $1.3.5.7$

. . . . . .

$\mu_{i+s,\,2}$ (s fixed) has differences of order $2s$ constantly equal to

$$(-1)^s\, 1.3.5. \ldots . (2s-1)$$

(3) $$\mu_{i+s,\,2} = \mu_{(i-1)+s,\,2} - i\,\mu_{i+(s-1),\,2}\,.$$

Property (2) enables us to find the values of $\mu_{i+s,\,2}$ (s fixed) for i greater than $2s$, when the values for i less than and equal to $2s$ are known.

TABLE I

VALUES OF $\mu_{i+s,\,2}$ $\begin{cases} i = 1, 2, \ldots, 10 \\ s = 1, 2, \ldots, 5 \end{cases}$

i \ s	1	2	3	4	5
1	−1	1	−1	1	−1
2	−3	7	−15	31	−63
3	−6	25	−90	301	−966
4	−10	65	−350	1701	−7770
5	−15	140	−1050	6951	−42525
6	−21	266	−2646	22827	−179487
7	−28	462	−5880	63987	−627396
8	−36	750	−11880	159027	−1899612
9	−45	1155	−22275	359502	−5135130
10	−55	1705	−39325	752752	−12662650

Property (3) enables us to find the values of $\mu_{i+s,\,2}$ (s fixed) when the values of $\mu_{i+(s-1),\,2}$ are known.

COMPARISON OF THE TWO FORMULAS

In order to compare the osculatory interpolation formulas obtained by the generalized Lidstone method with those formerly found by the generalized Sprague-King method, we set down side by side the corresponding formulas for $h \leqq 4$, and $k \leqq 4$. Only the terms which differ from Newton's formula are given, and the

Sprague and Lidstone results are marked (*S*) and (*L*) respectively. When $h=2$, and $k=2$

$$y_{2+x}= \ldots . + \frac{x^3(x-1)\ (5x-7)}{4!}\Delta^5 y_0 \qquad (S)$$

$$y_{2+x}= \ldots . + \frac{x^3(x-1)\ (5x-7)}{4!}\Delta^5 y_0 \,. \qquad (L)$$

When $h=3$, and $k=2$

$$y_{3+x}= \ldots . \frac{2x(x-1)\ (3x^3-x^2-8x-6)}{6!}\Delta^6 y_0$$

$$+\frac{4x^3(x-1)\ (-8x+11)}{6!}\Delta^7 y_0 \qquad (S)$$

$$y_{3+x}= \ldots\ldots\ldots\ldots\ldots + \frac{4x^3(x-1)\ (-8x+11)}{6!}\Delta^7 y_0 \qquad (L)$$

When $h=4$, and $k=2$

$$y_{4+x}= \ldots . + \frac{2x(x-1)\ (6x^3+17x^2+25x+12)}{6!}\Delta^6 y_0$$

$$+\frac{2x(x-1)\ (17x^3-15x^2-38x-24)}{7!}\Delta^7 y_0$$

$$+\frac{4x(x-1)\ (-16x^3-5x^2+45x+36)}{8!}\Delta^8 y_0$$

$$+\frac{36x^3\ (x-1)\ (11x-15)}{8!}\Delta^9 y_0 \qquad (S)$$

$$y_{4+x}= \ldots\ldots\ldots\ldots\ldots\ldots + \frac{36x^3(x-1)\ (11x-15)}{8!}\Delta^9 y_0 \qquad (L)$$

When $h=2$, and $k=3$

$$y_{2+x}= \ldots . + \frac{x^4(x-1)\ (-14x^2+35x-23)}{4!}\Delta^5 y_0 \qquad (S)$$

$$y_{2+x}= \ldots . + \frac{x^4(x-1)\ (-14x^2+35x-23)}{4!}\Delta^5 y_0 \qquad (L)$$

When $h=3$, and $k=3$

$$y_{3+x}= \ldots\ldots + \frac{x^4(x-1)\ (89x^2-222x+145)}{6!}\Delta^7 y_0 \qquad (S)$$

$$y_{3+x}= \ldots\ldots + \frac{x^4(x-1)\ (89x^2-222x+145)}{6!}\Delta^7 y_0 \qquad (L)$$

When $h=4$, and $k=3$

$$y_{4+x}= \ldots\ldots + \frac{4x(x-1)\ (-53x^5+107x^4-389x^3-4x^2+45x+36)}{8!}\Delta^8 y_0$$
$$+\frac{4x^4(x-1)\ (-275x^2+685x-446)}{8!}\Delta^9 y_0 \qquad (S)$$

$$y_{4+x}= \ldots\ldots\ldots\ldots + \frac{4x^4(x-1)\ (-275x^2+685x-446)}{8!}\Delta^9 y_0 \qquad (L)$$

When $h=2$, and $k=4$

$$y_{2+x}= \ldots\ldots + \frac{x^5(x-1)(46x^3-161x^2+193x-80)}{4!}\Delta^5 y_0 \qquad (S)$$

$$y_{2+x}= \ldots\ldots + \frac{x^5(x-1)(46x^3-161x^2+193x-80)}{4!}\Delta^5 y_0 \qquad (L)$$

When $h=3$, and $k=4$

$$y_{3+x}= \ldots\ldots + \frac{x^5(x-1)\ (-290x^3+1015x^2-1216x+503)}{6!}\Delta^7 y_0 \qquad (S)$$

$$y_{3+x}= \ldots\ldots + \frac{x^5(x-1)\ (-290x^3+1015x^2-1216x+503)}{6!}\Delta^7 y_0 \qquad (L)$$

When $h=4$, and $k=4$

$$y_{4+x}= \ldots\ldots + \frac{x^5(x-1)\ (3569x^3-12491x^2+14959x-6181)}{8!}\Delta^9 y_0 \qquad (S)$$

$$y_{4+x}= \ldots\ldots + \frac{x^5(x-1)\ (3569x^3-12491x^2+14959x-6181)}{8!}\Delta^9 y_0 \qquad (L)$$

The following conclusions can be drawn:

(1) The Sprague and Lidstone methods produce the same formula when $h \leqq k$, but different formulas when $h > k$.

(2) The final term is always the same in both formulas.

(3) By the Sprague method all terms of degree higher than $2k+1$ are modified so as to yield a formula of minimum degree, while by the Lidstone method only the final term is modified; and since one or more of the preceding terms will be of degree higher than $2k+1$ when $h > k$, it follows that the Lidstone method does not produce a formula of minimum degree when it differs from that produced by the Sprague method.

EXCESS INTEREST UNDER ANNUITY-CERTAIN

By E. G. FASSEL

Where policy proceeds are settled as an annuity-certain which pays principal and interest on outstanding balances at a guaranteed rate (i) in uniform guaranteed instalments, of which the first is immediately due, and where there is a provision for dividends, it is usual to declare an actual rate (i') and to increase the guaranteed instalments (after the first) by such excess interest dividend as represents the full surplus of the payment year arising from the fact that the actual rate is greater than the guaranteed rate.

Where the instalments are annual (I) no difficulty is encountered and the dividend to accompany the instalment (if there remain r to be paid, including the instalment immediately due) is $I \cdot a_{\overline{r}|} \cdot (i'-i)$, where the annuity value is taken at rate i.

Where payment is by instalments that are more frequent than once a year there are two difficulties—(1) the calculations involve interest for fractional years, and (2) the dividend is different for each instalment, whereas uniform dividends for each payment year are desirable to simplify home-office accounting.

The purpose of this paper is to derive a formula for the dividend where the excess interest surplus of the payment year is to be allotted in dividends that remain uniform throughout the payment year.

Let ${}^{(m)}I$ = guaranteed instalment payable at the beginning of each $\left(\frac{1}{m}\right)$th year of the payment period so that if the payments are guaranteed for n years there will be nm payments of ${}^{(m)}I$ in all.

i' and i = the declared and the guaranteed rates of interest, respectively.

r and t = integers, t being $< m$.

The surplus earned during the $\left(\frac{1}{m}\right)$th-year period at the end of which there are $(rm+t)$ remaining instalments (including the

instalment immediately due) is equal to ${}^{(m)}I \cdot m \cdot a\overline{\left|\frac{(m)}{r+\frac{i}{m}}\right.} \cdot \left\{ (1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}} \right\}$, if the surplus is evaluated as of the end of such $\left(\frac{1}{m}\right)$th period, the annuity value being taken at rate i. If such surplus is evaluated as of the end of the current payment year, the value is ${}^{(m)}I \cdot m \cdot a\overline{\left|\frac{(m)}{r+\frac{i}{m}}\right.} \cdot \left\{ (1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}} \right\} \cdot (1+i')^{\frac{t}{m}}$.

Accordingly, if we evaluate as of the end of the payment year the surplus earned during each period of the payment year and denote the aggregate amount by Δ, we have:

$$\Delta = {}^{(m)}I \cdot m \cdot a^{(m)}_{\overline{r+\frac{m-1}{m}|}} \cdot \left\{(1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}}\right\} \cdot (1+i')^{\frac{m-1}{m}}$$

$$+ {}^{(m)}I \cdot m \cdot a^{(m)}_{\overline{r+\frac{m-2}{m}|}} \cdot \left\{(1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}}\right\} \cdot (1+i')^{\frac{m-2}{m}}$$

$$\vdots$$

$$+ {}^{(m)}I \cdot m \cdot a^{(m)}_{\overline{r+\frac{1}{m}|}} \cdot \left\{(1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}}\right\} \cdot (1+i')^{\frac{1}{m}}$$

$$+ {}^{(m)}I \cdot m \cdot a^{(m)}_{\overline{r+\frac{0}{m}|}} \cdot \left\{(1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}}\right\} \cdot (1+i')^{\frac{0}{m}}$$

$$= {}^{(m)}I \cdot \left\{(1+i')^{\frac{1}{m}} - (1+i)^{\frac{1}{m}}\right\} \cdot \left[\begin{array}{l} \left(\frac{1+i'}{1+i}\right)^{\frac{m-1}{m}} \cdot \left\{(1+i)^{\frac{m-2}{m}} + (1+i)^{\frac{m-3}{m}} + \ldots + 1 + m \cdot a^{(m)}_{\overline{r}|}\right\} \\ + \left(\frac{1+i'}{1+i}\right)^{\frac{m-2}{m}} \cdot \left\{(1+i)^{\frac{m-3}{m}} + \ldots + 1 + m \cdot a^{(m)}_{\overline{r}|}\right\} \\ \vdots \\ + \left(\frac{1+i'}{1+i}\right)^{\frac{1}{m}} \cdot \left\{1 + m \cdot a^{(m)}_{\overline{r}|}\right\} \\ + \left(\frac{1+i'}{1+i}\right)^{\frac{0}{m}} \cdot \left\{m \cdot a^{(m)}_{\overline{r}|}\right\} \end{array}\right]$$

$$= \left\{ (1+i)^{\frac{1}{m}} - 1 \right\} \cdot I^{(m)} = \left\{ m(i^{(m)}+1)^{\frac{1}{m}} - m \right\} \cdot I^{(m)} =$$

$$\left[\begin{array}{l} \left\{ m(i^{(m)}+1)^{\frac{1}{m}} - m \right\} \cdot \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \\ + \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \\ - \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{1}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i^{(m)}+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{2}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \cdots \cdots \end{array} \right]$$

$$= \left\{ (i'+1)^{\frac{1}{m}} - 1 \right\} \cdot I^{(m)} = \left\{ m(i'+1)^{\frac{1}{m}} - m \right\} \cdot I^{(m)} =$$

$$\left[\begin{array}{l} \left\{ m(i'+1)^{\frac{1}{m}} - m \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \\ + \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \\ - \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{1}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{2}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \cdots \cdots \end{array} \right]$$

$$= \left\{ (i'+1)^{\frac{1}{m}} - m(i'+1)^{\frac{1}{m}} \right\} \cdot I^{(m)} =$$

$$\left[\begin{array}{l} \left\{ m(i'+1)^{\frac{1}{m}} - m \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \\ + \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \\ - \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{1}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{2}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \cdots \cdots \end{array} \right]$$

$$= \left\{ (i'+1)^{\frac{1}{m}} - m(i'+1)^{\frac{1}{m}} \right\} \cdot I^{(m)} =$$

$$\left[\begin{array}{l} \left\{ m(i'+1)^{\frac{1}{m}} - m \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \\ + \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \\ - \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{1}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ \dfrac{1}{m(i'+1)^{\frac{1}{m}} - 1} \right\} \cdot \left\{ m \cdot a_{\overline{r}|}^{(m)} \right\} \left\{ m(i'+1)^{\frac{2}{m}} + m \left(i'+1 \right)^{\frac{m-2}{m}} + \cdots + m \right\} \\ + \cdots \cdots \end{array} \right]$$

$$= \left\{ (i'+1)^{\frac{1}{m}} - m(i'+1)^{\frac{1}{m}} \right\} \cdot I^{(m)} =$$

Let

$$\theta_{i'}^{(m)}=\frac{i'}{(1+i')^{\frac{1}{m}}-1}, \qquad \theta_{i}^{(m)}=\frac{i}{(1+i)^{\frac{1}{m}}-1} \quad \text{and} \quad \theta_{i'\&i}^{(m)}=\frac{\left(\frac{1+i'}{1+i}\right)-1}{\left(\frac{1+i'}{1+i}\right)^{\frac{1}{m}}-1}.$$

Then

$$m\cdot a_{\overline{r}|}^{(m)}=a_{\overline{r}|}\cdot\left\{\frac{i}{(1+i)^{\frac{1}{m}}-1}\right\}=a_{\overline{r}|}\cdot\theta_{i}^{(m)}.$$

Also

$$\left\{(1+i')^{\frac{1}{m}}-(1+i)^{\frac{1}{m}}\right\}=(1+i)^{\frac{1}{m}}\left\{\left(\frac{1+i'}{1+i}\right)^{\frac{1}{m}}-1\right\}$$

$$=\frac{(1+i)^{m}}{\theta_{i'\&i}^{(m)}}\cdot\left\{\left(\frac{1+i'}{1+i}\right)-1\right\}=\frac{(1+i)^{m}\cdot(i'-i)}{(1+i)\cdot\theta_{i'\&i}^{(m)}}$$

Hence

$$\Delta=\frac{{}^{(m)}I\cdot(1+i)^{\frac{1}{m}}\cdot(i'-i)}{(1+i)\cdot\theta_{i'\&i}^{(m)}}\cdot\left[\frac{\theta_{i'}^{(m)}\cdot\theta_{i}^{(m)}}{i}+\left(a_{\overline{r}|}\cdot\theta_{i}^{(m)}-\frac{\theta_{i}^{(m)}}{i}\right)\theta_{i'\&i}^{(m)}\right]$$

$$={}^{(m)}I\cdot\left[a_{\overline{r}|}\cdot(i'-i)\cdot\theta_{i}^{(m)}\cdot\frac{(1+i)^{\frac{1}{m}}}{(1+i)}\right.$$

$$\left.+\frac{(i'-i)}{i}\left(\frac{\theta_{i'}^{(m)}\theta_{i}^{(m)}}{\theta_{i'\&i}^{(m)}}-\theta_{i}^{(m)}\right)\cdot\frac{(1+i)^{\frac{1}{m}}}{(1+i)}\right].$$

Let δ be the uniform dividend payable at end of each period of the payment year.

Then

$$\Delta=\delta\cdot\left[(1+i')^{\frac{m-1}{m}}+(1+i')^{\frac{m-2}{m}}+\ldots\ldots+1\right]=\delta\left[\frac{(1+i')^{m}-1}{(1+i')^{\frac{1}{m}}-1}\right],$$

whence

$$\Delta=\delta\cdot\theta_{i'}^{(m)}.$$

Substituting the foregoing value of Δ gives the formula:

$$\delta={}^{(m)}I\cdot\left[a_{\overline{r}|}\cdot(i'-i)\cdot\frac{\theta_{i}^{(m)}}{\theta_{i'}^{(m)}}\cdot\frac{(1+i)^{\frac{1}{m}}}{(1+i)}+\frac{i'-i}{i}\left(\frac{\theta_{i}^{(m)}}{\theta_{i'\&i}^{(m)}}-\frac{\theta_{i}^{(m)}}{\theta_{i'}^{(m)}}\right)\cdot\frac{(1+i)^{\frac{1}{m}}}{(1+i)}\right].$$

By successively putting $m=12$, 4, 2, and 1, four tables are produced which show the dividend per unit of monthly, quarterly, semi-

annual and annual guaranteed instalment. When $m=1$ it is seen that the formula reduces to the expression in the second paragraph.

The complete calculation for a table applicable to monthly instalments is given to show the ease with which the formula is applied.

CALCULATION FOR $m=12$, $i'=.048$ AND $i=.03$

$$\theta^{(12)}_{.048}=\frac{.048}{1.048^{\frac{1}{12}}-1}=\frac{.048}{1.0039146-1}=12.2618$$

$$\theta^{(12)}_{.03}=\frac{.03}{1.03^{\frac{1}{12}}-1}=\frac{.03}{1.0024663-1}=12.1640$$

$$\theta^{(12)}_{.048\ \&\ .03}=\frac{\frac{1.048}{1.03}-1}{\frac{1.048^{\frac{1}{12}}}{1.03^{\frac{1}{12}}}-1}=\frac{\frac{1.048}{1.03}-1}{\frac{1.0039146}{1.0024663}-1}=\frac{.0174757}{.00144474}=12.0961\ .$$

$$\text{Factor into } a_{\overline{r}|}=\frac{.018\times 12.1640\times 1.0024663}{12.2618\times 1.03}=.017379$$

$$\text{Constant factor}=\frac{.018}{.03}\times\left(\frac{12.1640}{12.0961}-\frac{12.1640}{12.2618}\right)\times\frac{1.0024663}{1.03}=.00794\ .$$

For $r=0$, which refers to the last payment year, the formula produces m uniform dividends, which means that the last dividend would fall due $\left(\frac{1}{m}\right)$th year after the last instalment payment, inasmuch as the instalments are payments in advance. In order that the last dividend may be payable along with the last guaranteed instalment the surplus of the last payment year should be allotted in $(m-1)$ uniform amounts and the normal dividend is therefore multiplied by the factor F, where

$$F=\frac{(1+i')^{\frac{m-1}{m}}+(1+i')^{\frac{m-2}{m}}+\ \ldots\ .\ +(1+i')^{\frac{1}{m}}+1}{(1+i')^{\frac{m-1}{m}}+(1+i')^{\frac{m-2}{m}}+\ \ldots\ .\ +(1+i')^{\frac{1}{m}}}$$

$$=\frac{\frac{i'}{(1+i')^{\frac{1}{m}}-1}}{\frac{i'}{(1+i')^{\frac{1}{m}}-1}}=\frac{\theta^{(m)}_{i'}}{\theta^{(m)}_{i'}-1}\ .$$

Hence for $r=0$, in the foregoing example, the required value $= \frac{.00794 \times 12.2618}{11.2618} = .00865$.

TABLE I

MONTHLY INSTALMENTS: EXCESS INTEREST DIVIDENDS FOR 4.8 PER CENT OVER 3 PER CENT

r	$a_{\overline{r}\rceil}$ at 3% (1)	(1)×.017379 (2)	(2)+.00794 (3)	Dividend for $100.00 Guaranteed Monthly Instalment	No. of Instalments Remaining, Including Instalment Due at Same Time as Dividend
29	19.1885	.33348	.34142	$34.14	359–348
28	18.7641	.32610	.33404	33.40	347–336
27	18.3270	.31850	.32644	32.64	335–324
26	17.8768	.31068	.31862	31.86	323–312
25	17.4131	.30262	.31056	31.06	311–300
24	16.9355	.29432	.30226	30.23	299–288
23	16.4436	.28577	.29371	29.37	287–276
22	15.9369	.27697	.28491	28.49	275–264
21	15.4150	.26790	.27584	27.58	263–252
20	14.8775	.25856	.26650	26.65	251–240
19	14.3238	.24893	.25687	25.69	239–228
18	13.7535	.23902	.24696	24.70	227–216
17	13.1661	.22881	.23675	23.68	215–204
16	12.5611	.21830	.22624	22.62	203–192
15	11.9379	.20747	.21541	21.54	191–180
14	11.2961	.19631	.20425	20.43	179–168
13	10.6350	.18483	.19277	19.28	167–156
12	9.9540	.17299	.18093	18.09	155–144
11	9.2526	.16080	.16874	16.87	143–132
10	8.5302	.14825	.15619	15.62	131–120
9	7.7861	.13531	.14325	14.33	119–108
8	7.0197	.12200	.12994	12.99	107– 96
7	6.2303	.10828	.11622	11.62	95– 84
6	5.4172	.09415	.10209	10.21	83– 72
5	4.5797	.07959	.08753	8.75	71– 60
4	3.7171	.06460	.07254	7.25	59– 48
3	2.8286	.04916	.05710	5.71	47– 36
2	1.9135	.03325	.04119	4.12	35– 24
1	.9709	.01687	.02481	2.48	23– 12
0	.....	.....	.00794 865	0.87	11– 1

The excess interest dividend payable with a particular instalment depends on the number of instalments remaining, so that,

whether the settlement is of long standing or only recently commenced, if less than thirty years of the payment period remain the foregoing table shows the dividends under all monthly instalment cases of a company paying excess interest at 4.8 per cent over 3 per cent.

VALUATION METHOD

By C. H. BECKETT

Inasmuch as this note presents no new theory nor a process for which originality is claimed, it is in order to explain why such a discussion is presented before a society of actuaries. The reason—or perhaps it might better be stated, the excuse—for its presentation is to bring to the attention of the Institute a method of so applying a well-known principle to one of our oldest office problems as to materially shorten processes and consequently reduce cost. As the title indicates, the method to be described deals with a process of ascertaining a company's reserve liability.

In passing I wish to explain that the subject treated herein is only one of several office problems which we have been endeavoring to solve by use of the punch cards and tabulating machines. Mr. George C. Van Duren, who is in charge of the Service Department, has contributed very largely to the practical working out of this valuation method, and I desire to give him full credit for his contribution.

The plan was worked out for our own company with the thought in mind of simplifying and shortening the processes which had previously been employed. Incidental, but yet of considerable importance, is the fact that by employing this method we have a record of our valuation which enables insurance departments to verify the work very quickly and therefore at small expense to the company.

In order to have our problem clearly before you, it should be explained that we do not make a seriatim valuation, but prior to this year followed the usual group-valuation method. This method, however, had become extremely laborious on account of the multiplicity of groups necessary in handling issues covering thirty-one insurance years, three different interest rates, a variety of policy forms, and the usual changes in methods of valuation such as are found in any company which has done business over a like period of time. Under the plan explained herein we immediately reduce our groups to those required for the details of the convention form of annual statement, namely, 4 per cent issued prior to 1901,

3 per cent and $3\frac{1}{2}$ per cent issued since said date, and two forms of disability benefits; one being the usual waiver of premium and the other the waiver of premium in connection with a monthly income of $10 per thousand during total and permanent disability.

A simple punch card was prepared upon which is punched the plan, year of issue, age at issue, reserve per thousand on life-portion, reserve per thousand on the disability waiver of premium portion, reserve per thousand on the disability waiver of premium combined with monthly income disability portion, and the amount of insurance in force corresponding to these three different reserve items. The plan code includes both the form of policy and the rate of interest. The reserve per thousand may be punched on these cards at any convenient time during the year. The amount of insurance in each actuarial group is punched directly from our groupbook. It should be explained that our insurance account is kept on punch cards, and the totals of the cards sorted actuarially are carried in the group book.

We are thus enabled to complete the valuation cards as soon as the insurance account is closed, and as rapidly as these cards are checked and released by the punch operators they can be sorted and tabulated by other operators and the work of ascertaining the full reserve liability is speedily accomplished. We thus eleminate entirely the use of multiplying machines and also the adding-machine work that is necessary in the usual methods of group valuation.

The principle employed is simply that whenever a multiplier is constant the sum of the products is equal to the product of the sum of the multiplicands by the multiplier. Of course, a group valuation is the accumulation of a large number of products, each one of which is the result of applying one of the multiplicands to one of the digits, one to nine, both inclusive, zero merely indicating position. Therefore the problem is to apply these operators in their proper positions and then count them.

Since in finding the total reserve corresponding to any given group of business we find the sum of the products of the reserves per thousand by the number of thousands, our problem resolves into finding the number of times the insurance account multiplier operates on each digit of the reserve factors and in addition the correct position of the multiplier.

The operation is as follows: The cards are sorted by the "Cents" column of the reserve factor. The amount of insurance whose corresponding reserve ends in one cent is added and the total entered under digits 1, sort 1; the amount of insurance whose reserve ends in two cents is entered under digit 2, sort 1, and so one to nine, inclusive. The final total of all the amounts appears in "Control," sort 1. Of course, the object of the "Control" is to give an automatic check which is accomplished since the "Control" is independent of the summation as it appears in the tabulation for digits. We next sort by the "Dimes" column and proceed in the same manner, placing the totals one decimal point to the left and similarly for all other positions in the reserve factors. The totals of each of the digits are then taken, and their grand total must be equal to the total in the "Control."

The actual reserve liability is then acertained by multiplying the total of the digits number 1 by 1, the total of the digits number 2 by 2, and so on, and then taking a grand total.

In order that the process may be understood more readily, I am furnishing herewith an exhibit of our valuation card, and also of the actual valuation of our business as per December 31, 1924, life-portion, American 3 per cent. The business which was involved in our last valuation included approximately ten thousand groups, speaking in terms of our former method, and had we been obliged to resort to the use of valuation sheets and cross-multiplication, it would probably have taken three operators about a month to have completed and checked the work in a satisfactory manner.

This year we were enabled to complete the work in about eight hours, using one sorter and one tabulator. In addition to the actual sorting and tabulating, one operator was employed about a week in punching the group cards, but, as a matter of fact, this work consumes practically no extra time since punch operators can be assigned to complete it at periods when they are not otherwise engaged. It will be noted that the amount of reserve in the exhibit shown is in excess of $25,000,000. This sheet was completed in approximately three hours with one sorter and one tabulator.

When the work has been completed we simply put all the cards in a box used by the manufacturer in shipping the cards to us, and nail them up. We then have everything in order for the department examiners when they wish to check our valuation. All they

need do is verify the insurance account as shown on the cards and run the cards through the tabulating machine so as to verify the amount set out in our detailed sheet of valuation.

The amount of work necessary in order to have the records in suitable order for such a plan of valuation should receive consideration. As previously explained, the account of insurance is kept by punch cards. The additions and deductions are sorted actuarially at at the end of each month and the groups adjusted. This is done monthly so as to reduce to a minimum work falling at valuation periods. Should a company desire to eliminate this monthly adjustment, the groups can be made up at the end of the year as follows:

Begin with the valuation cards as per December 31 of the preceding year. Punch all insurance-account accretions in duplicate with a two-card punch. File one card in the insurance account file and the other in an "accretion" file. Take cards from the insurance-account file for all terminations and file them in a "decretion" file. At the end of the year sort and tabulate the "accretion" and "decretion" files by groups and punch a card for the totals in each group. The "decretion" group card should be punched in complement. Finally, sort and tabulate the three sets of group cards: (1) valuation cards, December 31 of preceding year; (2) "accretion" totals; and (3) "decretion" totals punched in complement. The result will furnish a tape showing the correct groups from which the new valuation cards can be punched.

We have stated that this plan of valuation eliminates the use of a multiplying machine, but the schedule shows multiplication by the digits from one to nine. It is not necessary to perform any multiplication and we do as a matter of fact proceed in the following manner:

A double counter adding machine is used. Each total is added as many times as is indicated by the digit by using the repeat key, and a transfer total taken after each such operation. The total reserve then appears in the lower counter.

FORM OF PUNCH CARD

	PLAN	YEAR ISSUE	AGE ISSUE	RESERVE PER 1000	"A" RESERVE PER 1000	"B" RESERVE PER 1000	AMOUNT GROUP	AMOUNT "A"	AMOUNT "B"
0 0 0 0	0 0 0	0 0	0 0	0 0 0 0 0 0	0 0 0	0 0 0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0
1 1 1 1	1 1 1	1 1	1 1	1 1 1 1 1 1	1 1 1	1 1 1 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1
2 2 2 2	2 2 2	2 2	2 2	2 2 2 2 2 2	2 2 2	2 2 2 2	2 2 2 2 2 2 2	2 2 2 2 2 2 2	2 2 2 2 2 2 2
3 3 3 3	3 3 3	3 3	3 3	3 3 3 3 3 3	3 3 3	3 3 3 3	3 3 3 3 3 3 3	3 3 3 3 3 3 3	3 3 3 3 3 3 3
4 4 4 4	4 4 4	4 4	4 4	4 4 4 4 4 4	4 4 4	4 4 4 4	4 4 4 4 4 4 4	4 4 4 4 4 4 4	4 4 4 4 4 4 4
5 5 5 5	5 5 5	5 5	5 5	5 5 5 5 5 5	5 5 5	5 5 5 5	5 5 5 5 5 5 5	5 5 5 5 5 5 5	5 5 5 5 5 5 5
6 6 6 6	6 6 6	6 6	6 6	6 6 6 6 6 6	6 6 6	6 6 6 6	6 6 6 6 6 6 6	6 6 6 6 6 6 6	6 6 6 6 6 6 6
7 7 7 7	7 7 7	7 7	7 7	7 7 7 7 7 7	7 7 7	7 7 7 7	7 7 7 7 7 7 7	7 7 7 7 7 7 7	7 7 7 7 7 7 7
8 8 8 8	8 8 8	8 8	8 8	8 8 8 8 8 8	8 8 8	8 8 8 8	8 8 8 8 8 8 8	8 8 8 8 8 8 8	8 8 8 8 8 8 8
9 9 9 9	9 9 9	9 9	9 9	9 9 9 9 9 9	9 9 9	9 9 9 9	9 9 9 9 9 9 9	9 9 9 9 9 9 9	9 9 9 9 9 9 9
1 2 3 4	5 6 7	8 9	10 11	12 13 14 15 16 17	18 19 20	21 22 23 24	25 26 27 28 29 30 31	32 33 34 35 36 37 38	39 40 41 42 43 44 45

ILLUSTRATION OF ACTUAL WORK SHEET

Digit	Sort	Amount of Insurance			Reserve
	1	17868021			
	2	15756464			
	3	15074872			
1	4	17848928			
	5	29204075			
	6	2000			
	Total	312672597861	×	1	3 126 725 97 861
	1	14067812			
	2	14006308			
2	3	15978680			
	4	21948280			
	5	12814320			
	Total	151843478892	×	2	3 036 869 57 784
	1	14567518			
	2	18049251			
3	3	20316561			
	4	15328126			
	5	8929802			
	Total	106852862128	×	3	3 205 585 86 384
	1	16479204			
	2	18237759			
4	3	24357747			
	4	13616012			
	5	6930534			
	Total	85555983494	×	4	3 422 239 33 976
	1	15605486			
	2	17366651			
5	5	16750971			
	4	15008370			
	5	6594721			
	Total	82819949096	×	5	4 140 997 45 480
	1	17761231			
	2	14581758			
6	3	15075498			
	4	12200031			
	5	4117787			
	Total	55049029611	×	6	3 302 941 77 666
	1	17931459			
	2	17433978			
7	3	12839605			
	4	12691215			
	5	1759949			
	Total	31766936739	×	7	2 223 685 57 173
	1	17165071			
	2	17837361			
8	3	15219349			
	4	11450861			
	5	267083			
	Total	158391645 81	×	8	1 267 133 16 648
	1	16930509			
	2	15932116			
9	3	15935870			
	4	11712057			
	5	246388			
	Total	15945775669	×	9	1 435 119 81 021
All Digits		858345778071			25 161 298 53 993
		Control			
	1	148376311			
	2	149201646			
	3	151549153			
	4	131803880			
	5	70954659			
	6	2000			
	Total	858345778071			

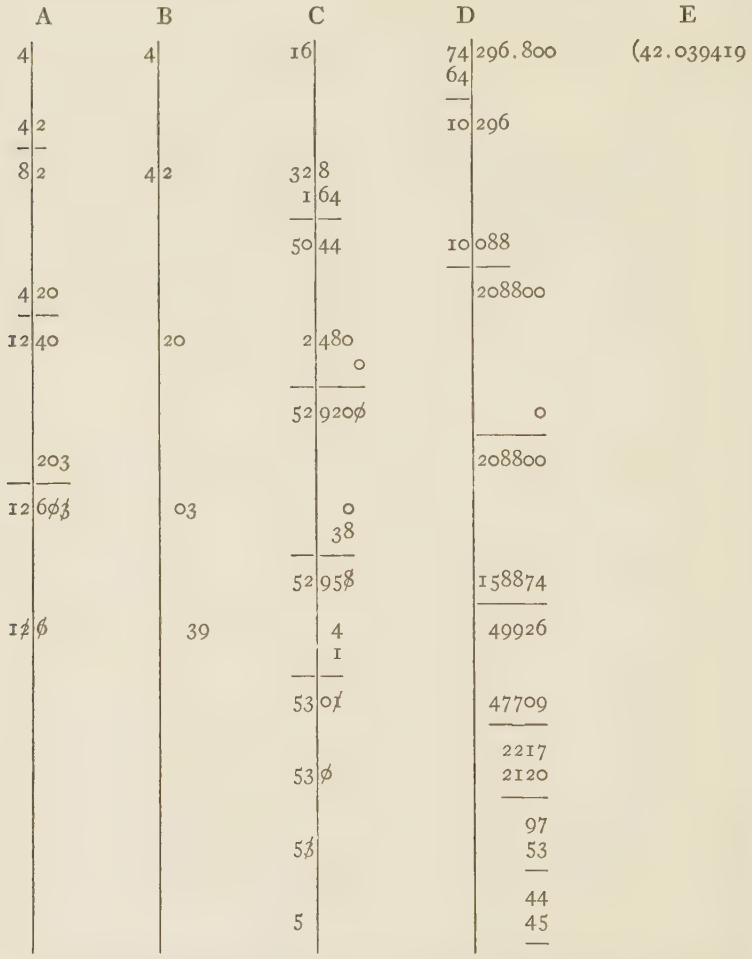
CUBE ROOT BY ABRIDGED DIVISION AND BY MACHINE

By E. G. FASSEL

A need recently, in connection with certain interest calculations, of the twelfth root accurate to more significant figures than available by ordinary logarithms led to the discovery of a method for cube root that is believed to be new. By this method the cube root is extracted to the required number of digits with a minimum of work; also, if desired the entire operation can readily be performed by machine.

CUBE ROOT BY ABRIDGED DIVISION

Example: Required the cube root of 74296.8. (Answer: 42.039419.)



(The vertical lines have been ruled to make it easier to see the relative positions in which the figures are entered as the work progresses.)

Procedure.—Write down the given number as for division and point off in 3's from the decimal point, supplying ciphers if necessary

to complete three groups. Inspect the left-hand group thus pointed off to determine the greatest integer that it contains as a cube. Enter this integer as the quotient to the right of the number; also on the left-hand side of the page write this integer twice and its square once. We then have five columns, A, B, C, D, and E, as in the foregoing example and proceed as follows:

Column		Operation
	D	C×E, subtract and bring down next group of D
	E	$\frac{1}{3}$ of times C goes into D (omitting last 2 digits of D)
	A	Last 2 digits of E and add (See Note 1)
	B	Last 2 digits of E (See Note 1)
	C	A×B and add (See Note 2[*a*])
(*r*)	D	C×last digit of E, subtract and bring down next group of D (See Note 3)
	E	Times C goes into D (omitting last 2 digits of D)
	A	Last 3 digits of E and add
	B	Last 2 digits of E
	C	A×B and add
(*s*)	D	C×last digit of E, subtract and cut last digit of C
	E	Times C goes into D
	A	Last 3 digits of E, add and cut last 2 digits
	B	Last 2 digits of E
	C	A×B and add (See Note 2[*b*])
	D	C×last digit of E, subtract and cut last digit of C
	E	Times C goes into D
	A	Bring down A and cut last 2 digits
	B	Last 2 digits of E
	C	A×B and add
(*t*)	D	C×last digit of E, subtract and cut last digit of C
	E	Times C goes into D
	A	—
	B	—
	C	Bring down C

Repeat from (*t*) until exhaustion

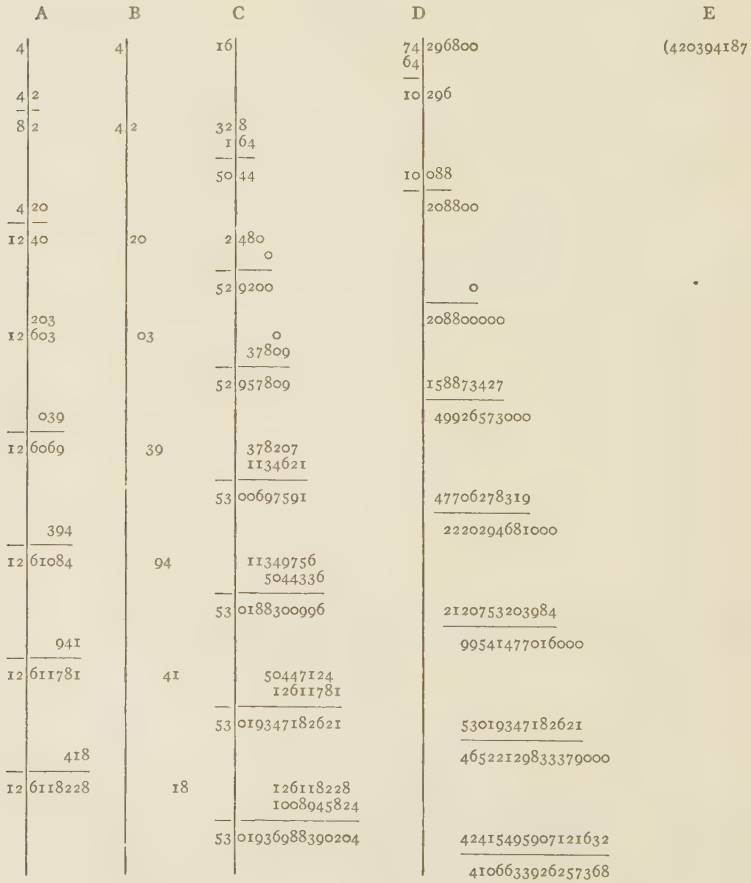
Note 1.—As the digits of E are repeated in A and in B, each digit is carried directly under its previous entry.

Note 2.—(*a*) In A×B, multiply A first by left-hand digit of B and carry result 1 decimal place right of existing C, then multiply A by right-hand digit of B and carry result 1 decimal place right of preceding. (*b*) After A has been cut, multiply first by left-hand of digit of B and carry result directly under existing C, then cut A mentally one place further, multiply by right-hand digit of B, and carry result directly under preceding.

Note 3.—In C×last digit of E the result is carried directly under D. If C×last digit of E is too great for subtraction from D, diminish E with corresponding corrections of A, B, and C.

The cube root of any number may be extracted by this method. More digits of the root would be obtained by providing another group of three ciphers in the initial value of D and by repeating from (*r*) before proceeding with (*s*).

The abridging process commences at (*s*) and has been explained because of the reduction in labor thereby obtained. The cube root may, however, be extracted by continued repetitions from (*r*) without proceeding with (*s*). The foregoing example worked by the Unabridged Method is as follows:



CUBE ROOT BY MACHINE

For mechanical extraction two machines, X and Y, are required. In the notation of the foregoing example, D is set up on the slide of Y, E appears in the quotient holes of Y, A is prepared by mental addition on the fixed plate of X, and B is prepared by continued multiplication on X with transfer of result as C on fixed plate of Y. Two machines of capacity 8×8 will give the cube root accurately to 11 digits, the last 3 digits being obtained by cutting and shifting the divisor to the right step by step as the dividend is exhausted. Slight or great accuracy in the result is obtained by stopping when the required number of digits have been determined.

Example: Required the cube root of 74296.8. (Answer: 42.039418774.)

A Fixed Plate of X	B Slide of X	C Fixed Plate of Y	D Slide of Y	E Quotient Holes of Y
			742968	(42039418774
4	16	16	102968	
82	5044	5044	2088	
1240	5292	5292	2088	
12603	52957809	52957809	49926573	
126069	53006976−	53006976	22202946	
1261084	53018830+	53018830	9954140	
12611781	53019347+	53019347	46522053	
12611823	53019370−	53019370	41065570	
		5301937	3952011	
		530194	240653	
		53019	28577	

Procedure.—On machine Y set up the given number as for division and determine what will be the left-hand group of integers if the number is pointed off in 3's from the decimal point. Inspect this left-hand group to determine the greatest integer that it contains as a cube. Note this integer mentally as if it appeared in quotient hole of machine Y. Set up this integer on fixed plate of machine X in second place from left, multiply by itself at left side of slide, and transfer result to fixed plate of Y at left side. Then proceed as follows:

Position	Operation
(D) Slide of Y	Divide by E (as mentally noted) and shift 1 place left
(E) Quotient holes of Y	Note mentally $\frac{1}{3}$ of times C goes into D
(A) Fixed plate of X	Add last 2 digits of E (including digit mentally noted) (See Note 1)

	Position	Operation
	(B) Slide of X	Multiply by second last digit of E (including digit mentally noted), shift 1 place left, and multiply by last digit of E (as mentally noted)
	(C) Fixed plate of Y	Fix at amount appearing on B
(*r*)	(D)	Divide by E (as mentally noted) and shift 1 place left (See Note 2)
	(E)	Note mentally times C goes into D
	(A)	Add last 3 digits of E (including digit mentally noted)
	(B)	Multiply by second last digit of E (including digit mentally noted), shift 1 place left, and multiply by last digit of E (as mentally noted)
	(C)	Fix at amount appearing on B
		Repeat from (*r*)

Note 1.—As the digits of E are repeated in A each digit is carried into same column as its previous entry.

Note 2.—If D cannot be divided by last digit of E, diminish E with corresponding corrections of A, B, and C.

ALGEBRAIC DEMONSTRATION

This method is based on the fact that the cube

$$(a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+\ \ldots\ )^3$$

can be expanded in the form

$$a_0 \cdot C_0+a_1xC_1+a_2x^2C_2+a_3x^3C_3+\ \ldots\ .$$

where

$$C_0=a_0^2$$

$$C_1-C_0=(2a_0+a_1x)\ (a_0+a_1x)$$

$$C_2-C_1=(3a_1+2a_1x+a_2x^2)\ (a_1x+a_2x^2)$$

$$C_3-C_2=(3a_0+3a_1x+2a_2x^2+a_3x^3)\ (a_2x^2+a_3x^3)$$

$$C_4-C_3=(3a_0+3a_1x+3a_2x^2+2a_3x^3+a_4x^4)\ (a_3x^3+a_4x^4)$$

If we put $x=.1$ and a_0, a_1, etc. = integers $\not<0$ and $\not>9$, we have a_0, a_1, a_2, etc. = the successive digits of E

(Left-hand bracket) = A, and shows why we add last 3 digits of E

(Right-hand bracket) = B multiplier, and shows why we use last 2 digits of E

C_n = total of bracket products, and shows why the divisor to be used with the successive digits of E is the result of an accumulated multiplication

SUBSTANDARD BUSINESS

By E. E. CAMMACK

One method of deriving the premium for a substandard risk is to charge a constant extra per $1,000 of insurance.

Another method is to rate up in age—that is to say, to treat the substandard applicant precisely as a standard applicant several years older.

A third plan is to base the premium on a substandard mortality table in which the rates of mortality are derived from the standard table by increasing them by a fixed percentage at all ages. The third method lies between the first and the second methods in that it assumes a more rapid increase in mortality from age to age than the first method but not so rapid as the second method.

Some companies employ two or more methods selecting for the various types of substandard risks the one reflecting, as near as can be judged, the incidence of the extra mortality expected. It is the most usual practice probably to charge a constant extra where the extra mortality results from occupational exposure to accident. The extra mortality from overweight is met closely by a rating up in age. Other types of impairments probably follow more closely a fixed percentage increase from year to year over the mortality rates for standard lives.

Some companies, however, use but one plan in rating substandard lives, and this appeals to the writer as a logical course.

In the life insurance business there are included in the so-called standard class many types of risks experiencing widely varying rates and types of mortality. Refinement in rating has neither been sought nor attained. Risks in extremely healthful occupations and without blemish in family or personal history and upon which as a class but 75 per cent of the standard mortality, for example, may be expected are placed in the same broad class as border-line risks belonging to a type upon which may be expected perhaps 125 per cent mortality. A clergyman is charged the same rate as a policeman, and yet there may be expected among the latter nearly double the rate of mortality among the former.

When this is the method employed in insuring lives at standard rates, it seems illogical to introduce greater refinements for the comparatively small proportion of business issued at extra premiums, and more especially so when the knowledge in regard to mortality on impaired lives is very limited.

Premium rates upon standard lives are based upon a mortality table constructed from the experience on lives insured at standard rates. It is customary to insure at standard rates types of risks upon which mortality not in excess of 125 per cent of a company's normal mortality for standard risks is expected. Consequently, the mortality table upon which rates are calculated is simply an average table.

A logical course would be to construct average substandard mortality tables for risks accepted with the expectation of higher rates of mortality, as, for example:

Class	Expected Mortality in Percentage
A	125–150
B	150–180
C	180–220
D	220–300

In the absence of such mortality tables based upon actual experience, tables may be constructed from the standard table with the rates of mortality increased by a constant percentage.

There is an advantage in rating in this way because of the convenience in determining the class to which a risk belongs. This is so because in investigations of any particular types of risks, it is usual to express the results in the form of ratios of actual to expected deaths.

If the method outlined above is to be followed, the thought arises as to whether a more convenient and less cumbersome method for deriving premiums approximately cannot be obtained. It will be found that approximately the same gross premiums can be derived by a fixed rating in age for each class together with the addition of a constant, all irrespective of age of insured or plan of insurance.

If a mortality table follows Makeham's law, then when the force of mortality is increased by a constant percentage, the new table will also follow Makeham's law.

$$u_x = A + Bc^x$$

and

$$u'_x = (1+K)(A+Bc^x) .$$

Then

$$u'_x = (1+K)A + Bc^{x+t}$$

where

$$c^t = 1+K .$$

In that a constant increase in the force of mortality does not result in the same constant increase in the premium, it is obvious that the plan proposed will not give rigidly accurate results, but none the less a rating in age together with the imposition of a constant gives, with our limited knowledge, what seem to be reasonably equitable premiums. For a test the writer has taken the gross non-participating premiums calculated in his paper, "Premiums for Non-Participating Life Assurances," appearing in the *Transactions of the Actuarial Society of America*, Volume XX, Part 2, Number 62. Premiums have been calculated in addition for specimen ages for the Ten-Payment Life and Ten-Year Endowment plans upon the same formulas as given in the paper with an allowance for commissions on the former plan of 30 per cent the first year and $7\frac{1}{2}$ per cent thereafter, and on the latter plan 20 per cent for the first year and 5 per cent thereafter.

In Table I appear two sets of gross premiums. The premiums in the first set are computed on the bases of $137\frac{1}{2}$, 165, 200, and 260 per cent of the A.M. Select Table of Mortality upon the formulas contained in the paper.

The second set of premiums is derived from the gross premiums appearing in the paper, increased in the following way:

Class A—rated	three years	plus $1.50
Class B—rated	five years	plus 2.00
Class C—rated	seven years	plus 3.00
Class D—rated	ten years	plus 5.00

RESERVES AND EXTENDED INSURANCE

The natural method under this plan is to carry a reserve for the rated age. This will be the terminal reserve. The mean reserve will be increased by one-half the constant. The reserves calculated on this method will be higher than the reserves calculated on a table

TABLE I

Age	Gross Premiums for Non-Participating Substandard Life Insurance*						Gross Premiums for Non-Participating Substandard Life Insurance Calculated from Premiums for Standard Insurance by an Age Rating and a Constant Extra						Differences						Age
	Annual Life	20-Pay't Life	10-Pay't Life	10-Yr. End't	20-Yr. End't	5 Yr. Term	Annual Life	20-Pay't Life	10-Pay't Life	10-Yr. End't	20-Yr. End't	5 Yr. Term	Annual Life	20-Pay't Life	10-Pay't Life	10-Yr. End't	20-Yr. End't	5 Yr. Term	
	137½ Per Cent Mortality						Class "A"—Age advanced 3 years +$1.50						Class "A"						
20	$ 14.78	$ 21.70	$ 35.81	$ 92.61	$ 41.00	$ 10.06	$ 15.00	$ 21.54	$ 34.78	$ 93.35	$ 41.54	$ 9.98	+$0.22	−$0.16	−$1.03	+$0.74	+$0.54	−$0.08	20
30	19.83	27.01	44.26	92.91	41.82	10.59	20.11	27.10	43.65	93.69	42.45	10.46	+ 0.28	+ 0.09	− 0.61	+ 0.78	+ 0.63	− 0.13	30
40	29.20	35.99	57.34	94.48	45.30	13.31	29.39	36.21	57.05	95.40	46.04	13.60	+ 0.19	+ 0.22	− 0.29	+ 0.92	+ 0.74	+ 0.29	40
50	40.37	51.26	76.29	99.74	55.22	22.96	45.92	51.08	75.84	100.49	55.48	23.07	− 0.45	− 0.18	− 0.45	+ 0.75	+ 0.26	+ 0.11	50
60	77.47	78.88	103.38	113.26	79.02	47.34	75.28	77.08	102.00	113.13	77.57	45.97	− 2.19	− 1.80	− 1.38	− 0.13	− 1.45	− 1.37	60
	165 Per Cent Mortality						Class "B"—Age advanced 5 years +$2.00						Class "B"						
20	$ 16.38	$ 23.61	$ 38.78	$ 93.22	$ 41.79	$ 11.32	$ 16.30	$ 22.96	$ 36.77	$ 93.90	$ 42.13	$ 10.57	−$0.08	−$0.65	−$2.01	+$0.68	+$0.34	−$0.75	20
30	21.90	29.21	47.56	93.58	42.78	11.96	22.07	29.10	46.44	94.36	43.39	11.28	+ 0.17	− 0.11	− 1.12	+ 0.78	+ 0.61	− 0.68	30
40	32.35	38.90	61.29	95.47	46.95	15.24	32.50	39.12	60.85	96.57	47.80	15.31	+ 0.15	+ 0.22	− 0.44	+ 1.10	+ 0.85	+ 0.07	40
50	51.76	55.96	81.45	101.81	58.90	26.80	51.02	55.62	80.85	102.73	59.09	26.72	− 0.74	− 0.34	− 0.60	+ 0.92	+ 0.19	− 0.08	50
60	87.34	87.87	111.11	118.20	.....	55.99	83.96	85.05	109.08	117.71	.....	53.70	− 3.38	− 2.82	− 2.03	− 0.49	.....	− 2.29	60
	200 Per Cent Mortality						Class "C"—Age advanced 7 years +$3.00						Class "C"						
20	$ 18.32	$ 25.88	$ 42.24	$ 93.99	$ 42.81	$ 12.94	$ 18.19	$ 24.96	$ 39.38	$ 94.94	$ 43.25	$ 11.62	−$0.13	−$0.92	−$2.86	+$0.95	+$0.44	−$1.32	20
30	24.40	31.76	51.32	94.43	44.01	13.72	24.71	31.74	49.91	95.60	44.95	12.70	+ 0.31	− 0.02	− 1.41	+ 1.17	+ 0.94	− 1.02	30
40	36.11	42.32	65.73	96.72	49.08	17.68	36.42	42.78	65.36	98.39	50.33	17.79	+ 0.31	+ 0.46	− 0.37	+ 1.67	+ 1.25	+ 0.11	40
50	58.24	61.64	87.33	104.46	63.59	31.68	57.17	61.17	86.68	105.80	63.76	31.47	− 1.07	− 0.47	− 0.65	+ 1.34	+ 0.17	− 0.21	50
60	99.33	.....	120.28	124.57	.....	.....	94.14	.....	117.30	123.51	.....	.....	− 5.19	.....	− 2.98	− 1.06	.....	.....	60
	260 Per Cent Mortality						Class "D"—Age advanced 10 years +$5.00						Class "D"						
20	$ 21.48	$ 29.43	$ 47.54	$ 95.32	$ 44.56	$ 15.72	$ 21.75	$ 28.64	$ 44.07	$ 97.01	$ 45.52	$ 13.71	+$0.27	−$0.79	−$3.47	+$1.69	+$0.96	−$2.01	20
30	28.38	35.72	56.95	95.90	46.13	16.72	29.53	36.52	56.01	98.14	48.04	15.69	+ 1.15	+ 0.80	− 0.94	+ 2.24	+ 1.91	− 1.03	30
40	42.11	47.67	72.33	98.90	52.74	21.87	43.44	49.29	73.07	101.94	55.22	22.72	+ 1.33	+ 1.62	+ 0.74	+ 3.04	+ 2.48	+ 0.85	40
50	68.65	70.85	96.22	109.06	71.64	40.02	68.12	71.07	96.61	111.64	72.51	40.51	− 0.53	+ 0.22	+ 0.39	+ 2.58	+ 0.87	+ 0.49	50
60	118.80	.....	134.96	135.71	.....	.....	112.24	.....	131.69	134.57	.....	.....	− 6.56	.....	− 3.27	− 1.14	.....	.....	60

* Based on formulas in "Premiums for Non-Participating Life Insurances," *Transactions of the Actuarial Society of America*, Vol. XX, Part II, No. 62.

of mortality with the rates of mortality increased by a constant percentage. This is so because a constant addition to the mortality rate decreases reserves.

On some types of risks it is admitted that the reserve under the plan submitted is too high, but on other types, as, for example, upon the overweight class, the reserve is too low. On the average it is probably reasonable. In Table II are shown the reserves for specimen ages and durations for both the Ordinary Life and Twenty-Payment Life plans, based firstly upon $137\frac{1}{2}$, 165, 200, and 260 per cent of the American Experience Table of Mortality; and, secondly, upon 100 per cent of the American Experience Table of Mortality with advance in age of three years, five years, seven years, and ten years. In this table is shown also the extended insurance corresponding to the two sets of reserves. There is also shown the extended insurance that would be purchased by the reserve based upon $137\frac{1}{2}$, 165, 200, and 260 per cent of the American Experience Table of Mortality if that reserve was applied on the basis of the American Men Table with $4\frac{1}{4}$ per cent interest and with the mortality rates increased to $137\frac{1}{2}$, 165, 200, and 260 per cent. It will be noted that the extended insurance granted under the combined rating in age and constant method is greater than would be allowed under a reserve method based upon a percentage addition to the rate of mortality by the American Experience Table of Mortality. But it is less than what the reserve under the latter method would purchase if applied on the basis of $4\frac{1}{4}$ per cent interest and $137\frac{1}{2}$, 165, 200, and 260 per cent mortality of the American Men Table.

It is submitted that the advantages of the method described are as follows:

1. It gives, upon the whole, premiums adequate but not excessive.

2. It furnishes as equitable treatment between individuals as is given in standard insurance.

3. It appeals to policyholders in that higher cash values are given in consideration of the extra premium charged. But the values are not as high as upon the simple rating up in age method where they are probably excessive for the average substandard risk.

4. It appeals to the agent as once the class of an applicant is known, he can quote rates and values for any plan of insurance without cumbersome rate schedules.

TABLE II

Ordinary Life

Age	Terminal Reserves, End of Year				Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year				Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year				Age
	5	10	15	20	5	10	15	20	5	10	15	20	
	Amer. Exp. $3\frac{1}{2}$–$137\frac{1}{2}$ Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–$137\frac{1}{2}$ Per Cent Mort.				A.M. Table $4\frac{1}{4}$–$137\frac{1}{2}$ Per Cent Mort.				
20	$ 35.43	$ 77.79	$128.19	$187.70	3	8	13	16	7	15	20	21	20
30	54.66	119.19	194.37	280.02	5	10	13	15	9	14	16	16	30
40	85.35	182.58	288.42	398.25	6	10	11	11	8	11	12	12	40
50	129.48	263.85	396.54	520.12	5	7	8	8	5	8	8	8	50
60	180.23	348.09	495.24	626.85	3	5	5	4	3	5	5	5	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 3 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 3 yrs.								
20	$ 37.62	$ 82.32	$135.13	$196.90	5	11	17	19	.....	.....	.....	.....	.....
30	57.54	124.86	202.49	289.22	7	13	16	16	.....	.....	.....	.....	.....
40	88.70	187.81	294.00	403.10	7	11	12	12	.....	.....	.....	.....	.....
50	130.74	265.07	396.63	518.42	6	8	8	8	.....	.....	.....	.....	.....
60	179.01	344.72	495.92	636.32	4	5	5	4	.....	.....	.....	.....	.....
	Difference in reserves												
20	$ 2.19	$ 4.53	$ 6.94	$ 9.20	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	2.88	5.67	8.12	9.20	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	3.35	5.23	5.58	4.85	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	1.26	1.22	.09	−1.70	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	−1.22	−3.37	.78	9.47	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

20–PAYMENT LIFE

Age	Terminal Reserves, End of Year 5	10	15	20	Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year 5	10	15	20	Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year 5	10	15	20	Age
	Amer. Exp. $3\frac{1}{2}$–$137\frac{1}{2}$ Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–$137\frac{1}{2}$ Per Cent Mort.				A.M. Table $4\frac{1}{6}$–$137\frac{1}{2}$ Per Cent Mort.				
20	$ 81.04	$180.78	$303.78	$455.90	9	21	32	Full pd.	19	33	44	Full pd.	20
30	100.12	222.34	371.73	555.25	9	19	25	.....	16	24	30	.....	30
40	126.51	277.23	455.78	672.59	9	15	18	.....	11	16	20	.....	40
50	159.28	335.60	535.06	786.56	6	9	11	.....	7	10	12	.....	50
60	192.59	382.13	579.78	877.86	4	5	6	.....	4	5	6	.....	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 3 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 3 yrs.								
20	$ 79.50	$176.12	$293.66	$436.81	12	26	34	Full pd.	.....	.....	.....	.....	.....
30	100.47	221.48	367.16	542.58	12	22	27	.....	.....	.....	.....	.....	.....
40	128.78	279.28	454.50	663.83	10	16	19	.....	.....	.....	.....	.....	.....
50	160.94	337.23	534.54	779.72	7	10	12	.....	.....	.....	.....	.....	.....
60	191.98	380.00	580.74	877.74	4	5	6	.....	.....	.....	.....	.....	.....
	Difference in reserves												
20	−$1.54	−$4.66	−$10.12	−$19.09	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	.35	− .86	− 4.57	− 12.67	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	2.27	2.05	− 1.28	− 8.76	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	1.66	1.63	− .52	− 6.84	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	− .61	− 2.13	.96	− .12	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

ORDINARY LIFE

Age	Terminal Reserves, End of Year 5	10	15	20	Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year 5	10	15	20	Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year 5	10	15	20	Age
	Amer. Exp. $3\frac{1}{2}$–165 Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–165 Per Cent Mort.				A.M. Table $4\frac{1}{4}$–165 Per Cent Mort.				
20	$ 36.53	$ 80.34	$132.65	$194.57	3	7	11	14	6	13	17	19	20
30	56.88	124.20	202.72	292.09	4	9	12	13	8	13	14	15	30
40	89.65	191.71	302.19	415.80	5	9	10	10	7	10	11	11	40
50	136.68	277.24	414.28	539.89	5	7	7	7	5	7	7	7	50
60	189.62	363.38	512.33	643.01	3	4	4	4	3	4	4	4	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 5 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 5 yrs.								
20	$ 40.91	$ 89.42	$146.54	$213.04	5	12	17	19	.....	.....	.....	.....	.....
30	62.73	135.76	219.15	310.75	7	13	15	16	.....	.....	.....	.....	.....
40	96.48	202.47	314.19	426.90	7	11	11	11	.....	.....	.....	.....	.....
50	140.08	281.41	416.82	540.11	5	7	8	7	.....	.....	.....	.....	.....
60	188.44	360.01	518.75	665.71	3	4	4	4	.....	.....	.....	.....	.....
	Difference in reserves												
20	$ 4.38	$ 9.08	$ 13.89	$ 8.47	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	5.85	11.56	16.45	18.66	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	6.83	10.73	12.00	11.10	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	3.40	4.17	2.54	.22	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	−1.18	−3.37	6.42	22.70	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

20-PAYMENT LIFE

AGE	TERMINAL RESERVES — End of Year				YEARS EXTENDED INSURANCE PURCHASED BY RESERVE IN PRECEDING COLUMNS — End of Year				YEARS EXTENDED INSURANCE PURCHASED BY RESERVE IN PRECEDING COLUMNS — End of Year				AGE
	5	10	15	20	5	10	15	20	5	10	15	20	
	Amer. Exp. $3\frac{1}{2}$–165 Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–165 Per Cent Mort.				A.M. Table $4\frac{1}{2}$–165 Per Cent Mort.				
20	$ 84.57	$189.54	$320.20	$483.51	7	19	29	Full pd.	16	30	40	Full pd.	20
30	103.50	230.79	387.67	582.52	8	17	23	.....	14	22	27	.....	30
40	130.01	285.47	470.49	698.26	8	13	16	.....	10	14	18	.....	40
50	163.42	342.93	545.53	807.91	5	8	10	.....	6	9	11	.....	50
60	198.56	389.31	583.13	892.28	3	4	5	.....	3	5	5	.....	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 5 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 5 yrs.								
20	$ 83.17	$184.14	$306.79	$456.00	12	25	33	Full pd.	.....	.....	.....	.....	.....
30	105.51	232.19	384.02	566.15	12	20	25	.....	.....	.....	.....	.....	.....
40	135.23	291.42	471.96	688.24	10	15	18	.....	.....	.....	.....	.....	.....
50	167.32	347.56	547.18	800.48	6	9	11	.....	.....	.....	.....	.....	.....
60	197.71	386.38	587.26	895.78	3	5	5	.....	.....	.....	.....	.....	.....
	Difference in reserves												
20	—$1.40	—$5.40	—$13.41	—$27.51	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	2.01	1.40	— 3.65	—16.37	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	5.22	5.95	1.47	—10.02	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	3.90	4.63	1.65	— 7.43	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	— .85	—2.93	4.13	3.50	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

ORDINARY LIFE

Age	Terminal Reserves, End of Year 5	10	15	20	Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year 5	10	15	20	Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year 5	10	15	20	Age
	Amer. Exp. $3\frac{1}{2}$–200 Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–200 Per Cent Mort.				A.M. Table $4\frac{1}{4}$–200 Per Cent Mort.				
20	$ 37.54	$ 82.76	$136.94	$201.27	2	6	9	12	5	11	15	17	20
30	59.07	129.20	211.14	304.36	4	7	11	12	7	11	13	13	30
40	94.11	201.17	316.71	434.14	5	8	9	9	6	9	10	9	40
50	144.63	291.64	433.09	560.53	4	6	6	6	4	6	6	6	50
60	199.69	379.59	530.13	659.26	3	4	4	3	3	4	4	4	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 7 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 7 yrs.								
20	$ 44.51	$ 97.17	$158.92	$230.40	6	12	17	18	.....	.....	.....	.....	.....
30	68.40	147.56	236.72	333.04	7	12	14	15	.....	.....	.....	.....	.....
40	104.59	217.58	334.68	450.55	7	10	11	10	.....	.....	.....	.....	.....
50	149.67	297.76	436.39	562.31	5	7	7	6	.....	.....	.....	.....	.....
60	197.42	376.73	543.55	698.77	3	4	4	3	.....	.....	.....	.....	.....
	Difference in reserves												
20	$ 6.97	$ 14.41	$ 21.98	$ 29.13	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	9.33	18.36	25.58	28.68	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	10.48	16.41	17.97	16.41	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	5.04	6.12	3.30	1.78	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	−2.27	−2.86	13.42	39.51	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

20-PAYMENT LIFE

AGE	TERMINAL RESERVES — End of Year				YEARS EXTENDED INSURANCE PURCHASED BY RESERVE IN PRECEDING COLUMNS — End of Year				YEARS EXTENDED INSURANCE PURCHASED BY RESERVE IN PRECEDING COLUMNS — End of Year				AGE
	5	10	15	20	5	10	15	20	5	10	15	20	
	Amer. Exp. $3\frac{1}{2}$–200 Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–200 Per Cent Mort.				A.M. Table $4\frac{1}{4}$–200 Per Cent Mort.				
20	$ 87.98	$198.36	$337.39	$513.52	6	16	26	Full pd.	14	27	37	Full pd.	20
30	106.57	238.85	403.67	611.38	7	15	21	.....	12	19	25	.....	30
40	133.09	292.94	484.63	724.72	6	11	15	.....	8	13	16	.....	40
50	167.70	349.84	554.70	829.21	5	7	9	.....	5	7	9	.....	50
60	205.47	397.52	585.73	906.20	3	4	4	.....	3	4	4	.....	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 7 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 7 yrs.								
20	$ 87.09	$192.67	$320.70	$476.26	12	24	31	Full pd.	.....	.....	.....	.....	.....
30	110.84	243.42	401.35	590.22	12	19	24	.....	.....	.....	.....	.....	.....
40	141.70	303.39	488.91	712.23	9	13	16	.....	.....	.....	.....	.....	.....
50	173.64	357.18	557.99	820.64	6	8	9	.....	.....	.....	.....	.....	.....
60	203.28	394.29	594.11	913.32	3	4	4	.....	.....	.....	.....	.....	.....
	Difference in reserves												
20	−$0.89	−$5.69	−$16.69	−$37.26	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	4.27	4.57	− 2.32	− 21.16	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	8.61	10.45	4.28	− 12.49	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	5.94	7.34	3.29	− 8.57	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	−2.19	−3.23	8.38	7.12	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

ORDINARY LIFE

Age	Terminal Reserves, End of Year				Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year				Years Extended Insurance Purchased by Reserve in Preceding Columns, End of Year				Age
	5	10	15	20	5	10	15	20	5	10	15	20	
	Amer. Exp. $3\frac{1}{2}$–260 Per Cent Mort.				Amer. Exp. $3\frac{1}{2}$–260 Per Cent Mort.				A.M. Table $4\frac{1}{4}$–260 Per Cent Mort.				
20	$ 38.61	$ 85.42	$141.91	$209.37	2	4	7	10	4	9	13	14	20
30	61.76	135.53	222.06	320.58	3	6	9	10	5	9	11	11	30
40	100.10	214.06	335.91	458.31	4	6	8	8	5	7	8	8	40
50	155.04	310.77	457.86	587.33	3	5	5	5	4	5	5	5	50
60	213.41	401.26	553.49	680.49	2	3	3	3	2	3	3	3	60
	Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 10 yrs.				Amer. Exp. $3\frac{1}{2}$–100 Per Cent Mort.—Age advanced 10 yrs.								
20	$ 50.58	$110.14	$179.47	$258.64	6	13	16	17	.....	.....	.....	.....	.....
30	77.92	166.89	264.62	367.63	7	12	13	13	.....	.....	.....	.....	.....
40	117.31	240.96	365.70	485.23	6	9	9	9	.....	.....	.....	.....	.....
50	164.34	321.81	465.19	597.84	4	6	6	5	.....	.....	.....	.....	.....
60	211.41	407.01	588.10	749.56	2	3	3	2	.....	.....	.....	.....	.....
	Difference in reserves												
20	$ 11.97	$ 24.72	$ 37.56	$ 49.27	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	16.16	31.36	42.56	47.05	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	17.21	26.90	29.79	26.93	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	9.30	11.04	17.33	10.51	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	−2.00	5.75	34.61	69.07	.....	.....	.....	.....	.....	.....	.....	.....	.....

TABLE II—*Continued*

20-PAYMENT LIFE

AGE	TERMINAL RESERVES — End of Year				YEARS EXTENDED INSURANCE PURCHASED BY RESERVE IN PRECEDING COLUMNS — End of Year				YEARS EXTENDED INSURANCE PURCHASED BY RESERVE IN PRECEDING COLUMNS — End of Year				AGE
	5	10	15	20	5	10	15	20	5	10	15	20	
	Amer. Exp. 3½–260 Per Cent Mort.				Amer. Exp. 3½- 260 Per Cent Mort.				A.M. Table 4½–260 Per Cent Mort.				
20	$ 91.81	$209.14	$360.01	$555.78	5	13	23	Full pd.	10	23	32	Full pd.	20
30	109.69	248.02	423.69	650.87	5	12	18	.....	10	16	21	.....	30
40	136.23	301.11	500.77	759.37	5	9	12	.....	7	10	13	.....	40
50	172.69	357.38	563.33	855.92	4	6	7	.....	4	6	7	.....	50
60	215.99	410.26	588.80	923.10	2	3	3	.....	2	3	3	.....	60
	Amer. Exp. 3½–100 Per Cent Mort.—Age advanced 10 yrs.				Amer. Exp. 3½–100 Per Cent Mort.—Age advanced 10 yrs.								
20	$ 93.46	$206.47	$343.07	$508.49	12	23	29	Full pd.	.....	.....	.....	.....	.....
30	119.46	261.10	427.87	626.92	11	18	21	.....	.....	.....	.....	.....	.....
40	151.35	320.77	512.92	746.98	8	12	14	.....	.....	.....	.....	.....	.....
50	182.97	369.84	570.69	849.97	5	7	8	.....	.....	.....	.....	.....	.....
60	213.40	413.57	611.63	936.64	2	3	3	.....	.....	.....	.....	.....	.....
	Difference in reserves												
20	$ 1.65	—$ 2.67	—$16.94	—$47.29	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	9.77	13.08	4.18	— 23.95	.....	.....	.....	.....	.....	.....	.....	.....	.....
40	15.12	19.66	12.15	— 12.39	.....	.....	.....	.....	.....	.....	.....	.....	.....
50	10.28	12.46	7.36	— 5.95	.....	.....	.....	.....	.....	.....	.....	.....	.....
60	— 2.59	3.31	22.83	13.54	.....	.....	.....	.....	.....	.....	.....	.....	.....

5. It avoids preparation of special tables, the labor of separate valuations, and is, in general, very simple in operation.

The practice of issuing a policy on a substandard life showing an age greater than the true age of the insured is not popular with either the public or the agents. Under the proposed plan, policies can be written showing the true age of the insured. A regular standard policy can be used with the following indorsement placed opposite the non-forfeiting values of the policy:

> This policy being issued at the Company's special Class _____ premium rates, the non-forfeiting values in Table A and all calculations with respect to non-forfeiting values are based on an age at issue _____ years greater than that of the insured.

The plan described is the one in operation in the company with which the writer is associated, and appended is the classification used in regard to the more important types of single impairments.

The same method is used for applicants substandard by reason of hazardous occupations, and it is also used for tropical and semi-tropical residents. The ratings for physical impairments are, of course, based largely upon the investigations made and published by the Actuarial Society and by individual actuaries of their own company's experience, particularly by Mr. Arthur Hunter and Dr. Rogers.

No claim for originality is made in regard to the method described, although the writer is not familiar with it having been actually adopted before. Moreover, the classification of certain impairments which is submitted is simply the classification used by the company with which the writer is associated. Necessarily it is largely based upon individual interpretation of published experiences, known practices, and individual judgment. The writer hopes, however, that this outline of a method used in writing substandard business may bring forth a useful discussion and constructive criticism.

APPENDIX

In the following tables *S* indicates Standard, *R* indicates Reject, and *A*, *B*, *C*, and *D* indicate the corresponding substandard classes.

BUILD

The build table shows the limiting weights of the various classes according to height and in some cases according to age. When the

weight falls exactly on a figure shown in the table the higher rating is used.

ABDOMINAL GIRTH

The foregoing table requires adjustment if it is to be applied to risks with an abdominal girth greater than the chest expanded. This adjustment is made by additions to the true weight before entering the build table to find the rating. The following additions are made for each inch of excess girth.

TABLE III

Weight	Ages under 40	Ages 40–50	Ages 50 and Over
	lbs.	lbs.	lbs.
180–200	2	2	3
201–220	3	4	5
221–250	5	7	9
251–300	7	10	13
301 and over	10	14	18

If the abdominal girth is 3 or 4 inches less than the expanded chest, the same number of pounds is subtracted as would be added for 1-inch excess, but not more than 10 pounds; if 5 or more inches less, the same number of pounds is subtracted as would be added for 2-inch excess, but not over 15 pounds.

ENDOWMENT INSURANCE FOR OVERWEIGHTS

The foregoing build table applies to ordinary life and most other forms of policies. The following modifications are made in the case of short-term endowments:

TABLE IV

Ordinary Life Rating	Rating for Endowments Maturing at or before Age 50	Rating for Endowments Maturing at or before Age 55
S	S	S
A	S	S
B	S–A	A
C	A	A–B
D	B	C

FAMILY HISTORY OF TUBERCULOSIS

Since the ratings depend upon the weights, there is a special build table for applicants with a family history of tuberculosis.

TABLE V

Build

Weights at 150% Mort.

Ht.	Ages 15–19	Ages 20–24	Ages 25–29	Ages 30–34	Rating
5′0″					
1					
2					
3					
4	79				
5	82	81			A
6	86	85	80		
7	90	89	83		
8	95	93	87	81	
9	99	97	91	84	A
10	103	101	94	87	
11	107	105	97	90	
6 0	112	109	101	94	
1	116	113	105	98	A
2	122	118	111	104	
3	127	124	117	111	
6 4	134	131	125	119	

Underweight Limit of Standard Insurance

Ages 15–19	Ages 20–24	Ages 25–29	Ages 30–34	Rating
83	82	80	78	
86	85	83	80	S
89	88	86	83	
93	92	89	86	
97	96	93	89	
101	100	96	91	S
105	104	99	94	
109	108	102	97	
114	112	106	100	
118	116	110	103	S
122	120	113	106	
126	124	116	109	
131	128	120	113	
136	133	125	118	S
142	138	131	125	
148	144	138	132	
155	152	146	140	

Underweight — Ages: 15–19, 20–24, 25–29, 30–34 — Ages: 15–19, 20–24, 25–29, 30–34

Build

Overweight Limit of Standard Insurance

Ht.	Ages Und. 40	Ages 40–44	Ages 45–49	Ages 50–54	Ages 55 and up	Rating
5′0″	165	165	165	165	165	
1	167	167	167	167	167	
2	170	170	170	170	170	A
3	173	173	173	173	173	
4	177	177	177	177	177	
5	181	181	181	181	181	
6	185	185	185	185	185	A
7	190	190	190	190	190	
8	195	195	195	195	196	
9	200	200	200	200	202	
10	206	206	206	207	208	A
11	211	211	212	213	214	
6 0	217	217	219	220	221	
1	224	225	227	228	229	
2	231	233	235	237	238	A
3	238	241	243	245	246	
6 4	245	249	252	253	254	

Overweight — Ages: Und. 40, 40–44, 45–49, 50–54, 55 and up

Wt. at 150% Mortality	Rating	Wt. at 180% Mortality	Rating	Wt. at 220% Mortality	Rating	Ht.
181		200		221		5′0″
183	B	202	C	224	D or R	1
186		205		228		2
190		208		232		3
194		213		236		4
198	B	218	C	241	D or R	5
203		223		246		6
208		228		252		7
214		234		258		8
219	B	240	C	264	D or R	9
225		246		270		10
230		252		277		11
236		258		284		6 0
243	B	265	C	291	D or R	1
250		273		299		2
257		280		306		3
264		287		313		6 4

All ages — All ages — All ages

TABLE VI

Tuberculosis Family History

Limiting Weights

Ht.	Weights at 180% Mort. at Ages				Rating	Weights at 150% Mortality at Ages						Rating	Weights at 125% Mortality at Ages						Rating	Ht.
	15–19	20–24	25–29	30–34		15–19	20–24	25–29	30–34	35–39	40–44		15–19	20–24	25–29	30–34	35–39	40–44		
	One Death																			
5′0″	77	75	72	68		93	91	88	84	79	71		110	108	105	101	96	88		5′0″
2	83	81	78	73	B	99	97	94	89	83	74	A	116	114	111	106	100	91	S	2
5 4	91	89	85	79		107	105	101	95	87	77		124	122	118	112	104	94		5 4
6	99	97	91	84		115	113	107	100	92	81		132	130	124	117	109	98		6
5 8	108	105	98	90		124	121	114	106	97	85		141	138	131	123	114	102		5 8
10	116	113	105	95	B	132	129	121	111	102	90	A	149	146	138	128	119	107	S	10
6 0	125	121	112	103		141	137	128	119	109	96		158	154	145	136	126	113		6 0
2	136	131	123	115		152	147	139	131	121	108		169	164	156	148	138	125		2
6 4	149	145	138	130		165	161	154	146	137	124		182	178	171	163	154	141		6 4
	Three Deaths																			
5 0	93	91	88	84		108	106	103	99	94	86		120	118	115	111	106	98		5 0
2	99	97	94	89	B	114	112	109	104	98	89	A	126	124	121	116	110	101	S	2
5 4	107	105	101	95		122	120	116	110	102	92		134	132	128	122	114	104		5 4
6	115	113	107	100		130	128	122	115	107	96		142	140	134	127	119	108		6
5 8	124	121	114	106		139	136	129	121	112	100		151	148	141	133	124	112		5 8
10	132	129	121	111	B	147	144	136	126	117	105	A	159	156	148	138	129	117	S	10
6 0	141	137	128	119		156	152	143	134	124	111		168	164	155	146	136	123		6 0
2	152	147	139	131		167	162	154	146	136	123		179	174	166	158	148	135		2
6 4	165	161	154	146		180	176	169	161	152	139		192	188	181	173	164	151		6 4
	15–19	20–24	25–29	30–34		15–19	20–24	25–29	30–34	35–39	40–44		15–19	20–24	25–29	30–34	35–39	40–44		

TABLE VII

HEART MURMURS

	Very Best No Hypertrophy	Average No Hypertrophy	With Slight Hypertrophy	With Moderate Hypertrophy	With Considerable Hypertrophy
Mitral regurgitation	B	B	C	D	R
Aortic obstruction	B	C	D	D	R
Aortic regurgitation and mitral obstruction	R	.....	.....	.....	.....
Age at issue	15–35		36–45		46 and over
Functional murmur	S		A		B

TABLE VIII

ALBUMINURIA*

	Intermittent			Constant		
Age at issue	16–30	30–45	45 up	16–30	30–45	45 up
Faint trace	S	S	S	S	S	A
Trace	S	S	S	A	A	B
Moderate	S	S	A	A	B	B
Large amount	B–R	B–R	C–R	C–R	D–R	C–R

* Definitions: Accidental—if found once in several tests, disregarded; intermittent—if found in two out of three or four tests; constant—if found in two out of two, or three out of three or four tests.

TABLE IX

IRREGULAR OR INTERMITTENT PULSE*

	Ages at Entry			
Degree of irregularity or intermittence	15–30	30–45	45–55	55 up
5 or less per minute	S	A	B	C
Over 5 per minute	A	B	C	C
Irregular and intermittent	A	B	C	C

* The ratings apply to cases where the irregularity or intermittence is probably not merely accidental.

TABLE X*

RAPID PULSE

Pulse Rate	Rating
90–100	B
100–110	B–R

* The ratings apply to cases where the high-pulse reading is probably not merely accidental.

TABLE XI*

High Blood Pressure

Age	Class	mm.	Class	mm.	Class	mm.	Class	mm.	Class
22 or under		143		151		158		166	
23–27	S	145	A	154	B	161	C	167	D or R
28–32		148		156		163		169	
33–37		150		158		165		170	
38–42	S	151	A	160	B	167	C	172	D or R
43–47		153		161		168		174	
48–52		155		162		170		175	
53–57	S	156	A	164	B	171	C	177	D or R
58 and over		158		165		173		179	

* The table shows the limits of high systolic blood pressure. An additional rating is necessary when the diastolic is not normal.

TABLE XII*

Syphilis

Cured	Thoroughly Treated	Not Thoroughly Treated
Initial lesion only	A	B
Slight secondaries	A	B
Marked saturations	B	C
Tertiary symptoms	R	R

* In the table "cured" means freedom from all symptoms, "thoroughly treated" means adequate supervision for at least two years including mixed treatment and freedom from symptoms for at least one year after discontinuance of treatment.

TABLE XIII

Gastric and Duodenal Ulcers

Year since Operation or Attack	Gastric Ulcer with or without Operation	Duodenal Ulcer	
		Without Operation	With Operation
First	R	R	R
Second	R	D	C
Third	D	C	S–A
Fourth	C	B	S
Fifth	B	A	S
Sixth	A	S–A	S
Seventh	S	S	S

TABLE XIV*

GALL STONES OR BILIARY COLIC

Time Elapsed since Attack	Less than 1 Year	1–3 Years	3–5 Years	Over 5 Years
One attack	R	B	A	S
Two attacks	R	B	B	A
Repeated attacks	R	D–R	C	B

* Drainage of the gall bladder is treated the same as one attack, but if the gall bladder has been extirpated, the rating is reduced to one-half.

TABLE XV

RENAL COLIC, GRAVEL OR CALCULUS

Time Elapsed since Attack	Less than 1 Year	1–3 Years	3–5 Years	Over 5 Years
One attack	B	A	S	S
Two attacks	B	B	A	S
Repeated attacks	C–R	B	B	A

TABLE XVI

GLYCOSURIA

With diet or treatment—reject.
Without diet or treatment:

Intermittent	B–C
	50–100
Persistent but less than 1 per cent	C–R
	100–150
More than 1 per cent	R

HISTORY OF LUNG TROUBLE

Table XVII *A* should be applied to applicants giving a history of blood spitting or haemoptysis, but without physical signs of disease. Table XVII *B* should apply to applicants who present abnormal physical signs such as dulness, suspicious apices, rales, area of consolidation, and the like.

TABLE XVII *A*

WITHOUT PHYSICAL SIGNS OF OLD LUNG TROUBLE

AGE AT ENTRY	NO. OF YEARS SINCE ATTACK	PERCENTAGE UNDER- OR OVERWEIGHT				
		−20 PER CENT	−10 Per Cent	°	+10 Per Cent	+20 Per Cent
25	2–5	R	D–R	D	B	B
	5–10	D–R	D	B	B	A
	Over 10	C	B	A	S	A
35	2–5	D–R	D	C	B	B
	5–10	D	C	B	A	A
	Over 10	B	A	A	S	A
45	2–5	C	B	A	A	A
	5–10	B	A	A	A	A
	Over 10	A	S	S	S	A

TABLE XVII *B*

WITH PHYSICAL SIGNS OF OLD LUNG TROUBLE

AGE AT ENTRY	NO. OF YEARS SINCE ATTACK	PERCENTAGE UNDER- OR OVERWEIGHT				
		−20 PER CENT	−10 Per Cent	°	+10 Per Cent	+20 Per Cent
25	2–5	R	R	D	D	C
	5–10	R	D–R	C	B	B
	Over 10	D	C	B	A	A
35	2–5	R	D–R	D	C	B
	5–10	D–R	D	B	B	B
	Over 10	C	B	A	A	A
45	2–5	D	C	B	A	B
	5–10	C	B	A	A	A
	Over 10	B	A	S	A	A

TABLE XVIII

History of Pleurisy, Dry or with Effusion

Age	Time of Last Attack	Percentage Underweight			
		−30 Per Cent	−20 Per Cent	−10 Per Cent	0
25	Within 2 years	D	C	B	A
	2–5 years	C	B	A	A
	5–10 years	B	A	A	S
35	Within 2 years	C	B	A	S
	2–5 years	B	A	A	S
	5–10 years	A	A	S	S
45	Within 2 years	A	A	S	S
	2–5 years	A	S	S	S
	5–10 years	S	S	S	S

TABLE XIX

Inflammatory Rheumatism

Period since Last Attack	Less than 1 Year	1–5 Years	Over 5 Years
One attack	A	S	S
Two or more attacks	A	A	S

TABLE XX

Goiter

When present, simple goiter may be handled either as standard or Class A. Exophthalmic involves rejection. History of goiter operated will be rated as follows:

Period since Operation	Simple	Exophthalmic
Under 1 year	R	R
1–3 years	S–B	R
3–5 years	S	A–B
Over 5 years	S	S–A

TABLE XXI

Gout

Period since Last Attack	Rating
Under 1 year	R
1–5 years	A–D
Over 5 years	S–B

TABLE XXII

APPENDICITIS

If operated, may take standard after three months. Where no operation has been performed, rating will be as follows:

Period since Last Attack	One Attack	Two or More Attacks
Under 1 year	R	R
1–3 years	S–A	S–B
Over 3 years	S	S–A

TABLE XXIII*

ALCOHOL HABITS

	Present Practice	Within 2 Years	3–5 Years	6–10 Years	Over 10 Years Ago
Steady free user, never intoxicated	B–C	A–C	A–B	S–A	S–A
Occasional excesses; day or evening only:					
Not over six times a year	B–C	A–C	A–B	S–A	S
Once a month	B–D	B–D	A–C	A–B	S–A
Once a week	R	C–R	B–D	B–C	S–B
Lasting two or more days:					
Three times a year	D–R	C–R	B–D	A–C	S–B
More than three times a year	R	R	C–R	B–D	S–C

* The table is applied with a great deal of latitude in both directions.

BLINDNESS

Ordinary cases—Class B.

Especially good cases—Class A.

When applicant has not adjusted himself satisfactorily to his environment—reject.

NEPHRECTOMY

Operation within five years—reject.

Operation between five and ten years ago—rated according to reason for and time since operation. Operation after ten years—rate A.

TOTAL DEAFNESS

Rate A without disability income provision or double indemnity for accidental death.

LAMENESS REQUIRING USE OF CRUTCHES

Rate A without disability income provision or double indemnity for accidental death.

ASTHMA (BRONCHIAL)

Where present, light attacks, where applicant is young and conditions are otherwise satisfactory, may be disregarded or in some cases met by an A rating. In more serious cases risks will be rejected until after the attacks have ceased.

In cases with a history of repeated attacks, after not less than one month of freedom, rating will be as follows:

TABLE XXIV

Age	Class
Under 30	S–B
From age 30–45	A–B
Over age 45	A–R

TABLE XXV

EXPOSURE TO TUBERCULOUS INFECTION

NATURE OF EXPOSURE	CONDITION EXISTING NOW OR WITHIN SIX MONTHS		CONDITION TERMINATED MORE THAN SIX MONTHS AGO
	To Age 30	Over 30	All Ages
Wife to husband	B	A	S
Husband to wife	B	A	S
Parent to child	B	A	S
Daughter to mother	B	A	S
Daughter to father	A	A	S
Son to parent	A	S–A	S
Brother to brother	B	A	S
Brother to sister	A	S–A	S
Sister to brother	A	S–A	S
Sister to sister	B–D	S–B	S–A

Exposure to more remote relatives or non-relatives will be rated on the "son exposed to parent" basis.

TABLE XXVI

HISTORY OF CANCER

Periods since Apparent Cure	Skin Cancers	Cancer of Tongue, Breast, or Internal Organs
Less than 5 years	R	R
Between 5 and 10 years	A–B	B–C
Over 10 years	S	S–A

TABLE XXVII

HISTORY OF BENIGN TUMORS

Periods since (Observed) or Removed	External	Breast and Internal
Three months to 1 year	S–R	R
Between 1 year and 5 years	S–B	A–C
Over 5 years	S	S–A

TABLE XXVIII

HISTORY OF TUBERCULOSIS OF BONE

Periods since Cured	Rating
Under 5 years	R
Between 5 and 10 years	A–C
Over 10 years	S–A

TABLE XXIX

HISTORY OF TUBERCULOSIS OF GLAND (OPERATED)

Periods since Cured	Rating
Under 1 year	R
Between 1 and 5 years	A–C
Between 5 and 10 years	S–B
Over 10 years	S

TABLE XXX

DISABILITY AND DOUBLE-INDEMNITY BENEFITS

Cause of Rating	Limitations
ALBUMINURIA	No disability
Rated Class C or higher	O.K. for double indemnity
BLOOD PRESSURE	No disability
All ratings	O.K. for double indemnity
ASTHMA	No disability
All Ratings	O.K. for double indemnity
FAMILY HISTORY OF TUBERCULOSIS	No disability
All ratings	O.K. for double indemnity
GOITER	No disability
All ratings	O.K. for double indemnity
HABITS	No disability
Class B or higher	No double indemnity
HEART MURMURS	No disability
Rated Class B or higher	No double indemnity
HISTORY OF TUBERCULOSIS	No disability
All ratings	O.K. for double indemnity
HYPERTROPHY OF THE HEART	No disability
All ratings	No double indemnity age 40 and over
PLEURISY	No disability
All ratings	O.K. for double indemnity
PULSE	No disability
All ratings	O.K. for double indemnity
RHEUMATISM	No disability
All ratings	O.K. for double indemnity
SYPHILIS	No disability
All ratings	O.K. for double indemnity

FINANCIAL STATEMENTS

By VALENTINE HOWELL

The following notes enlarge on several points that have arisen in dealing, first, with the Convention Blank form of annual statement in use by companies located in the United States, and, second, with periodical estimates of financial position generally required between years in connection with dividend scales or for other reasons.

BOOKKEEPING PRINCIPLES

It is the exceptional company in which the actuary is not the final resort on questions dealing with accounting, yet, prior to the publication in the *Proceedings of the Insurance Institute of Toronto for the Year 1922* of a very excellent paper on the subject by Mr. Reburn, there had been, in the literature dealing with accounts, somewhat of a break between accounting as treated in the general texts and the accounts and statements of the Life Company. That paper bridged the gulf in a very satisfactory manner, so there needs only to be introduced here such additional matter as may be of aid in understanding the form in use in the United States, together with one or two points perhaps worthy of some additional emphasis.

Accounting in general.—It is not possible to compress the theory of accounting into a paragraph. However, the student will remember that in bookkeeping all accounts are classified into the two divisions of "Revenue" or "Proprietorship" accounts and "Asset-Liability" accounts. "Asset-Liability" accounts obviously represent values, positive or negative. "Revenue" accounts may be taken to represent increases or decreases in value. The particular name of the account indicates, where possible, the source of the increase or decrease. "Merchandise Sales," for example, is a revenue account because, *if the value of the merchandise itself is temporarily ignored*, a sale is the cause of an increase in the cash or other assets. "Merchandise Purchase" is similarly the cause of a decrease in cash, or an increase in liabilities. Since the value of the merchandise has been heretofore ignored, the books of account will not show correct results until such value has been incorporated in them by means of a third account, "Merchandise Inventory."

One of the main objects of accounting, which is a species of shorthand for depicting the history of a business in brief space, is to furnish a statement of financial position at the end of any period, generally termed the "Balance Sheet," together with an analysis of the change in position as compared with the beginning of the period, which statement goes by a number of names, the more usual being "Profit and Loss Statement," or "Revenue Statement." For this purpose the system of bookkeeping by double entry is a simple yet wonderfully effective instrument, and life insurance accounts, together with practically all other modern accounts, are based on double entry.

So far as the Convention Blank is concerned, it will be obvious that the "Assets and Liabilities" section is identical with a Balance Sheet, and the "Receipts and Disbursements" forms the nearest approach to a Revenue Statement, although with points of difference to be noted later.

When the books are kept by double entry, every entry in the journal (which is eventually posted to the ledger) has two parts: a debit, which when posted into the ledger represents an asset, or a decrease in value (loss) and a corresponding credit which is either a liability, or an increase in value (gain). From this it follows that the two sides of the ledger must always balance, which equality is shown in the trial balance.

From the original entries when books are opened, we have the equation:

A. Assets = Liabilities + Surplus Account,

and a year from commencement, the trial balance, which embodies both Asset-Liability accounts and Revenue accounts, must show the equality:

B. Assets + Losses = Liabilities + Gain + Surplus Account,

when "Surplus Account" is unchanged, no entry having been made affecting it during the period. This trial balance may then be divided into two statements: one, the Revenue Account, will be of the form:

C. Surplus Account (Beginning) + Gains = Losses + Surplus (End),

and the other, the Balance Sheet, will show:

D. Assets = Liabilities + Surplus (End),

which two statements upon combination mathematically reduce to the trial balance.

General ledger.—As in commercial bookkeeping, the first step in preparing the annual statement of a life company is to obtain a trial balance of the accounts in the General Ledger. The General Ledger itself is made up largely of controlling accounts posted from agency and home-office cashbooks the details of which are to be found in subsidiary books or on cards, and these accounts in general bear the same titles as the items of the Convention Blank, although the different items of expense may be grouped under one controlling account, "Expenses."

General balance account.—Life insurance accounting in the United States is peculiar in that the form of annual return required apparently contemplates that no liability accounts and no asset accounts of the type periodically written on to the books are to be carried in the ledger, and consequently that no attempt will be made to show the surplus on the books. The surplus account must then be replaced by some ledger account which will serve to keep the books in balance, and into which accounts representing receipts and disbursements may be closed at the end of each period. Such an account goes under a variety of names, such as "Consolidated Balance Account," "Total Ledger Assets," or "General Balance Account," which latter title will be used here. As there are no other ledger liabilities (except, in some cases, "Capital Stock"), this account prior to the closing out into it of income and disbursement accounts will be seen upon reflection to be equal to the total of the ledger assets at the beginning of the year. The relation of the Convention Blank to the trial balance, when accounts are kept in this fashion, thus appears. The trial balance shows the following equality:

E. Assets+Disbursements=Income+General Balance Account,
and substituting Ledger Assets (beginning of year) for General Balance Account, and transposing, there results:

F. Assets (Beginning)+Income−Disbursements=Assets (End),
which is the method of presentation used in the Convention Blank.

From the foregoing it will be obvious that the first step in compiling the statement from the trial balance is to reconcile the General Balance Account, by whatever name included, with the ledger assets at the beginning of the year. Inexperienced bookkeepers have been known to charge off directly some item through the General

Balance Account during the year, thus causing considerable confusion when the statement has to be prepared.

This transformation of the trial balance into the Convention Form, while probably a commonplace of insurance bookkeeping, has heretofore received so little attention that it may be worth while to illustrate the process by a numerical example. A skeleton form of balance sheet as of the first of the year may be taken to be:

	Dr.		Cr.
Cash	$ 100,000	General Balance Account	$1,200,000
Agents' balances	100,000		
Other ledger assets	1,000,000		
	$1,200,000		$1,200,000

Assume that the transactions of the year consist of the receipt of agency reports indicating the collection of $200,000 of premiums, and that there was $50,000 claimed by the agencies for commission, $150,000 being remitted in cash from the agencies during the year. The entries in the general ledger are sufficiently obvious, and the resulting trial balance for December 31 would be:

	Dr.		Cr.
Cash	$ 250,000	Premiums	$ 200,000
Agents' balances	100,000	General Balance Account	1,200,000
Commission	50,000		
Other ledger assets	1,000,000		
	$1,400,000		$1,400,000

The Convention Blank entries made from this trial balance would be as follows:

Ledger assets, January 1	$1,200,000
Plus receipts: Premiums	200,000
	$1,400,000
Less: Disbursement, Commissions	50,000
Ledger assets, December 31	$1,350,000
Cash	$ 250,000
Agents' balances	100,000
Other ledger assets	1,000,000
Ledger assets, December 31, as above	$1,350,000

The revenue accounts would then be closed out into the General Balance Account and we should have a balance sheet showing, on the one hand, the ledger assets as of December 31, and, on the other, the General Balance Account equal to their sum of $1,350,000.

Incorporation of reserves and accruals in books of account.—In accounting generally, liabilities to be set up and assets to be taken credit for, if they do not already show in the accounts, are formally incorporated on the books, journal entries being made, and appropriate ledger accounts being opened for them. For example, suppose at the beginning of the year a fire insurance premium of $300 had been paid, covering a period of three years. At the end of the year a journal entry

Premiums paid in advance, Dr..... $200
to
Fire insurance premiums, Cr..... $200

would be made and posted, the result being to show an asset of $200 for premiums paid in advance, and a net cost for insurance premiums for the year of $100. But the entries in the Convention Blank indicate that non-ledger assets taken credit for, non-admitted assets deducted, and liabilities set up are not to be formally incorporated on the books, so that no actual "Surplus" account is carried, and it may be that this direct listing in the "Assets and Liabilities" page of the annual statement without preliminary entries in journal and ledger is largely responsible for the break in continuity, in the average student's mind, between commercial bookkeeping and life insurance accounting.

Theoretical entries.—While the subject is elementary in its nature, it may be of advantage to consider some of the entries that are made when the entire life insurance statement is arrived at according to formal accounting methods. Take the matter of premiums, for example: Two new accounts must be opened in the ledger—"Outstanding Premiums" and "Premiums Paid in Advance." Assume there were no outstanding or advance premiums at the beginning of the year, that the premiums paid in cash for the year totaled $100,000, that the outstanding, end of year, was $15,000, and paid in advance $2,000. If there were no other items, the ledger, prior to the making of the year-end entries, would show $100,000 debit under "Cash," $100,000 credit under "Premiums," and a General Balance Account of zero. Entries would then be:

Outstanding premiums, Dr..... $15,000
to
Premiums, Cr..... 15,000

and

Premiums, Dr..... 2,000
to
Premiums paid in advance, Cr..... 2,000

This, when posted, would result in the Premium Account showing a debit balance of $113,000, which would represent the true premiums arising out of the business of the year; a new asset account of outstanding premiums of $15,000 and a liability account of premiums paid in advance of $2,000.

Further, suppose the Policy Reserve be computed to be $110,000, a ledger account, "Policy Reserve," should be opened and credited with $110,000, the corresponding debit being "General Balance Account." Then, when the revenue account of "Premiums" is closed out into the Balance Account, we have for this account a credit of $110,000 from Policy Reserve and a debit of $113,000 from "Premiums," and a balance will be seen to represent surplus for the purpose of this example. Furthermore, the balance sheet shows directly the so-called non-ledger "Assets and the Liabilities," being identical with pages 3 and 4 of the Convention Blank.

In the keeping of such a set of books, it is further necessary to close these new ledger accounts as of January 1 by reversing entries. This means that the Premium Account, for example, having been previously closed out into General Balance Account or surplus, would receive a debit balance on January 1 of the amount of the outstanding less the paid in advance, which results in this figure being apportioned to the previous accounting period, where it belongs. The amount of the policy reserve would go back into the General Balance Account which would lose its meaning considered as surplus.

Revenue versus cash basis.—As has been indicated, books kept on the foregoing principles cause the emerging income and outgo to be on a revenue basis instead of the present cash basis and in point of theory this is an improvement, but the more usual bookkeeping system has the great merit of simplicity in so far as its bookkeeping side is concerned, for the bookkeeper has nothing to do with the items whose amount has to be separately computed,

so that the work of the Actuarial Department begins where that of the Bookkeeping Department leaves off, with little interlocking. Consequently, no change in the present system seems called for, at least so far as the first four pages of the statement are concerned.

GAIN AND LOSS

But it is believed that a form of revenue statement should find a place in the Convention Blank; furthermore, it is believed that by a rearrangement of the present "Gain and Loss Exhibit," such end may be accomplished without additional work or space. This is because, as is evident to those familiar with the "Exhibit," the "Gain and Loss" in its present form is nothing more than a combination of a revenue statement with an analysis of the accumulation of the policy and disability—double-indemnity reserves between years. What should be done in the opinion of the writer is to replace the present "Exhibit" by, first, a revenue statement, preferably arranged in columnar form; and, second, a reserve analysis arranged to show separately the accumulation of the regular and the disability and double-indemnity reserves. Then the combination of these two statements in order to show gains and losses from the separate sources might be left to the insurance-department officials interested.

FORM OF REVENUE STATEMENT

The columnar form of revenue statement, an example of which is shown in the Appendix, is believed to be of advantage because figures appearing in it may be taken directly from those in the main body of the statement, and because the correctness of each column may be separately verified. The statement should provide for columns carrying the following headings:

1. Cash receipts and disbursements
2. Outstanding and paid in advance, present year. The paid in advance should be entered in red, showing that a deduction is to be made
3. Cash plus outstanding
4. Outstanding and paid in advance, previous year
5. Income and outgo, revenue basis. This column evidently represents simply the difference between columns 3 and 4

The items at the side group income accounts, showing the total, and underneath, the disbursements accounts, similarly totaled. The grouping may be simply that necessary to provide the present

"Gain and Loss" information, but preferably should be sufficiently detailed to allow other information to be obtained. For example, income items may profitably include: "New Premiums," "Regular," "Renewal Premiums," "Regular" (including single premiums for paid-up insurance and dividend additions), "Disability and Double Indemnity Premiums," "Annuity Premiums" (including supplementary contracts involving life contingencies), "Supplementary Contracts" not involving life contingencies and dividends left at interest (one item), "Interest and Rents" less amortization and including renewals, "Change in Investment Values," "Other Income." For disbursements we will have: "Death Claims," "Maturities," "Annuity Payments," "Double Indemnity Claims," "Disability Claims" (including as a liability the present value of "Annuities on Disabled Lives"), "Surrender Values Paid in Cash," "Surrender Values Applied to Purchase Paid-Up Insurance," "Payments under Supplementary Contracts and Dividends Left at Interest Surrendered," "Dividends to Policyholders," "First-Year Expenses," "Renewal Expenses," "Investment Expenses," "Disability Expenses," and "Double Indemnity Expenses," "Investment Changes," "Changes in Non-Admitted Assets" (except securities), "Other Outgo." If all the receipts and disbursements, and all the non-ledger assets, non-admitted assets, and liabilities except policy reserves and special reserves have been included, the difference between the income and outgo of the final column will represent the increase in reserves, special funds and surplus, or what would be termed in England, unless I am mistaken, the "Life Insurance Fund."

ACCUMULATION OF RESERVE

This statement, differing perhaps as to detail, will be familiar to everyone. It is based on the equally familiar formula dealing with the accumulation of reserves (suitably modified for mean reserves and for terminations of the year):

$$({}_nV_x + P_x)\ (1+i) - q_{x+n}(1 - {}_{n+1}V_x) = {}_{n+1}V_x$$

In practice, however, it is not as simple to assemble as would appear, due to the ramifications of modern insurance practice. Its composition will therefore be described in some detail:

1. Mean reserve, beginning of year, less reinsurance reserve, excluding reserve on disability and double indemnity, including reserve on annuities and paid-up additions
2. Plus net premiums of year. The calculation of the net premiums will, of course, vary with the methods of the company. The item should include single premium for paid-up additions, single premiums applied to purchase paid-up insurance, annuity premiums, supplementary contracts involving life contingencies, and should be diminished by the net premiums on reinsurance
3. One year's interest at the tabular rate on (1) and one-half year's interest on (2). If reserves at two different rates of interest are involved, a composite rate, based on the composition of the reserves at the beginning of the year, should be used
4. The sum of the foregoing amounts should be diminished by:
   a) Reserves released by death
   b) Maturities, from the revenue statement
   c) Annuity claims, from the revenue statement
   d) Reserve released on surrender for cash or loan
   e) Reserve released on surrender for paid-up
   f) Reserve released on lapse less reserve set up (loss) on change or restoration
   g) One-half year's interest at the tabular rate on the foregoing deductions
   h) The expected mortality for the year, which should include the mortality on paid-up additions, the extra mortality provided for in substandard premiums, the mortality on new business for one-half year only, and should be net after the deduction of the expected mortality on reinsurances
   i) The difference between the reserve expected to be released and that actually released by death of annuitants
5. The final result should be an approximation to the reserve at the end of the year, and should agree to a reasonable extent with the figure brought out by the valuation. In one case in a reserve of approximately forty million, the errors have been for two successive years $4,000 and $2,000. This is believed to be closer than will usually obtain. Note that annuities have been included in with the regular business for convenience. A separate calculation is made for them for the "Gain and Loss," but they can, of course, be separately treated from the commencement.

The analogy between this accumulation of reserve and the revenue statement is interesting. Forebearing for the moment to complicate the situation by the introduction of any items except

those pertaining to the life coverage, we have the following outline for a logical presentation of the present "Gain and Loss Exhibit":

Life "Fund," Beg. of Year	Reserve Beg. of Year	Surplus Beg. of Year
Plus:		
Premiums less Expenses	Net Premiums	Gain or Loss, Loading
Interest Earned	Interest Required	Gain, Interest
Less:		
Death Claims	Reserve Released; Expected Mortality	Gain from Mortality
Matured Endowments	Matured Endowments	
Surrender Values Paid	Reserve Released	Gain from Surrenders
.....	Reserve Released on Lapse	Gain from Lapses
Dividends	.....	Loss, Dividends
Life "Fund," End of Year	Reserve, End of Year	Surplus, End of Year

Disability and double indemnity.—Based on the same principles but in a slightly different form, there may also be shown a statement of gain or loss on disability and double indemnity. As the "Expected Loss" is not here available, the reserve as calculated is deducted from the funds (commencing with the reserve at the beginning of the year and accumulated at the tabular rate of interest) to show the gain or loss for the period. The details of the statement need not be entered into.

Complete information furnished.—From the statements given all the information yielded by the present "Gain and Loss Exhibit" is immediately available. For example, the interest required to maintain reserves for the "Gain and Loss" is the balance of the interest items from the accumulation of reserve and disability and double-indemnity "Gain and Loss" statements, plus interest at the tabular rate on the mean amount of the reserves for supplementary contracts and dividends at interest. The gain from interest is the interest and rents from the revenue statement less the investment expenses and less the interest required as above. Again, the gain from loading is the gross premiums (revenue statement) less net premiums (accumulation of reserve statement) less expenses.

Moreover, it will be noted that since the difference between the

revenue income and outgo is equal to the increase in reserves, in surplus, and in special reserves, and the other statements analyze the increase in reserves, the total error in the resulting "Gain and Loss" if completed, must be composed simply of the unaccounted-for balance in the reserve statement plus any gain or loss that may emerge on supplementary contracts and dividends at interest.

Summary.—Enumerating the points raised in connection with the "Gain and Loss Exhibit," it may be stated as follows: (1) It is desirable that the Convention Blank include a statement of income and outgo on a revenue basis. (2) Such a statement is at present incorporated in the "Gain and Loss," which is a combination of revenue statement and analysis of accumulation of reserve. (3) The present "Gain and Loss" should be so revised as to separate these two elements, (4) thus obtaining without additional work a revenue statement, which may be separately checked and found to be correct, and an estimate of reserve which will be of value, first, as a guide to the correctness of the year-end direct computation of reserve, and, second, as a basis of estimation of reserves for quarterly statements and other purposes. (5) While the gains and losses of the present exhibit will be easily available if required, they will not be subject to misuse by ignorant or malicious persons.

ESTIMATION OF EARNINGS

At the end of each quarter throughout the year, it is usually necessary to obtain some idea of the financial operations of the year to date and an approximation to the present status as regards surplus and earnings. A comparison with the corresponding period of the previous year is here of much value, and such a comparison, in one form or another, is undoubtedly made by all companies.

Receipts and disbursements.—The precise form such comparison takes is dependent upon the material available in the particular company. A usual form is a comparative statement of receipts and disbursements for the period. Such a statement is on a cash basis, consequently a change in time of making payments may affect the figures. For instance, what appeared to be a considerable saving in medical fees was ascertained to be due to the fact that the company, during the current accounting period, had put the payment of medical fees on a quarterly in place of a monthly basis. The amount of first year's commission and of medical fees and inspections natur-

ally bears an intimate relation to the first year's premiums collected. The amount of taxes paid may fluctuate and each large payment should be separately investigated in order to determine the true relation to the previous year. The information given regarding claims is not of much value, and a supplementary statement from the claim department showing the claims incurred with the corresponding reserves released, double-indemnity claims incurred, and disability annuities set up, less recoveries, is usually required.

Summary statement.—The foregoing statements make no attempt to arrive at or summarize the insurance earnings for the period, and a form of comparative statement which attempts to furnish this information is here described (see Appendix). The statement is based on the "Gain and Loss Exhibit" and shows in parallel columns the figures for the present period, the corresponding period of the preceding year, and the final figures of the "Gain and Loss" for that year. However, the actual position of the company, for example, at the end of the first quarter, will best be shown by a comparison of the earnings as shown, with those of the first quarter of the previous year, rather than with one quarter of the total earnings of the previous year.

The gain or loss from loading is on a cash basis; any change in the outstanding due to single large items, such as taxes, having been previously investigated. A fixed percentage, taken from the "Gain and Loss Exhibit" of the previous year, is applied to the cash premiums of the period. From this is subtracted the expenses paid, less a percentage of the assets for general-investment expenses, the result being the gain or loss from loading.

The total of interest and rents is on an incurred basis, as an exact computation of the outstanding interest and rents (except for policy loans) is available quarterly, and as the outstanding varies considerably. From this is deducted investment expenses, and interest required to maintain reserve, which is, for the previous period, one quarter (for example) of the total for that year from the "Gain and Loss," and for the current year a percentage of this last figure, the amount of such percentage being equal to the increase per cent in assets. The result is the gain from interest. For my own company, there is further deducted the interest required to maintain the deferred dividend fund, such interest being implicitly included in the dividend item of the "Gain and Loss."

Mortality gain is simply the difference between the net death losses incurred, reserves deducted, and the estimated expected mortality for the quarter. Disability and double-indemnity loss consists of the present value of claims admitted, less disability recoveries, plus payments made, plus estimated increase in liability on existing claims less the "expected loss" figure, which is based on the accumulation of reserves statement of the previous year.

As the claim department of my own company tabulates the reserves released by surrender for cash of paid-up in their claim-books, the gain from surrenders is immediately available. If this were not the case, I believe it would be quite sufficiently accurate to use ratios of reserves released to values allowed, based on the experience of previous years. The gain from lapses, less revivals and changes, is an estimate, based on the average gain per thousand of lapsed business; gain or loss from annuities is ignored. The total of these items, compared with the total of those of the same period of last year, should furnish a fair approximation to the increase in insurance earnings, and if account is taken of the changes in value of investments, of the agents' balances (non-admitted assets), and of miscellaneous gains and losses, the increase in total earnings to date is obtained.

QUARTERLY RESERVE ESTIMATES

A quarterly statement of this description is useful in connection with the statement at the end of each quarter required by the New York law. The method is that of the accumulation of reserve statement, except that the reserve, net premiums, and "expected" losses for disability and double indemnity are included, and in addition to the cash net premiums from the statement of the previous paragraph, the net increase in the outstanding as estimated is included. This means that in the New York quarterly statement prepared in this manner, the amount of the outstanding premiums does not affect the surplus as the increase appears as an asset and as an increase in reserve. As this figure, including as it does the due and unpaid, is difficult to estimate, this is probably on the side of accuracy.

ESTIMATES OF FUTURE EARNINGS

With regard to estimates in general, one elementary principle impressed by practice is to avoid, where possible, the estimation of

large figures. Company surplus may evidently be looked upon as a balance of much larger sums; that, in fact, is the method by which it is ultimately computed. Again, change in surplus is the net result of the operation of receipts and disbursements and of changes in assets and liabilities, or looked at from the viewpoint of the "Gain and Loss Exhibit," it is the resultant of gains or losses in loading, mortality, interest, surrenders, dividend payments, changes in non-ledger assets, and so forth. It is, however, difficult to obtain dependable results from a method which estimates separately these component parts for the obvious reason that a small separate percentage error in each of two large figures may lead to a large error in their difference.

When, therefore, it is required to obtain in advance some idea of the probable year's change in surplus, it would seem to be best to gain, from a study of surplus changes and their causes for the last two or three years, some idea of the "normal" increase, and then to make allowance for any special events of the year in question or for any premises that may be laid down.

NEW BUSINESS

The cost in surplus of a given amount of new business is often a matter of interest to life insurance executives. The first question, obviously, is to determine what amount of new business may be absorbed without special change in surplus. The difference between this figure and the given amount, multiplied by the estimated loss per thousand caused by extra new business, will give the loss in surplus. The following points call for attention: (1) average premium per thousand of business paid for; (2) average first year's commission, considered as a percentage of premiums; (3) compensation other than commission, if any, paid the agent for new business; (4) average cost per thousand of medical and inspection fees. Note that this cannot be obtained by dividing the medical and inspection fees allowed by the average policy, as cost of rejected business must be included. (5) Premium taxes, say $2\frac{1}{2}$ per cent of the premium. (6) First year's dividend. While this is not disbursed, a reserve for it must be set up, which comes out of surplus. (7) The average reserve to be set up at the end of the year, considered as a percentage of the premium. (8) The true mortality cost for the average duration of six months.

A further point is the effect of the business written during the previous year, if of abnormal amount, on the profits from lapse of the following year. While this effect is not difficult to estimate, the thought occurs in passing that a possible basis of estimation might be a combination of these two factors; that is, new business paid for, less lapses.

In closing it may be well again to emphasize the fact that the paper considers reserve and earning estimates more particularly from the viewpoint of the smaller company. I am inclined to believe that different methods of treatment are necessary for the large companies, possibly differing with the individual company. As an instance, it must be admitted that the "Check of Reserves" statement previously described is of very slight value as a check on the computation of the reserve for a large company, as even the usual variation is generally so large as to obscure all but the largest and consequently most palpable errors. Again, analyses of the kind discussed become much more complicated with the larger companies, due to the more varied scope of their activities, and consequently one or more factors are almost inevitably lost sight of and the results correspondingly distorted.

REVENUE STATEMENT

	Cash	Outstanding '24	Cash plus Outstanding	Outstanding '23	Revenue
Reg. Prems. incl. Divs. and Surr. Values applied excl. Supp. C. *not.* involving					
Premiums for Disability					
Premiums for Double Indemnity					
Premiums for Supp. C. and Divs. at Interest (incl. Accruals and Amortization)					
Interest and Rents (less Interest Disbursement Items)					
Investments					
Exchange					
Total Income					
Death Claims, not incl. Double Indemnity Claims					
Maturities of Endowments					
Annuity Claims					
Disability Claims (incl. present val. of claims set up)					
Double Indemnity Claims					
Surrender Values, paid in cash or applied to Paid-Up Insurance					
Dividends Policyholders					
Dividends to Stockholders					
Supplementary Contracts and Dividends of Interest					
Insurance Expenses					
Investment Expenses					
Disability Expenses					
Double Indemnity Expenses					
Exchange					
Investments					
Special Funds					
Non-admitted Assets					
Agents' Balances charged off					
Total Disbursements					

CHECK OF RESERVE

Reserve, beginning of year			
Net Premiums of year			
Interest required for Reserve			
Interest required for Net Premiums, one half-year			
Less: Reserves released by: Death			
Maturity			
Annuity Payments			
Surrender, net			
Lapse			
Interest on above, one half-year			
Expected Mortality			
Estimated Reserve, end of year			
Regular Reserve as calculated			
Unaccounted for			

STATEMENT OF GAINS AND LOSSES

	First Six Months 1924		First Six Months 1925		Year 1924
New Business paid for, not incl. revivals.					
Loading:					
Premiums paid in cash or by div. . . .					
Estimated Loading (_____%)					
Expenses per expense sheet					
Less general-investment expenses					
Estimated Loss in Loading					
Interest:					
Total Interest Received					
Outstanding, beginning of period					
Outstanding, end of period					
Interest Income					
Investment Expenses, from above					
Investment Expenses, real estate					
Net Interest Income					
Estimated Interest req. for Res					
Gross Interest Gain					
Less Div. Acc. Interest					
Gain from Interest, net					
Mortality:					
Net Death Losses incurred, per Claim Dept					
Expected Mortality					
Gain from Mortality					
Disability and Double Indemnity:					
Disability Claims admitted, less Recoveries					
Payments made					
Double Indemnity Claims incurred . . .					
Increase in Liability on existing claims					
Expected Losses					
Gain on Disability and Double Ind . .					
Surrenders:					
Reserve Released on Surrender					
Allowance made					
Gain on Surrender					
Lapses:					
Business Lapsed					
Estimated Gain on Lapse, less Revivals					
Other Gains or Losses					
Total Insurance Earnings					

AVIATION HAZARD

By W. F. POORMAN

Aviation is the newest form of transportation. It is not as yet generally established, but having proved its value in military life, it will doubtless prove its necessity to commercial life. The possibility of commercial aviation of the future may be indicated by the fact that 327,510 flights were made and 278,668 passengers carried in the three years 1921–23 by the aviation companies in the United States. Since there are hazards involved, those who participate in aviation are desirous of insurance protection. Where there is a demand for this protection it is our duty to prepare to supply it.

A careful search of insurance literature reveals a general treatment of the subject by Walter G. Cowles[1] and the informal discussion of the subject in *The Record*, Volume XIII. It has been my endeavor to determine what experience is available on this subject, and by the courtesy of the Post-Office Department, the War Department, the Navy Department, and the aviation companies operating commercially, I am enabled to offer five tables of fatalities.

Briefly the results may be summarized as in Table I:

TABLE I

	Period	Total Miles Flown	Fatalities	Average Miles Flown per Fatality
Itinerant flyers	1921–23	9,000,000*	196	48,300
War Department	1921–24	22,057,096*	211	104,536
Navy Department	1923–24	4,108,500*	27	152,166
Post-Office Department	1918–24	9,213,318	40	230,333
Commercial companies	1921–23	8,767,800	25	350,715

* Estimated.

The experience of the Post-Office Department is given in Table II in detail:

[1] *P.C.A.C.*, VI, 13.

TABLE II

Year	Miles Flown	Fatalities of Pilots Flying Ships	Average Miles Flown per Fatality	All Fatalities	Average Miles Flown per Fatality
1918*.....	102,548	1	102,548	1	102,548
1919.....	461,295	4	115,324	5	92,259
1920.....	1,048,444	8	131,055	15	69,896
1921.....	1,912,733	7	273,247	9	212,414
1922.....	1,756,803	1	1,756,803	1	1,756,803
1923.....	1,870,422	5	354,084	6	311,737
1924.....	2,061,073	3	687,024	3	687,024
1918–24.....	9,213,318	29	317,700	40	230,333

* May 15 to December 31—seven and one-half months.

From May 15, 1918, to December 31, 1924, a period of six years, seven and one-half months, a total of 9,213,318 miles was flown. During this period there were 40 fatalities of which 38 were in the air and 2 on the ground. Thirty-one pilots lost their lives; 20 while flying ships with mail; 9 on test and ferry trips; and 2 as passengers. Seven mechanics were fatally injured while flying as passengers and 2 were killed by propellers on the ground. During the period 1921–24 an average of 43 pilots was continuously employed.

TABLE III

Experience of the War Department

Fiscal Year	Total Number of Fatal Crashes	Number of Fatalities	Number of Hours Flown	Number of Hours* Flown per Fatality
1921.....	45	73	76,858	1,053
1922.....	26	44	65,214	1,482
1923.....	34	60	65,750	1,096
1924.....	23	34	94,351	2,775
1921–24.....	128	211	302,173	1,432

* Approximate number of miles flown may be obtained by multiplying the hours by 73.

The War Department figures are given in Table III. These figures show a more favorable average experience than that for the years 1918–20, as given by Mr. Hoskins.[1]

The hazard was apparently reduced slightly in this group due to the fact that forced landings on water terminate fatally less fre-

[1] *R.A.I.A.*, XIII, 392.

TABLE IV

EXPERIENCE OF THE NAVY DEPARTMENT

Fiscal Year	Total Hours Flown	Fatalities	Average Hours* Flown per Fatality
1922.....	.....	.....	2,622
1923.....	35,785	17	2,105
1924.....	32,690	10	3,269
1923–24.....	68,475	27	2,536

* Approximate number of miles flown may be obtained by multiplying the hours by 60.

quently than do forced landings on land. In this experience the fatality ratio among student aviators was approximately twice that of regularly qualified aviators.

TABLE V

EXPERIENCE OF COMMERCIAL-AVIATION COMPANIES*

Year	Miles Flown	Fatalities	Average Miles Flown per Fatality
1921.....	2,907,200	6	484,561
1922.....	2,846,000	7	406,577
1923.....	3,014,600	12	251,218
1921–23.....	8,767,800	25	350,715

* *Aircraft Year Book*, 1924.

Table V gives the experience of commercial-aviation companies in the United States. Mr. Gunn has given the experience of the British commercial-aviation companies for the years 1919–22.[1] During 1923 these companies reported that 943,000 miles were flown; 15,502 passengers carried with 5 fatalities (2 pilots and 3 passengers. Passenger rates average fifty cents a mile, which confines this transportation to people of means. Needless to say, it is people of means who carry substantial amounts of insurance.

Table VI shows the mortality experienced among gypsy or itinerant flyers. There are estimated to be 500–600 nondescript craft in this country, consisting mainly of obsolete surplus army supplies and homemade machines with little semblance of stability

[1] *Ibid.*, p. 394.

or air-worthiness. A careful review of the causes of death in this group for the past three years shows 84 deaths due to faulty equipment, 56 deaths due to stunt flying, and 8 deaths due to carelessness on the field.

TABLE VI

EXPERIENCE OF OCCASIONAL FLYERS*

Year	Estimated Miles Flown	Fatalities	Average Miles Flown per Fatality
1921.....	3,000,000	49	61,222
1922.....	3,000,000	62	48,387
1923.....	3,000,000	85	35,294
1921–23.....	9,000,000	196	48,301

* *Aircraft Year Book*, 1924.

Diligent inquiry was made in regard to the experience in the various European countries but no information would be released for publication.

CONCLUSIONS

Scanty material available makes definite conclusions impossible, but apparently those participating in aeronautics can be divided into three classes:

1. *Occasionals: the amateur, sportsman, or ex-Army aviators.*—They fly irregularly with obsolete and untested equipment and without the aid of expert mechanics. The fatalities in this group are very high, 1 in 48,000 miles.

2. *Army and Navy aviators, who are constantly experimenting with equipment, weather conditions, capacity loads, etc.*—They do stunt work because that is a military necessity. In spite of the best machines and excellent facilities for mechanical repairs, their fatality likelihood is still high, 1 in 104,000 and 1 in 152,000 miles, respectively.

3. *Commercial flyers and passengers.*—Aviation companies are endeavoring to prove aeronautics as something practical and safe. They have the best of facilities and avoid unusual hazards, but in spite of all carefulness the fatality likelihood is 1 in 350,000 miles. Possibly post-office aviators belong in this group but experience a somewhat higher mortality, probably due to attempting flight regardless of climate conditions.

DISCUSSION OF PAPERS READ BUT NOT DISCUSSED AT PREVIOUS MEETING

A STATISTICAL STUDY OF BLOOD PRESSURE

BY ROBERTSON G. HUNTER

MR. F. B. MEAD:

There is little to be said on this paper except to express appreciation for the new data brought out.

Mr. Hunter calls attention to the fact that the average blood pressure for the various age groups is less, according to the experience of his company, than that of companies which have previously published their data. This probably is because as men become more expert in taking blood pressure their readings are inclined to be lower than when they were less experienced. We find that our more experienced men report lower blood pressures than the less experienced.

Blood pressure, I am sure, and this is indicated in this paper, will be demonstrated in the future to have an influence on mortality just as weight has. All other factors being equal, an increase in blood pressure will mean an increase in mortality, just as, all the factors being equal, an increase in weight will increase the mortality except for age incidence in connection with weight.

Mr. Hunter, on page 207, refers to the fact that after age forty-five there was evidently a larger variation from the average pressure, especially in connection with overweights, but up to the point where the data were large enough to be thoroughly representative, overweight to the extent of 16–20 per cent did not increase the systolic pressure more than 4 points. That probably is as might be expected. It really is quite a large variation when you consider that they took cases within certain narrow limits.

If you took the extreme weights they have accepted and took their blood pressure after a period following the examination, you would probably find the variation very great because overweights

develop the degenerative diseases of which blood pressure is a symptom. It would probably be found among unexamined lives that the variation is quite great.

Toward the close of his paper Mr. Hunter says that one of the interesting things about the table is that the systolic pressure seems to be as significant as the diastolic pressure.

Recently in our own company we have gathered statistics to determine the relation of the diastolic to the systolic in substandard lives, or lives that have been rejected; that is, those who had very high blood pressures with other impairments. We took only the data from the very best examiners, and we found that almost a constant relation held between the average systolic and the average diastolic; however, the diastolic did not increase quite as rapidly as the systolic. It is almost two-thirds on healthy lives, but when the systolic increases, the diastolic tends to be, as it gets higher and higher, about 59 or 60 per cent of the systolic.

Now, the average diastolic bearing about the same relation to the average systolic, it would be natural to expect that in a large group of lives a high systolic would be just as significant as a high diastolic. However, if you pick out the high systolics and classify them into high diastolics, low diastolics, and average diastolics, it would probably be found that the mortality is high where the diastolic is high, and not so high where the diastolic is average. Hence the statement that the systolic is as significant as the diastolic may not be true in general.

MR. E. G. FASSEL:

Regarding the point that Mr. Mead referred to, saying that there is a tendency toward lower blood-pressure readings, one of our medical directors in a recent conversation pointed out the same situation and offered the same explanation. He said that in the early days the doctors had less experience in blood-pressure readings and would make too much of a business of it with the result that the applicant would become somewhat excited and his blood pressure would go up. There is still a tendency for the pressure to rise at first owing to excitement, but nowadays an experienced doctor ignores probably the first two readings and takes later readings as being more nearly the correct systolic blood pressure. As a result, we get a lower reading than formerly.

MR. J. R. LARUS:

We have been contributing to the general information that is being gathered on blood pressure. To that end we have not made any investigation in our own company.

I was interested, as I came in, in one thing Mr. Mead said. He may have answered it before I got in here but I would like, if he did not, to bring up the question of what a diastolic should be if the systolic is high? In other words, we will say the systolic normally should at a certain age be 130 and the diastolic 90. Now, if the systolic jumps to 150, what is the perfect diastolic to go with it? Should that also jump 10 or 15 of the 20 points that the systolic has jumped, or should it stay low and give you a very high pulse pressure? Did you answer that, Mr. Mead, before I came in?

MR. MEAD: I intimated it would be better to have a lower diastolic.

MR. LARUS: And have the high pulse pressure?

MR. MEAD: And have the high pulse pressure.

MR. LARUS: I understood from talking with a very prominent medical director of one of our large companies that a pulse pressure of over 55 mm might be considered pathological. If the diastolic stays low you get a high pulse pressure. If the diastolic comes up we are told that that is unfortunate. What is the ideal diastolic for a systolic 20 points above the normal?

MR. L. A. ANDERSON:

Since this discussion seems to run somewhat to the question where doctors disagree, I just want to call attention to one little discussion we had at a joint meeting of the Medical Directors Association and the Actuaries in Des Moines.

A doctor read a paper on blood pressure, and I raised the question as to what difference it would make if the test was taken immediately after meals, after drinking coffee, after vigorous exercise, after rest and so forth, and he answered positively that it made no practical difference. I found other doctors who are decidedly of the other opinion.

It seems to me that if we are to rely on their figures we should have somewhat of a uniformity in the time and method of taking the pressure.

MR. R. G. HUNTER:

(Written discussion submitted after the meeting)

What discussion there has been bearing directly on the paper seems to have been brought forth by the statement that "systolic pressure appears to be as significant as diastolic pressure." The conclusion was reached from the examination of tables based upon the cause of death. Generally speaking, I carefully avoid drawing any conclusions from such material, but I was prompted in this case to do so as it ran contrary to the opinion generally expressed by medical directors. Mr. Mead defends this opinion of medical directors, although I cannot quite follow his reasoning.

There seems to be no doubt that emotional excitement, the intake of stimulants, etc., cause the systolic pressure to vary but have little or no influence on the diastolic pressure. Accordingly, the diastolic is a better index of an individual's normal blood pressure. I have even heard medical men go so far as to say that if the diastolic pressure is normal, it makes little or no difference what the systolic is within fairly wide limits.

However, the tables that I have presented do not confirm this opinion. I am not willing to go so far as to say that the evidence that I have presented refutes it, but until further statistical evidence is produced I shall be governed in my judgment of risks by the belief that systolic pressure is as significant as diastolic.

ON CERTAIN APPLICATIONS OF MATHEMATICAL STATISTICS TO ACTUARIAL DATA

By PROFESSOR H. L. RIETZ

MR. H. H. JACKSON:

When Beowulf was about to enter on his terrific combat with Grendel's mother, a kind friend lent him an especially famous sword named Hrunting. It is related, however, that when the actual struggle came the hero made use of an old mighty sword which had been hanging on the wall. Returning victorious to his friends, he remarked in explanation, "I really couldn't do a thing with Hrunting in battle," then added magnanimously, "But it's a good sword." I wonder if this episode does not illustrate the attitude of a great many actuaries regarding the application to actuarial studies of

specialized mathematical statistical methods. In the actual heat of investigation they fall back on the old and mighty weapon referred to in the paper as "common-sense methods of analysis," and politely but firmly refrain from employing the newer statistical technique, although admitting that it is an excellent weapon. Thus, in concluding a discussion of a fairly recent paper of Mr. Elderton on this subject, the President of the British Institute remarked that he thought rough-and-ready, general, and easily understood methods of conducting business were being adopted more and more, but that might be because the actuaries had not yet brought their scientific analysis up to the point at which the necessity of greater refinement had been clearly established.

Possibly there are other reasons why the actuarial student frequently neglects this portion of the statistical field. For one thing, the ideal textbook on the subject has yet to be written. I suppose Yule has almost satisfied this requirement, but Yule deliberately avoids demonstrations through the infinitesimal calculus. Yet the student before going far in the subject is bound to come upon formulas dependent on the calculus, and is too often expected to take them for granted or else to accept a demonstration which his mathematical equipment has not prepared him to follow. The *Handbook of Mathematical Statistics* recently edited by Professor Rietz reveals this difficulty. The individual chapters are very helpful to the student, but he occasionally comes upon undemonstrated formulas which he would be wholly unable to derive for himself and for which he may find it difficult to turn to demonstrations.

A second difficulty is that so many statistical discussions are written by specialists for specialists, all human interest having been sedulously squeezed out before the proofs reached the printer. Probably the American volumes which most completely refute this charge are those of Raymond Pearl. In spite of its title, his *Medical Biometry and Statistics* makes interesting reading for the actuary. Julian Lowell Coolidge's recently published *Introduction to Mathematical Probability*, too, is a charmingly human book. Here again Professor Rietz has come to our assistance, his discussion in the present paper of the skewness of frequency curves for weight in pounds as contrasted with those for height in inches being a delightful example.

A third difficulty results from what might paradoxically be

termed "the inconclusiveness of conclusions." For example, suppose we have, after a somewhat laborious process, arrived at the coefficient of correlation for given variates. We may still be far from a correct interpretation of our result. So standard a work as Brunt's suggests that "it seems doubtful whether any serious meaning can be attached to values of r which are less than .5." Yet Mr. Elderton's paper referred to by Dr. Rietz contains great numbers of coefficients of correlation treated as significant though much smaller than .5. Indeed, I judge that most laborers in this field regard the coefficient as significant even with a pretty small absolute value, provided the value is relatively large compared with its probable error. This fundamental difficulty of interpretation of results cannot, of course, be removed, although the paper under discussion admirably illustrates the care necessary in reaching conclusions.

A fourth difficulty confronts the actuarial student. Except for Elderton's textbook, which he is usually eager to forget as soon as examinations are over if he has not had the misfortune to forget it before entering the examination-room, and for papers very sparsely scattered through the *Journal of the Institute*, he will find almost no applications of modern statistical methods to practical actuarial problems. The present paper is therefore particularly timely and helpful. Indeed, my chief criticism of it is that it modestly omits reference to an earlier paper by the same author, which in an interesting way acquainted the actuarial student with the application of the statistical method to chance fluctuations in mortality and the related problem of the maximum. This paper in *The Record*, II, 1, should be read by all students of the present paper.

These difficulties (and my enumeration of them has barely scratched the surface) being what they are, it is hardly to be anticipated that most actuaries will ever employ the methods widely, in preference to those of "simple subclassification and comparison." But, undoubtedly, actuarial investigations and results will in future be subjected more and more critically by members of the profession to technical statistical tests. As a result, I think mere accumulated data, which at best form but a part of statistical history unlikely to repeat itself, will be, for practical purposes, more and more freely dealt with, while educated judgment and mathematical statistical tests will help to prevent our going too far, or very wrong. An interesting example of this tendency in the hands of an accomplished

statistician has already appeared in the publication for the Institute and faculty of *The Mortality of Annuitants*, 1900–1920.

If the rôle of modern statistical methods is to be that suggested, of checking up results arrived at by more rough-and-ready processes, it may be of some interest to turn the tables by attempting to check certain of Professor Rietz's statistical results by common-sense analysis. Suppose an actuarial student unfamiliar with these statistical methods were asked whether in his judgment there would be fairly close agreement in general between ordinary ages at entry and corresponding mean ages at death; would he not immediately realize that a table of the complete expectation of life on a select basis would readily supply him with the mean age at death, and that even lacking the select basis he could arrive at roughly helpful results by turning to the American Men ultimate expectations? These he would quickly find give mean ages at death which are very nearly a linear function of the age at entry, and a test shows that the select complete expectation still gives the mean age at death as approximately a linear function almost unappreciably steeper. A reference to such graphs as 39 and 41 in Professor Glover's invaluable *United States Life Tables* shows that this condition is not at all peculiar to the American Men Table. It seems likely, then, that the condition illustrated in Table VII and Table VIII of the paper would be found to hold for more extensive data, and that the linear regression of Table VII is not due in any important degree to the rapid growth of the company within the past twenty years. As a minor suggestion it may be pointed out that the conventional scheme of indicating impossible combinations in such tables as VII and VIII by a diagonal ruling might emphasize to the eye the explanation carefully made in the text, that many of the blank spaces can have no significance.

To make a rough-and-ready test of one other conclusion of the paper may be worth while. Indeed, Professor Rietz has already indicated the test and the conclusion in this case. Suppose a practical actuary, widely experienced in the selection of risks, should be approached with this hypothetical question:

Suppose you are supplied with adequate data concerning three features of deceased male policyholders, namely, for each, his age at entry; the age of his father at death, death having preceded the son's entry; the age of the son at death. Suppose you are able by

ingenious statistical devices to eliminate the effect of age at entry upon age at death so that your results can be treated as if a body of entrants, all of a given age at entry with fathers dead at specified ages, supply adequate data for consideration. Assume that no other significant figures are tabulated. You have only paternal age at death instead of age at death of what statisticians call the "mid-parent." All data concerning grandparental and sibling histories are ignored. Furthermore, all accidental or quasi-accidental causes of death, whether among fathers or sons, have been indiscriminately included. Remember, finally, that where the nature of the father's death was such as seemed very likely to influence disadvantageously the son's expectation of life, the entire case was automatically excluded from the data by means of a declination, while no subdivision has been made for build, occupation, personal history, or the other qualities dear to committees on selection. Under these circumstances, would it be possible to trace any significant degree of similarity between the ages at death of the fathers and the ages at death of the sons?

Bearing in mind the inconclusive results of a far more general investigation concerning the influence of family history on mortality, as reported in *Transactions of the Actuarial Society of America*, XXII, 538–40, would any of you hesitate to answer that hypothetical question with an emphatic "No," thereby once more checking the statistical conclusion of the present paper?

For his very patient and illuminating efforts to remove from the path of the actuary some of the difficulties between him and a reasonable understanding of modern statistical methods, every actuarial student is grateful to Professor Rietz, and, indeed, "Aren't we all . . . ."?

PROFESSOR J. W. GLOVER:

(Written discussion presented by Mr. C. F. Cross)

Within the present century a transformation has been taking place before our eyes which has a most significant bearing upon our subject. I refer to the tremendous strides which have been made in the analytical theory of statistics. Before the advent of this child in our midst, we might have flattered ourselves with the pleasing thought that most of the problems in actuarial analysis, or at least our point of view toward them, had been settled. Happily the new

era does not mean that they are now unsettled—it means a deeper and more satisfying insight into what we thought we quite fully understood before.

The theorem of Bernouilli is not mentioned in the first edition (1887) of the *Text Book of the Institute of Actuaries*, nor in any of its reprints, indeed, in the recent revision by Spurgeon and *Calculus and Probability* by Henry, no slightest hint is given of the fine distinctions of modern analysis between a priori probability and statistical relative frequency. This is due to the meta-empirical trend of thought in the English school wherein the postulated probability was lost to sight in the maze of ghostlike forms pointing the way to a composite limit. But even this limit was not taken seriously—it was replaced by the empirical facts, a step quite justified, perhaps, but through this emphasis serious analysis of the deviation between empirical fact and postulated limit received almost no attention, although it now appears that it is at the core of the whole doctrine of chance.

After all, we obtain our first ideas on probability from the text-books, and if they do not show the light, only a few will have the time or the curiosity to wander beyond their pages, and in one way or another absorb different and perhaps new points of view. Perhaps Dr. Rietz has done more than anyone else in this country to call the attention of mathematicians to the genuine importance of mathematical statistics, and it is fitting that he should be the first one to present some ideas concerning the applications of this type of analysis to actuarial data. He has done well, I believe, to select illustrations under the headings of random sampling, frequency curves, and correlation. I wonder if actuaries have not been prone to spend too much time in measuring fluctuations in terms of smoothness, as indicated by the run of some order of differences, rather than from the point of view of the deviations lying within prescribed limits of random sampling. Pearson's test of "Goodness of Fit" applied to the actual and adjusted deaths seems much more to the point. Here we reach a definite conclusion that on the basis of random sampling about 45 cases in 100 trials on the average would give deviations as great or greater than those actually observed. This is much more satisfactory than knowing that the third differences are small and frequently change sign.

In passing I would like to refer to several papers which may

prove useful in applying the test. The first is "Goodness of Fit in Statistics and Physics," by Pearson, in *Biometrika*, II, 239–61; "On Criteria for the Existence of Differential Death Rates," by Pearson and Tocher, in *ibid.*, pages 159–84; "Some Experiments to Test the Theory of Goodness of Fit," by Brownlee, in the *Journal of the Royal Statistical Society*, LXXXVII (1924), 76–82.

The stochastic or probability concept of actuarial functions and the relation of statistical relative frequency thereto should be carefully thought out and the values of such functions should be studied in connection with their probable limits of variation. Reversions, annuities, in fact most of the functions with which we have to deal are of this character. It would be difficult to find better illustrations of mathematical expectation and averages than those furnished by single premiums, and their full treatment, including probable errors, or probability of fluctuations within prescribed limits.

With regard to frequency curves I think Elderton has placed the work of Pearson fairly well before actuarial readers; also a number of papers have been published in various actuarial journals based on the application of the method of moments to frequency curves to actuarial data. The work of Charlier and others in the Scandinavian school is now available, not only in the original memoirs of Charlier in the *Proceedings of the Lund Astronomical Observatory*, but also in the *Skandinavisk Aktuarie-Tidskrift* (*Scandinavian Actuarial Journal*), 1918–24, now in its eighth year. Many of these papers are in English, and I most urgently commend this *Journal* to those who have the time and inclination to follow up the most recent developments in statistical analysis.

It is under the heading of correlation that Dr. Rietz has pointed out the most interesting applications. This theory occupies a broad field; it attempts to measure in a quantitative way the dependence or degree of association existing between two variates. The limits between which it operates are from no dependence (in the probability sense) to absolute functional relationship. This embraces a field potentially rich in important problems and with data already accumulated and ready to be analyzed. I have no doubt it is destined to become a most valuable adjunct to what Dr. Rietz so tactfully describes as the "educated judgment" of the actuary. Undoubtedly the judgment which comes with long experience is of first importance, but I advise the young man who is ambitious to

get the last drop out of his data to leave no stone unturned to acquire the breadth of view which this fascinating subject of mathematical statistics will surely give him.

MR. E. A. PORTER:

There is really a moot question as to which is the least reliable statistical worker—the man who has mastered the mathematical methods and applies them mechanically to any and every kind of data, or the so-called "common-sense worker" who believes, for some reason, that he can get results without an adequate kit of tools. This paper should be of substantial help in combating any tendency we may have in either of these directions, and in confirming us in the proper ideal of the use of good common sense, complemented with thorough equipment to do the best and most effective work.

The most common statistical fault of our profession is a tendency to assume homogeneity where such, if it exists, certainly calls for demonstration. Our largest statistical activity is the investigation of mortality among various classes of insured lives. As ordinarily carried out, the largest possible group is obtained showing the certain kind and degree of impairment about which information is needed; the death-rate in this group is observed over a period of years and compared with the death-rate in another group presumed to be made up of normal lives. Although there have been noteworthy exceptions, the general tendency has been to assume the difference in death-rate observed to be largely or entirely due to the impairment under investigation. Great expense and labor have been incurred in the effort to eliminate error arising from accidental variations by the use of large groups, but relatively little attention has been paid to securing the extremely high degree of homogeneity which should exist in them if the comparison is to be on a sound basis.

I would like to venture the opinion that the mortality experience of a small group, the individuals composing which having been so picked as to make up a homogeneous group, will be a more reliable basis for estimating the extra mortality due to a certain kind and degree of impairment common to the members than the experience of a far larger group, which merely constitutes the run of acceptances showing such impairment. Our problem in the study

of impaired lives is essentially one of partial correlation, and we have, to a very large extent, been treating it as simple correlation.

To be specific—suppose we examine the mortality experience among the run of standard office acceptances showing blood pressure 122.5 to 127.4 and 142.5 to 147.4. We are already wise enough to subdivide such experience by age, and we know definitely that height and weight are correlated with the resulting mortality rate. Considering these taken care of, we should still find, I believe, a marked difference in the two groups as to occupations, habits, family histories, nationalities, etc. It would seem better, instead of employing the entire experience in bulk, to handpick a small group to represent each blood-pressure range, subjecting each individual included to the same requirements in order to insure homogeneity.

Now if we begin to follow such a plan as I have here suggested, we shall have need of mathematical statistical theory to test our conclusions, inasmuch as the smaller groups employed will necessitate close guard against the results of chance fluctuations.

Furthermore, even though we adhere to all our present ways and practices, surely we can in much of our work safely reduce the volume of material to be used, and consequently the cost of research. Mathematical statistics will furnish a guide for planning investigations by random sampling, and will fix fairly reliable minimum amounts of data to be obtained.

We have one problem which falls somewhat in the field of random-sampling theory which is of predominant importance. I refer to the determination of limits of company retention. Too large limits place a company in danger of disaster from mortality fluctuation, too low limits mean a constant drain on surplus to carry reinsurance on risks. This problem involves so many different elements that it is of course incapable of a strictly mathematical treatment. However, the literature of the subject, although extensive, still fails to give acceptable guidance, and it remains a very inviting field to some actuary possessed of rare common sense and high mathematical attainment.

The mathematico-statistical method in combination with biological hypotheses may eventually lighten very materially our statistical burdens. The Swedish Mortality Investigation of 1921[1] is an example of an investigation of mortality in classes which was

[1] See *TASA*, XXII, 209.

made from deaths alone, no exposures being calculated. Mr. Arne Fisher[1] has also produced some excellent mortality tables by the use of death records only, without computation of the exposed to risk. If there are underlying biological laws governing in gross the life and death of humanity, surely these laws when known will largely obviate the necessity of the compilation of vast masses of statistics. Perhaps even the present dimly apprehended relationships may do stalwart service, if properly handled.

We owe a great deal to Mr. Rietz for his illuminating and interesting paper. Especially interesting is his discovery of an average duration from issue to death which appears to be the same for all ages. If confirmed by larger experiences, this relationship might prove of value in theoretical work on mortality and lapse rates. The final conclusion of his investigation of correlation between age at death of father and son is not so very surprising. For many years, considerations of heredity, per se, have been receiving less and less consideration in practical underwriting. It is pleasant to note that common sense, without statistics, finally got around to about the same conclusion as was obtained by highly perfected analysis of statistics.

I certainly trust that this paper will be the forerunner of many others along the same line.

DR. H. L. RIETZ:

It is doubtful whether I can add anything to the very able discussion that has been given of this paper. But I do not wish to let the opportunity pass without saying that I appreciate very greatly the discussions and that I feel very much encouraged from what has been said.

I find myself in agreement with practically everything in the discussion.

I acknowledge again that this paper would not have been written except for the suggestion by our President that I write a paper involving the methods of mathematical statistics. I was not very sure after the paper was written and read that he felt convinced that his suggestion was worth while, but I am sure he feels after this discussion that his suggestion was very much worth while. I am encouraged to think that the paper has set some of us to thinking

[1] See *Frequency Curves*, 1922.

about the possibility of supplementing our methods of common-sense judgment by more technical tools for doing certain things.

There were two or three points brought out in discussion on which I might possibly comment briefly, although I think it would require a precision of statement that I cannot perhaps command off-hand.

The first point to which I wish to direct attention is one made by Mr. Jackson, in pointing out the differences of opinion as to the meaning of a correlation coefficient, say of .5 or smaller. He quotes a capable mathematician, Mr. Brunt, as expressing doubt as to giving any serious meaning to a correlation coefficient smaller than .5.

I think Mr. Jackson will agree with me that the meaning to be attached to such small correlations depends upon the purpose we have in mind. If the purpose is to establish the existence or non-existence of correlations, then a very much smaller correlation than .5 may be significant. In this case the comparison of the correlation coefficient should be with its probable error—its value is how many times that of its probable error? That is to say, our purpose in this case would be simply expressed by the question: Does correlation exist or is there no significant correlation?

Our purpose next might be to predict something about an individual corresponding to an assigned value. For example, if we wish to predict something about the age at which an individual son would die because we have found that the father had died at age twenty-five, I dare say that a correlation coefficient of .5 would not give a degree of accuracy that is worth while. The standard error in the son's age at death would be reduced but little on account of that small correlation.

On the other hand, if we had a large number of men, let us say that we are going to take one hundred men, all of whose fathers died at age twenty-five, then let us ask the question: What would be the predicted average age at which those sons would die? You may then tell a great deal with a correlation coefficient of .5, and tell it with a moderate degree of accuracy which for many purposes would give real satisfaction in prediction.

So here are three cases set up in which our purposes are different, and depending on the purpose we have in mind, the correlation coefficient of .5 may or may not be of any significance.

I was very greatly interested in the discussion of homogeneity

brought out by Mr. Porter. There is an important thought in what he said. It is the most difficult thing I know about in statistics to say what we mean by a homogeneous statistical group. There is nothing to be encouraged more in statistical analysis than the very thing that Mr. Porter brings out—subclassification and common-sense methods applied to these subclassifications.

Unfortunately, I did not feel I had the space in this paper to bring out the technical things that are associated with certain kinds of a subclassification. The thing I have in mind is found in the Lexis theory of dispersion. You will find a chapter on that subject in the *Handbook on Mathematical Statistics* that I edited, and which was published about a year ago. I refer freely to that book because it brings the authors no royalties at all. The royalties, if there are any, go to the National Research Council.

Hence, I venture to say, if you are interested in subclassifying certain data so as to detect in what respects they are not homogeneous for a particular purpose, that you may be materially assisted on the technical side by a study of the Lexis theory of dispersion. To indicate the high regard for the Lexis theory in certain quarters, let me say that Charlier, the great statistician of Sweden, said in his recent book (1920)[1] that the Lexis theory which was brought out in 1877 was the first essential advance in statistics since the day of Laplace, about 1820. J. M. Keynes in his critical *Treatise on Probability*, published in 1921, makes a somewhat similar statement. He praises the Lexis theory, and he seems to me to be too critical toward the Karl Pearson theory. His main point is practically in line with the matter to which Mr. Porter refers in respect to subclassification of data to secure a better type of statistical analysis.

MR. P. H. EVANS:

Dr. Rietz has referred to the fact that he wrote this paper because I suggested that a paper of the sort should be written. He has said that he had some doubt at one time after the paper was written and read as to whether I still felt that way. But I want to assure the Doctor that some of my friends think that my loyalty to my own ideas amounts almost to stubbornness, that I never had any doubt at all at any time as to the timeliness and value of this paper. I hope that the subject will be given further attention by members of the Institute.

[1] *Vorlesungen über Mathematische Statistik.*

A STUDY OF THE EFFECT OF USING STANDARD RESERVES AND SURRENDER VALUES FOR SUBSTANDARD POLICIES

BY F. S. WITHINGTON

MR. L. A. ANDERSON:

I welcome the present paper because I feel it is timely, interesting, and useful. It is timely because practically all companies today are interested more or less in substandard business. It is interesting because the comparisons have been made in a way not heretofore found in actuarial literature, and I believe it is useful because it gives us the actual amount of excess or deficiency in reserves and surrender values under certain conditions.

The spirit of the paper is scientific—not dogmatic. It does not pretend to be orthodox except in so far as the author seems to assume that the multiple q_x is the proper basis on which to build substandard rates and values. There is no question but that this is a proper basis for some classes of risks, but to assume that it is a suitable basis for all classes of substandard risks is hardly warranted. We have only to look about us for example. Where, for instance, the extra risk is in all probability fairly constant from age to age at least within certain age groups, it would not be fair to charge the increasing cost resulting from the multiple q_x. In other cases the extra hazard is decreasing like that of tuberculosis after a certain age. In other cases and age groups the extra risk is progressive even beyond the progressive increase of the q_x or multiple q_x. To assume, therefore, that the extra cost of carrying the risk is in all cases increasing along the line of a multiple q_x is a rather far-fetched assumption. As Ruskin said: "The work of science is to substitute facts for appearances and demonstrations for impressions." I feel that the author has complied pretty well with this rule as far as he has gone. The paper, therefore, is suggestive rather than comprehensive although it does cover a wide range of examples given in the voluminous tables that he has given us. This wide range of examples gives us an excellent check on conclusions that might easily be drawn from less extensive investigations. The author sets out to show that standard values should not be used in policies covering substandard lives, and this I feel he has accom-

plished very thoroughly. The tables are arranged in two groups, namely, from I to VII, giving a comparison of standard values with substandard values at the true age at issue; and second, Tables VIII-XIV, giving the comparison between the standard values at the rated-up ages and the substandard at the true ages, based, of course, on a multiple q_x table.

The tables are valuable in showing that at certain ages and plans the plus or minus sign will run through a long series in which conclusions might easily be drawn on a superficial investigation that they will hold throughout, but on examination of the tables we find that the values change rather suddenly to the opposite sign in some cases where I, at least, did not expect the sign to change.

Another striking difference is that the paid-up values show plus signs while extended insurance shows minus signs, and while the cash value shows minus signs the pure endowment shows plus signs.

Again, there is a rather wide variety of differences when we come to the higher ages. For example, on page 282, "Ten-Year Endowment," age fifty, 200 per cent q_x shows plus signs throughout, while the 225 per cent q_x shows a plus sign the second year, minus signs the third and fifth, and a plus sign the ninth. Similar curvatures and fluctuations appear in the figures even if the sign does not change. The change of signs, however, is very helpful in bringing to our attention those peculiar fluctuations.

I feel that an extensive review of the figures and comparisons would be of very little use at this time. I would recommend that the student refer to the tables as printed in the *Record*.

The company with which I am connected has been using Mr. Withington's tables as an auxiliary source of information in attempting to fix upon rates for substandard risks. So far we have only a small amount of rated-up business, but like most companies I take it we are practically forced to consider some of it in order to take care of our agents.

I believe it can be said without fear of contradiction that the best fundamental basis for substandard rating is separate mortality tables based upon actual experience in the various classes to be rated. As examples I would mention the tropical, subtropical, standard industrial, and substandard industrial tables, but the adoption of special tables and all the necessary derived tables for working purposes would be too laborious and expensive.

Referring again to the company that I am connected with, we have made rather free use of the flat extra premium rather than rating up the age. I am fully aware that a theoretical danger exists in that because when a policy is lapsed we have to give extended and paid-up insurance on a standard reserve basis. I feel that this danger is more theoretical than practical, however, because in all my connection with and study of the life insurance business I have yet to find a single case where a substandard or impaired life deliberately lapsed a policy into extended insurance on the assumption that it would outlast him. On the contrary, I have seen and heard of a considerable number of cases where persons in the advanced stages of pulmonary tuberculosis have asked to reinstate a policy that has been running under extended insurance and still has as much as nineteen years to run. To assume that a person in the advanced stages of pulmonary tuberculosis would live more than nineteen years is, of course, absurd.

In other cases that have come to my observation, the policyholder and his relatives have literally scratched and scraped to raise the money to pay the premiums because they knew the insured was in poor health and might have allowed the policy to lapse into extended insurance with a reasonable certainty that the insurance would outlast the insured.

I would suggest that a study be made of surrender values based on special substandard mortality tables with those of the standard 100 per cent q_x table.

MR. ARTHUR COBURN:

Mr. Withington's paper gives us a basis for comparison of the three more common methods of issuing substandard insurance.

The paper calls our attention to the rather high reserves called for by the advance-in-age method. I believe that, especially for a company with a comparatively limited surplus, they should hesitate, before embarking on substandard business, in adopting the advance-in-age method of issuing substandard business.

What are the proper reserves to hold for substandard business? I do not believe we know yet the proper answer to that question. I do not believe we will know the proper answer to that question until we have extensive mortality tables made up on the basis of substandard experiences.

Meanwhile, the companies are interested in practical and simple methods of issuing substandard business. For that reason many of them have adopted the system of charging extra premiums and allowing standard cash values.

The paper shows that it may be unwise to allow extended term insurance under that system. Some of the companies have been eliminating extended insurance where the mortality rating was more than 150 per cent. I think this paper shows that this is a wise step to take.

MR. F. B. MEAD:

There are at the present time three principal methods of reserving in this country under substandard business. The methods of rating practically coincide.

One of the valuation methods which seems to me to be out of the question, particularly for small companies, is to use the rated-up-in-age method, because the reserves are unquestionably too high. For instance, on a 150 per cent risk at age thirty-five, the mortality provided by the rating-up-in-age method in the earlier years is only about 115 per cent. The company does not get enough provision for mortality and puts too much in the reserve fund.

As a consequence, some companies have adopted standard reserves. Other companies have adopted the system of multiple tables, based largely on the American Experience Table. That is an intermediate method and coincides closely with the method of assessing the risks.

Recently, Mr. Cammack has introduced in his company a compromise method which he described today and which is intended to approximate closely the multiple-table system. By that method he employs a compromise between the extra-premium method and the rated-up-in-age method, and it is a very ingenious artifice with many advantages in connection with the preparation of the policies, supplying the agents with the values and the ease in application in the Actuarial work of valuation.

The smaller companies, of course, are interested in maintaining their surpluses in these times of strong competition, and the method of charging standard reserves has many advantages. However, the extended insurance and the paid-up values are too high, but if a company has the automatic premium-loan feature, that question

may be entirely eliminated for the reason that very few people apply for extended insurance when the policies provide for automatic-premium loans.

The disadvantage, I think, of Mr. Cammack's method is that it still further increases the reserves over the multiple-table basis; and it has some of the disadvantages of the standard reserves in that the extended insurances are increased over the multiple-table method; but it might be that the advantages of the ease of administration, both in the field and at the home office, may make up for the disadvantages of the increased reserves and the increased extended-insurance values. It is a method that is well worth while looking into. I believe that the rated-up-in-age method is to be avoided.

Mr. Anderson has referred to the fact that while a great many companies use the multiple-table method, yet that does not apply to all types of substandard risks. I might say that that criticism is true of standard policies. We have just as great a variation there in the types of risks, almost as great, as we have with substandard insurance. For instance, possibly we have 100 per cent constant risks. We have risks which are 80 per cent of the table, and we have many risks which are 115 or 120 per cent of the table, some types of mechanics, and numerous other types that might be mentioned.

And again, we have the overweight type where the mortality might be 80 per cent of the American Men Table in the earlier years, and that mortality gradually increase until it became 130 or 140 per cent at the advanced age, and the average would be about 100 per cent. There might be some further extremes where the mortality over a long period of years would be 120 per cent.

And then we have the light-weight and the consumptive family-history type where the mortality in the earlier years might be 120 or 130 per cent of the standard and in later life be 80 or 70 per cent of the standard.

So the same criticism applies to standard insurance as to substandard insurance, and it seems to me that the criticism is not a just one, and that on the basis of the law of averages, just as we apply that law in connection with standard insurance, it is just as consistent to use the multiple-table system.

Furthermore, the advantage of the multiple-table system is that it fits in with the matter of selection. For the most part, risks

are based on assessments, 125, 150, 175, 200 per cent, and so forth, and as we assess the risks and grade them that way, and our mortality statistics are gotten out on that basis and viewed from that standard and they probably can be classed that way, it seems consistent that we should compute our premiums and reserves on that or similar bases.

In our own company, in last June we got out our entire substandard experience collectively up to the anniversaries in 1923, and all of our risks have been assessed on the percentage basis. The experiences I am speaking of are those which have been assessed and rated on the percentage basis, and the mortality was 103 per cent of the American Men Multiple Select, and the various groupings, 125, 150, 175, and 200 per cent, show the same tendencies, although, of course, in the smaller groups there were some variations. Some were below 100 per cent of our expected and some were above. But the whole, collectively, were 103. As our mortality was still more favorable last year I think the mortality is even closer at the present time.

So there seems to be some evidence on every hand that the best method we have at the present time is the percentage method.

MR. C. H. BECKETT:

Simple methods of classifying underaverage lives for life insurance are desirable, but a tendency to go too far in this direction should be resisted. We cannot hold that all classes for which the probable mortality is such that standard insurance cannot be issued should be treated in the same way without ignoring a fundamental factor, viz., the incidence of mortality.

When we say, as has been suggested here, that we will reduce our classifications to four groups and issue policies in each group by a combination of advance-in-age and flat extra premium, we have improved the now generally discredited advance-in-age method but have not provided either premiums or policy values such as would be arrived at if more careful attention were given to the mortality element. This subject is now so well understood that it is not necessary to go into details.

My plea is for greater care in classifications of risks at some possible sacrifice of ease in office detail. As a matter of fact, improvements in office methods by the use of modern machinery have

largely eliminated the old idea of extra cost on account of additions to the number of groups.

I am convinced that the best results from field men will be achieved whenever the agents are convinced that the company is making a real effort to grant insurance to their applicants on plans and at premium rates which appeal to them from a common-sense viewpoint as being reasonable and fair.

The actuary's attitude toward this entire subject should be favorable to a more careful classification of applicants into suitable groups, bearing in mind all the while that too great refinement is neither necessary nor desirable, but that such differentiation be made as will give just and considerate treatment to every type of extra-mortality group. So-called office and agency difficulties should be subordinated since they are easily overcome.

MR. C. O. SHEPHERD:

(Written discussion presented by the secretary)

Mr. Withington has made a comparison of cash surrender values, paid-up life values, and extended-term insurance values based on what he calls, for convenience, "true substandard reserves," with similar values based on (1) standard reserves and (2) standard reserves for an equivalent rated-up age. This comparison is made for seven different plans and at ages twenty, thirty-five, fifty, and sixty.

Mr. Withington uses the expression "true reserves" to signify reserves computed on the multiple q_x mortality table, explaining that the term is used as a matter of convenience as he realizes that there are sincere differences of opinion as to what constitute true values. Elsewhere in his paper he shows his own convictions in these words:

> I believe these differences are sufficiently large to indicate clearly that the use of standard values for substandard policies should be regarded as inadmissible not only on the basis of the American Experience Table of Mortality but on the basis of any other general experience covering standard lives in any company or group of companies.

The value of any measurement depends on the measuring rod, and the vital question involved in this paper is the suitability of a multiple q_x mortality table for measuring substandard mortality. I

have never been satisfied that it was suitable. I might have been less disposed to question this system if I had not felt that it was a development along the lines of least resistance. It is such a comfortable way of disposing of the many troublesome questions that arise. Once a single assumption is made, all else follows mathematically. No other method yet proposed seems to satisfy so nearly the natural desire for this "mathematical" accuracy.

Substandard premium rates came before substandard business, and they had to be determined many years before we had dependable knowledge of the mortality which the business would show. It was convenient and natural to determine premium rates on the multiple q_x basis, and with reasonable loading margins to cover uncertainties and contingencies they were accurate enough. Adequacy of rates was all important in the experimental stage—from which we have not yet graduated. It was natural but not necessary, and (as I believe) not wise, to adhere rigidly to the same assumption when reserves had to be computed. The assumption having been made for one purpose, it was hard, perhaps especially hard, for the mathematical mind, to see any reason why the same assumption should not be adopted for another purpose. And yet I believe the process had more of the form than the substance of a scientific development. It mattered little whether the incidence of mortality assumed was correct so long as rates were being computed, because the error introduced could not be great and it was legitimate to use the assumption even though we could not believe that it closely represented the facts; but when it came to the determination of reserves, the incidence of mortality was a matter of much greater importance and on account of our non-forfeiture laws a much more serious matter.

Mr. Withington cites an instance in which cash values based on true reserves exceed cash values based on standard reserve by $90 per thousand as an argument in favor of his plan. I cite it as one of the dangers of his plan. If this excess is not justified by fact, and I do not believe it is, it shows how we may increase the cost of substandard insurance by paying unnecessarily high surrender values. This excess is found at the end of the nineteenth year on the "Twenty-Payment Life" at age thirty-five for the 200 per cent group. Is it reasonable to assume that we need a reserve of nearly $100 extra on a group of lives about which our knowledge is limited

to the fact that they were known to be impaired twenty years before?

I should take this situation less seriously perhaps if I did not believe that we had already erred in allowing high surrender values in our standard business. We have been both led and forced into this situation by the statutes of our various states basing values on the American Experience Table with $3\frac{1}{2}$ per cent interest. Mr. Withington mentions the "apparent tendency toward greater liberality" in respect to surrender values. I hope that this is "apparent" or at the most temporary. With mortality improving as rapidly as it is, the possibilities of reducing the cost of insurance are encouraging, but the same cannot be said for the cost of high surrender values, and I believe that reduction in premium rates cannot go much farther without a reduction in surrender values.

The old principle that there is no such thing as the reserves on an individual policy does not often find a place today in our actuarial reasoning. The counter-principle of individual equities constantly presses in on us from all sides. The latter principle has been carried far enough, and there is less need for its elaboration and refinement than there is for a reversion to broad, underlying truths.

Undoubtedly there are some, if not many, types of impairments resulting in relatively heavy mortality in early policy years. Policyholders suffering from such impairments should receive lower, not higher non-forfeiture values than standard policies. In my judgment and to my knowledge, no one has yet made out a case to show that any other system of surrender values is superior to standard values for substandard policies. The great purpose of non-forfeiture provisions is to avoid serious forfeitures, and it is not a subject that lends itself to great refinement.

There are so many factors affecting substandard mortality on insured lives that by the time statistical experience has accumulated it is out of date. Tuberculosis does not affect substandard mortality today as it once did. Glycosuria may have a different effect in the future from that which it has had in the past. Even overweight may have a different significance now that its evils are known and the blood-pressure test is commonly used. We are dealing with a group of classes that are likely to change rapidly. Then, too, there is the effect produced by a change in the types of impairments which the company will accept and changes in the classification

of different types due to new and improved tests. All these things point to the indesirability of too great refinements in the matter of surrender values.

Substandard policies providing for values based on standard mortality tables do not prevent a company from carrying contingent reserves for additional security. It is one thing to accumulate funds to swell individual equities subject to withdrawal and another thing to accumulate funds for additional protection against unfavorable contingencies. The most serious objection to the higher reserves that Mr. Withington proposes is the higher surrender values which the law requires to go with them. Substandard business is being shopped and reshopped, and the right to surrender may involve a serious selection against the company, particularly if we pay higher surrender values than the facts warrant. We hurt the company and ultimately the substandard policyholder because the cost of his insurance will go up. If, on the other hand, in our effort to deal justly we pay slightly less than the facts warrant, little or no harm will be done and at worst we will be exercising a correcting influence on a practice that has already been carried to an extreme.

MR. F. S. WITHINGTON:

I am certainly gratified at the amount of discussion which this paper has called forth, but I want to say that it was not my purpose, in writing the paper, to defend the multiple or any system of valuation. It was written to demonstrate what I regarded as being the wrong method, in using standard surrender values for substandard policies, on any method or basis.

The discussion has taken a rather wider range than I expected, but I suppose this is natural and inevitable.

I want to state, however, in answer to Mr. Shepherd particularly, that it was not my purpose to advocate larger surrender values for substandard policies. In fact, I think my paper shows conclusively that on the basis of the comparison used therein, in a great many cases, the use of substandard reserves results in smaller surrender values than those based on standard reserves, particularly in the case of endowment policies.

I repeat that the primary purpose of the paper was to point out the inconsistency of the use of standard reserves for substandard policies.

There are numerous fluctuations, mentioned by Mr. Anderson, in the differences between the reserves on the two bases, and some remarkable inconsistencies in these differences, which I traced down; and in thus tracing down the differences for each year between the five-year intervals, it was shown that there were some very sharp curves in these differences, but these are not because of error. They are due to abrupt variations in the incidence of difference; also, no doubt, largely to the two antagonistic elements of term insurance and pure endowment, in the various forms of policies.

I am gratified that this paper has also called forth the discussion of the general matter of basis of valuation of substandard business, in which I feel great interest.

While I did not expect to go into the subject of the relative importance or advisability of the adoption of the multiple system, I will say that I recognize, of course, that no system will exactly fit all cases, but I advocate the general adoption, eventually, of the multiple system, principally in order to simplify the handling of substandard business. I realize that there are a great many cases where a flat extra premium seems more preferable than the multiple system, but it seems to me that we must get to a more simple way of handling this business if we are going to continue to do it successfully and practically.

I tried to make the paper as comprehensive as possible, that is, to cover as wide a field as possible in order to see what the trend of difference was between standard and substandard reserves. The paper was intended to be informative rather than argumentative.

INFORMAL DISCUSSION

NON-MEDICAL BUSINESS

A. Solicitation—what agency regulations should be adopted?
B. Underwriting—to what extent is home-office medical supervision necessary?
C. Valuation, etc.—should non-medical business be segregated for gain-and-loss accounting, dividend allotment, and expense account?
D. What is the probable future of non-medical business in the United States?

MR. J. G. PARKER:

Mr. President, there is not very much which I can say in connection with the non-medical business which has not been said at many of the previous meetings of the Institute.

Under the heading of "solicitation," the question is asked, "What agency regulations should be adopted?" I am not certain of this question. I think, however, that one important regulation is to insist upon all business within the defined limits governing non-medical business being submitted on a non-medical basis. The agent should not have the option of submitting the risk medically or non-medically according to the inclination he may have at the time.

In our own company we have modified this rule to some extent in that we give authority to our agents to have certain specified cases examined without first referring the case to the head office. Among other things specified on our non-medical blank a personal history of rheumatism or pleurisy given by the applicant to the agent would give the agent authority to have the risk examined.

Many companies have insisted that all of their agents do a non-medical business, making no exception. In the case of my own company we have made one exception. This is a city branch, the agents in which were averse to the proposition. They, accordingly, have no option but submit all their business on the medical basis.

In regard to the underwriting practice at the head office, a non-medical case goes through exactly the same routine as a medical case. On account of the primary selection of the risk on the part of

the agent most of the cases are dealt with by the lay underwriters, very few of the non-medical cases having to be submitted to the medical referee.

MR. J. F. LITTLE:

I imagine the idea in the mind of the person who suggested this subject was rather non-medical business in the United States than non-medical business in Canada.

The conditions in the two countries are in many respects widely different. In the case of Canada we have a population who are living under laws that are interpreted much more strictly, in the matter of the protection afforded life insurance companies in case of fraudulent misrepresentation, than in the United States. As a consequence, the state of mind of an applicant, in filling out and signing an application, as to the responsibility that rests on him to give precise facts is not similar in the two countries.

It must therefore be borne in mind, in doing non-medical business in the States, that we cannot assume because of the—I think I may say—entire success in Canada that we shall be equally successful in handling such business in the States. We may very likely find that there is a definitely worse experience.

In regard to the solicitation, I do not propose to say anything, nor as to the underwriting, except that I imagine in all companies the medical department will often be consulted as to indications in the application of possible unsatisfactory conditions.

What I am more interested in particularly is Section C, which reads: "Valuation, etc.—should non-medical business be segregated for gain-and-loss accounting, dividend allotment, and expense account?"

That all depends on what is meant by "segregating." If it is proposed to keep an entirely separate account, and entirely separate funds from the rest of the company's business in the same way, for instance, as we separate our industrial and ordinary business, the result would be simply that the medical fees saved would be thrown away and that the business would undoubtedy be more costly to carry than medically examined business.

There is absolutely no necessity for segregating the business in this way, for keeping the premium income separate, the interest income separate, and the reserves separate, and having a separate gain-and-loss exhibit for these items. It would be a serious waste

of money, and it would not be in the interests of the policyholders, either in the medical or non-medical section.

It is possible for any company to analyze its non-medical business so far as the one point of importance is concerned, that is, the mortality experience. If it finds that the mortality on the non-medical business is in excess of the medical business to a degree greater than is offset by the savings in the medical fees, and if it finds that that excess is significant, is enough to make it worth while distinguishing between the two, then I think the company should do one of two things: Either modify its non-medical underwriting rules, which might go as far as ceasing to write non-medical business altogether; or give different dividends on the two classes of business, which would probably amount to the same thing, as it would tend to abolish non-medical business if it were known that the dividends thereunder would be smaller.

Practically, however, we may easily find that where non-medical business is written, the cost of a full separation between medical and non-medical may be so great that the class that showed the higher profits—let us assume it is the medical class—may have dividends no greater than without the separation—may even have smaller dividends—while the non-medical class would have still smaller dividends. In other words, if a common dividend plan allotted $5.00 each to a non-medical policy and to a medical policy, and it was decided that a separation of accounts would be more just, if such separation disclosed that the medical dividend should be 10 per cent greater than the non-medical but $1.00 of the $10.00 went in extra expenses, the result would be that the medical policy's dividend was *reduced* to $4.72 and non-medical to $4.28—a result unsatisfactory to both.

There is a point beyond which I think precision in calculations should not go. It is the point where the expense of making the detailed calculations is certain to be so great that no policyholder can secure any advantage but some or all will suffer a loss.

The necessary facts as to the relative earning-power of medical and non-medical business can be ascertained by a simple analysis of the mortality in the two sections. If that analysis indicates that the non-medical business, after taking into account the savings in medical fees, is worse than the medical, the company may readily make a proper adjustment to meet the situation. But, so long as the

difference is very small, the company is justified in handling the business as a whole in the most economical fashion, and thus give the best service to the community.

After all, the thing we should consider first, to my mind, is not so much the convenience of the company or some theoretical idea as to strict justice as this: What is the plan, what is the method of carrying business that is really of most service to the community as a whole? Now, the non-medical business will have an advantage in one direction, it will occasionally insure a man not insurable on the medical basis. From the community standpoint that is an advantage, and if in the final result it does not involve any financial loss to the medically examined policyholders of the company, then the non-medical business is justified in every way. If there were a financial loss, but so small as to be inconsiderable, even then the better service to the community may justify a continuance of the plan, but if we arrive at the point where there is a serious financial loss, then of course we must take account of it and make such arrangements as will protect the interests of medically examined policyholders.

MR. E. A. PORTER:

I just want to call attention to the remarkable reactions that the agency men have to this non-medical proposition. We have sounded out our agency staff quite extensively on this question in various ways with some surprising results.

We find that the most trustworthy and the most successful type of agent often does not want to do non-medical business. He has a variety of reasons.

In the first place, he fears that the set limit to non-medical business may incline him to write cases for $2500, coming just within the limit, when if the medical examination were required in all cases he might be able to write from the beginning a $5,000 application.

Furthermore, the lack of a medical examination makes it impossible to order out an extra policy without application and to endeavor to make delivery and obtain an application for it.

A still further idea that some of the biggest men have stressed on the matter of non-medical is the well-known fact that the life insurance agent ordinarily is not selling the idea of life insurance but inducing men to take it out now. Many of the prospects are taking hundreds of thousands in the remote future and perfectly willing

to say so, but the agent's job is mainly to get them to take some out today instead of next year or the year after next. One of the big levers which the agent has had, one of the big talking points, has been the possibility of becoming uninsurable. A great many, as I say, of the bigger and more successful types of agents have a general fear of the growth of non-medical business, that it may to a certain extent weaken them in this particular way.

I have also heard some very disturbing reports of various kinds through subterranean channels that certain agents not wholly trustworthy have said, "Well, it doesn't make much difference. I know so and so who represents another company that is doing a non-medical business and if I have any cases that I need to put over there, why I can put them all right." Which indicates very strongly that in the start, at least, there is going to be a rather large number of poor risks presented under the non-medical plan.

As I understand it, one of our larger companies took a vote of its most successful agents and found them overwhelmingly against the adoption of the non-medical plan.

I also understand that one of the smaller companies of the West which has begun the plan has found a remarkable number of M.I.B. impairments against the applicants and has had something like 50 per cent of the non-medical applicants examined, which I regard as virtually abandoning the non-medical principle.

MR. J. R. LARUS:

I have been very much interested in what Mr. Porter has said because the company with which I am connected, the Phoenix Mutual, last fall presented the question of non-medical insurance to its agency force, and the response was just as overwhelmingly in favor of going into a non-medical business. I doubt whether there is any great difference in the personnel of the two agency forces but possibly the difference lies in the way in which the presentation was made from the home office. I am perfectly frank to say that at that time we in the home office were already prejudiced in its favor and that influenced our presentation; there were practically no important negative reactions.

Of the two negative reactions that were expressed the first, as Mr. Porter said, was that there would be a chance to lose a larger policy or the opportunity for ordering out additional policies. The

other question that was brought up negatively was that it might gain the ill-will of the medical examiners and that there would be some difficulty encountered because the medical examiner would lose almost half his fees. The consensus of opinion, however, among the field force was that they wanted it.

While I do not like to bring up differences in company practices here, yet I think there is one point in our own organization that should be mentioned. We have no part-time men. I think with part-time men the situation is quite different, if the privilege of non-medical business is given to them. We do not do a brokerage business. We will not take this business from agents of other companies. I think that is a safeguard that should be thrown around this matter. We ask in the application: "Did the applicant come to you or was he suggested to you by a man of another company?" and if the answer to either of these questions is "Yes," we are very careful in the investigation.

As against 50 per cent on which medical examination is called for, we are examining 4 per cent. Now, I personally do not know whether that is a dangerous sign or not. I rather imagine that the fact that we are examining only 4 per cent means that we are getting an occasional heart case, albumin case, or other type of case that would be caught and thrown out on examination. We have carefully tried to watch to see if there is a higher percentage of those with M.I.B. records against them, and we cannot find them.

We have watched our business to see that the occupational distribution is satisfactory. We do not want to run into a lower grade, bordering on the industrial, and each month we are going over our non-medical cases and dividing them according to occupations. We cannot find that we are getting into a lower grade, though there is that fear always before us, that the agent changes the type of the group he is soliciting.

We cautioned our agents very particularly at the start that it was not a means to go out into places where they previously had not gone and into a lower type of risks and enter the industrial grade; that it was a method for them to save time in disposing of the small cases which agents normally meet and an opportunity for them to build up a clientèle among people who could at the present time take only small policies but who would later on be eligible for larger policies.

We felt quite a little hesitancy about offering this non-medical privilege to our new agents, and for a time we felt we might have to restrict it to agents who had been with us over a certain period or had a certain favorable type of business, but we finally went ahead, throwing all the safeguards around it that we could. We were undoubtedly influenced in this decision by the fact that all our agents have gone through a required course of training in our home office.

A home-office official visited each agency when we first entered the non-medical field, instructing the men and cautioning them that the business would be watched very closely.

Under those conditions, so far we have had no trouble. We have had three cases, out of some three thousand that have come to our attention where we know now that the man definitely was uninsurable at standard rates; and in two of those cases there is no question but that the impairment was not known to him. In the other case there is some question. But I think even in that case there is no doubt that it was not known to the agent. That does not mean we have not possibly had a selection against us, and there is no definite proof that occasionally an impairment may have been known to the agent. But in the six months we have been doing this business, no information has come to us that would make us definitely know that a fraud or a questionable proceeding was worked against us. It is too early yet to have any figures indicative of what the mortality will be.

I was very much interested in what Mr. Little said about segregating this group. I agree entirely with him in any question of gain-and-loss exhibit. The only thing that seems necessary is to keep the mortality of that group separate so that in a year or two (because the selection against us, if any, will come almost immediately) we can tell what that selection is worth compared with the medical fees that we are saving.

I do not quite follow him in his suggestion that if there be an excess mortality there is a possibility that a company may adopt the method of special dividends. I think in that case that the company should stop issuing such business; we would feel there was no object in continuing it if the mortality considerably overbalanced the saving in medical fees.

I am not quite sure how we would stand legally about giving lower dividends per thousand on our $1,000 and $2,000 cases than

on our $10,000 policies. I should be inclined to feel we would be running pretty close to the anti-discriminatory laws, because we force a person applying for less than a certain amount to take a non-medical form if he is between the age limits, and it would almost seem as if we would be selling insurance at different net rates between people of presumably the same expectancy.

There certainly should be a word of caution sounded, though I think it is in the minds of all of us, that this business, which is now being carried on in an experimental way, should not get under the wheels of competition so that the various companies would be competing with one another as to their age limits and as to their amounts and not be guided largely or almost entirely by the forces within their own company and the type of their own agency force. If developed soundly and conservatively, there seems to be every chance that this business will be to the benefit of the public and the community. But if competition enters and there is a mad race to see who will give the most liberal benefits, I think there is no question but what it will mean disaster. And to my mind that word of caution should be in the minds of actuaries because we may have to meet in our own companies the question of increasing the limits beyond what we think are safe for our own particular case. If competition enters and drives us faster than we are prepared to go, faster than we have the information necessary, it would be extremely unwise for the general program of non-medical insurance and the country itself.

MR. F. R. JORDAN:

I was very much surprised that one company found it necessary to examine 50 per cent of the non-medical business. Our experience coincides with what Mr. Larus has stated, only our ratio is about 6 per cent instead of 4 per cent. We have not found any undue number of impairments on non-medical business.

You might be interested in knowing what the saving is in medical fees. It is about 50 per cent, notwithstanding the fact that our business has increased, our new business, about 19 per cent over the corresponding period of last year.

MR. LARUS: What are your limits? $2,000 or $2,500?

MR. JORDAN: $2,000.

MR. LARUS: And your age limits?

MR. JORDAN: Between the ages of fifteen and forty-five.

I quite agree with Mr. Little that no segregation should be made of non-medical and medical business so far as the gain-and-loss and dividend allotment are concerned.

I can see no more reason for making such a segregation between medical and non-medical business than a similar segregation between Ordinary Life and Endowment policies. We are all aware that the mortality under Ordinary Life and Endowment policies will vary to considerable extent in a given company, especially if such company follows the practice of issuing Endowment policies to a considerable number of border-line risks.

The company with which I am connected did not consult its agency force before adopting the non-medical plan. The plan shortly after it was adopted was explained thoroughly at a meeting of the general agents. I venture to say that very few of our agents would be willing to return to the old order.

MR. J. C. CAMERON:

I should like to testify as to the experience of a young southern company in this field.

Before I do that I want to say a word about the question of writing to your agents and examiners and getting their opinion about the plan.

In our company we decided among ourselves. Two of our officers are field men, and we had all studied the question for about two years. Finally, in the spring of this year we decided to put on non-medical. We then advised the agents as to our decision and asked their co-operation, explaining to them that we were putting the responsibility upon their shoulders and, of course, giving a full description of the requirements, the nature of the application, etc.

In our case the age limits are fourteen to forty-five, both sexes, with a $2,500 maximum. We allow double indemnity to both sexes and disability benefits to male lives only. Our rates are the same as on our regular policies, and the policy forms are the same. We make the non-medical compulsory to all agents, not exempting part-time men.

We believe that the responsibility of the first selection lies with the agent more than with anyone else, more with the agent than with the examiner. Now, for the first time in our history, we are

placing the responsibility fair and square on the agent's shoulders. We are making him understand that.

When we announced our plans to the agents we received, out of a total organization of four or five hundred men, about twelve adverse replies. We had these agents come into the home office, and the result was that practically all were satisfied with the plan.

As to our experience, we started non-medical about the end of March. In April, our total applications were 1,555 by number and $3,790,000 by amount. Of that business the non-medical was 789 by number of applications and $1,317,000 by amount.

In May the plan began to come into full force in all our field except Mississippi. In May our total applications for all business were in number 1,892 and in amount $4,326,000, of which the non-medical were in number 1,341 and in amount $2,221,000. In other words, two-thirds of the applications were non-medical, rather more than one-half of the business.

There is one point I want to bring to your attention, especially in regard to a company operating largely in rural districts. Our own business is chiefly a country one. We operate principally in Texas and Oklahoma and among farmers and small tradesmen of these states. In these country districts, it is often hard to get the examiner to see your prospect. The agents have to wait sometimes ten to twelve days or longer, as the doctor is perhaps 30 miles away. Under the non-medical plan, therefore, they obtain much more rapid service.

One other point in regard to the agent's work. We did not tell our agents to go to the prospect and say, "Our company gives you insurance without examination." The method that we emphasize is for the agent to approach the prospect and write the insurance in the ordinary way. At the end he says to his applicant, "Well, you seem to me to be a pretty good fellow and a high-class risk. I believe the company may waive examination if they agree with me." By this mode of approach, we do not hold out the want of examination as an inducement to insurance.

Now, as to the examiner's reaction. We had a few examiners who objected strongly to non-medical insurance but we told them frankly that, in our opinion, non-medical was here to stay, not only in our company but in all companies at some day not very far distant, and that we would still have a considerable amount of business

on which we would require their help. We find when we talk to them that most of these examiners are quite satisfied to go along with us and to examine our business over $2,500.

As to the segregation of records, I entirely agree with Mr. Little that it would be a vast waste of time to make any attempt to have a complete accounting.

Our method is to put a special code mark in the Hollerith card so that we can pick out for any period the mortality on non-medical business, and we also have record of the medical fees saved.

I entirely agree with Mr. Larus in his word of caution. We cannot afford to be competitive in this business. If we are, I think that our mortality will be pretty apt to overcome any saving we may effect.

MR. F. B. MEAD:

We have begun in our company a non-medical business to a limited extent. Before taking up non-medical in any state we are making a survey of the past business in the state as to the sources of our information whereby we acted adversely, either by declination or modification. And we at the present time have done this business in only four states—North Dakota, Minnesota, Michigan, and Indiana.

We require the general agent to indorse the soliciting agent for this. It is also a very good plan to get a certification from each agent that he has carefully read the rules and is going to comply with them and also that he is going to give the company any special information that he may have regarding the risk.

Of course, the non-medical is obligatory within our age limits and the limit of amount.

We have made very little change in our blanks. We consider that the non-medical information should be the same. We require the same information in passing upon a non-medical risk that we do in passing upon a medical risk. We are using the same Part I for both forms. There have been some modifications in that form, both from a legal standpoint and also from the way the questions are arranged, but very little.

Then we have Part II to be used in connection with a medical case, and then Part III and also the special form for the non-medical case. There has been very little modification in Part II for the reason that we felt that we would require the same information.

We have been very much amused at the letters we have been receiving from inspection companies in regard to the special requirements of inspection in connection with the non-medical. We believe we require the same type of inspection on non-medical as we do on medical. That is, if there is any unfavorable information, we want it.

One company wrote us and told us that in view of the fact that we were going into the non-medical business they were going to give our business special attention. We wrote them and told them that if there was anything they had not done in the past that it was most reprehensible and they should at once undertake careful, special work in connection with our medical as well as our non-medical.

It might be of interest to give the results of the first 995 applications. Our limit is $2500. The average application is about $1500. In these first 995 cases we modified 19. In other words, we are doing a substandard, non-medical business. The modified policies were about the same average size as the regular. There were 19 cases totaling $30,000.

Sixteen of these modifications were brought about by the statements of the applicants. One modification was brought about by the statement of the agent; two because of codes found in the Bureau. The codes were of very little moment in these cases.

Eight of these modifications were because of medical information; that is, given by the applicant. Ten were because of the occupation of the insured, and one was unaccounted for when the report was complete.

We requested examinations only in connection with nine cases. That is about nine-tenths of 1 per cent, for $12,500.

The average case upon which we requested examination was less than the average of the non-medical, which is a very good barometer. Of these 995 cases we declined one after examination and three without requesting an examination.

I might say that in this section of the country, eliminating brokerage business, our rate of rejection has been running around 1 per cent.

Five of the examined cases were accepted as standard and three cases were issued as substandard.

All of the requests for examination were made upon statements

given by the applicant to the agent. All of the declinations were based upon statements of the applicant to the agent and supplementary information obtained through the family physician.

Only one case was affected by the inspection report.

MR. L. A. ANDERSON:

There is one question that has come to my mind that may seem to have only a remote connection with the subject but I believe it has a bearing upon it. It is not entirely my own argument.

In some places there are complaints that the doctors move to the large cities, and it has been proposed to hold out some inducement or other to young doctors to locate in some of the smaller places. The argument is that if you take away from the local doctors the income they get from medical examinations there would be a tendency further to drive them to the cities, because many of them find it hard work now to make a living.

That would probably make it a little bit harder to get your medical business examined for one thing and then, of course, it increases the difficulty of getting the doctor in time in case of injury or acute illness.

MR. V. R. SMITH:

Some of the remarks that have been made regarding the hazards of doing a non-medical business recall to mind some of the fears we had in 1921 when we began to write life insurance without a medical examination. If you think of all the things that may happen to you with a non-medical business, perhaps there is a justification for being a bit fearful. As a matter of fact, the result has been excellent, and of the seventeen Canadian companies doing this kind of business, I know of no company that is not perfectly satisfied with its non-medical business.

The mortality has been first class. The experience in 1924, of the three or four companies who are keeping separate records, shows that the mortality runs from 50 to 75 per cent of the Canadian Men's Select Table, a very severe standard, and, you will agree, a very excellent result.

From the agency standpoint, I think the general opinion is that the agents of the companies would be dissatisfied if the non-medical business were discontinued and the companies were to do nothing but a medically examined business.

From the standpoint of the doctors, there was a little difficulty at first, but when the situation was explained nothing more was heard about it.

Perhaps the question of limits of age and amount tells how well the Canadian companies are satisfied with the result of their experiment. In 1921 the limit of amount was $1,000 and of ages, twenty to forty-five. Recently one of the companies changed its limits to age fifteen or seventeen and fifty, taking $5,000, together with disability and double-indemnity benefits under one application. Apparently, there is only one company that has gone so far, but all the others have increased the limit, both as to age and amount, beyond the limit of 1921.

Perhaps the principal danger lies in the granting of a disability benefit, which is really a non-cancelable sickness policy with the first three months of sickness eliminated. This danger may not be serious when the amount of each application is small or limited to $2,500, but when the amount is increased to $5,000, the danger is much increased when the disability benefit is added. Doubtless there would be less hesitation and a greater feeling of security if net premiums for the disability benefit were more readily determined.

There has been some comment this morning with regard to the small policy, but perhaps the small policy is one of the reasons for the success of the business. Undoubtedly, in the beginning, many people thought that they would be able to obtain insurance without medical examination no matter how badly they were impaired, and at first we received many applications of this character. Accordingly, the small policy has practically eliminated the risk of fraudulent claims. On the other hand, the unfavorable feature connected with the small amount has been overcome to some extent by some companies allowing their agents to take further policies each year, at a yearly interval. Indeed, in most cases there is no limit to the total amount that will be taken in this manner on a single life.

With regard to the remarks I understood Mr. Porter to make as to the agent's opinion, I quite agree with him and some of the other speakers that practically any opinion desired may be obtained from the agency force on this question, and it depends absolutely on the point of view and the state of mind of the head office. If the head office is enthusiastic, the agents will be enthusiastic also; and if the

head office feels hesitant and diffident about it, the agency force will feel exactly the same way. One of the Canadian companies endeavored to obtain the opinion of their agents, and not liking the non-medical business, found their agency force hostile. Eventually the company was forced by competition into the business, and now their agency force is quite satisfied.

There has been some comment with regard to segregating the business and paying a different scale of profits. The question that comes to my mind is, Are the companies going to pay more profits on the non-medical business, should the non-medical business show a lighter death-strain than the business as a whole, in addition to showing a saving in the expense of the medical fees, which, during the four years' experience in our company, have been almost sufficient to pay the death claims we have received in the non-medical branch? I do not believe it is proper to segregate the non-medical business for surplus distribution purposes.

We are issuing about 50 per cent by amount, of our business on the non-medical plan, and about two-thirds, or 70 per cent, by number, and these percentages are increasing year by year as the limits are increased.

Mr. Beckett: Before Mr. Smith leaves the floor I should like to ask him a question. Would you be kind enough to explain this: Many of us here in the States have thought a great deal, in considering the matter from the Canadian viewpoint, as to the effect your intercompany exchange had on your selection in non-medical business. Of course, I have been told that has been discontinued, but that is a matter we folks have talked about a great deal. I know how you originally started it, but it would be of very great interest to me to know to what extent that seemed to affect your business.

Mr. Smith: I take it that is the Bureau that we started at the beginning, under which each company reported each day every case of a non-medical application, giving the name and occupation, and so forth, for identification purposes.

After running the Bureau for three years, we discontinued it for the reason that men did not apply to the various companies for non-medical insurance. In the beginning we were afraid that a man who was impaired would shop around to the various companies and obtain a considerable amount of non-medical insurance, but such is not the case. The companies are protected, first, because we

report all impairments through the Bureau, if there are any, and, secondly, by the fact that we found only one applicant in forty applying to two companies, one in one thousand to three companies, one in eighty thousand to four companies, and only one man out of one hundred and seventy thousand to six companies in a period of almost three years. We found by experience that there was no selection against us on this score and something that we were afraid of did not exist.

MR. HENRY MOIR:

If we are going largely into non-medical business we should take steps to amend one particular law in the United States. It is that law which makes a policy indisputable after two years. We ought to get the words "except for fraud" inserted; this becomes very important when life insurance is written without medical examination.

In Canada the companies do not have that limitation—nor in Great Britain. In those countries a carefully drafted policy taken out fraudulently can be contested at any time, and this seems the proper condition. It does not seem right that a policy, obtained by fraud, should become incontestible after two years. My suggestion is that we bend our energies to get the words "except for fraud" inserted to meet this situation if we relax our rules for selection.

When the law was first proposed in New York in 1906, I suggested to the legislative leaders that the words "except for fraud" be put in the law. They advised and insisted that the words were unnecessary because a fraudulent contract is void *ab initio*, therefore can always be contested. Unfortunately, the supreme courts have not sustained that opinion.

I am not so hopeless about getting this "insurmountable" thing done although I agree that it is difficult. Nevertheless, I think we should try, as the object is for justice.

MR. L. K. FILE:

There are one or two points that occurred to me in this discussion this morning.

I represent a Canadian company but we also do business in a great many of the United States.

We adopted non-medical several years ago and have been perfectly satisfied with the result, but we have not seen our way clear to adopt it in the United States chiefly for the reason that Mr. Little

pointed out, that the conditions are somewhat different—particularly in the larger cities, I think. You have there a large mass of people, a portion of whom are foreign born, who have only recently come to the country and have not yet assimilated the ideas of this side of the Atlantic. We have hesitated for that reason about introducing it into the United States. We have no intention of doing it at the present time. We are, however, watching the experiment being made by the United States companies in that regard.

We have seen, however, a tendency in certain quarters in the United States to increase the amount to be taken on non-medical basis beyond what we think to be safe at this early stage. And to us it is a matter of regret that there should be a tendency in that direction.

One speaker referred to the fact that in one company 50 per cent of the non-medical cases submitted were sent back for examination. As Mr. Smith pointed out, at the time non-medical was started in Canada there was a similar state of mind on the part of the public as also exists here. People thought that they could get insurance merely by asking for it. When it was advertised that they could get insurance without medical examination they jumped to the conclusion that they could get it without any restriction whatever. When the agents were properly educated, however, and the people understood that they had to submit to a very strict questionnaire, a better understanding prevailed and misconceptions were cleared away. But I fancy that that has accounted largely for the large percentage of cases sent back for examination by this particular company. The conditions were not properly understood.

As a matter of fact, I think in many cases we get better evidence of insurability in non-medical cases than we do in medical.

We must not forget the fact that at the present day we are having trouble with the medical situation in a great many parts of the country. Our medical examiners are satisfactory in a great many places but in a great many other places, with all due respect to the profession, in many sections of the country the medical situation is not satisfactory, and we are satisfied that in many cases we are getting better evidence of insurability on our non-medical form than we do on our medical.

One speaker made reference to the fact that some of the agents felt the introduction of non-medical would lead to a reduction in

the amount of business written. In the number of years so far that we have been doing this business we see no evidence of any decrease in the size of the average policy. As a matter of fact, the average policy has rather increased.

MR. G. B. PATTISON:

I should like to ask some of those gentlemen in the United States who have adopted the non-medical plan if they made any modification in their incontestibility clause of from one year to two years. I wonder if any of them have considered that.

MR. MEAD: Our company has a two-year clause with a one-year suicide clause. We have had it for many years. We found that the one-year clause was not sufficient because a large proportion of the misrepresentations show up about the time the second premium is due. A large number of the companies are already doing business on a two-year incontestible clause. It seems to me to be very advisable in your non-medical business to have a two-year incontestible clause.

MR. CAMERON: We have a two-year clause in our policies.

MR. MEAD: The two-year clause is not a matter of competition in the field at all. It is not felt.

MR. LARUS: I might say that our own company has the one-year clause in its regular business, and we would be very unwilling to put a different clause in the non-medical than we have in the medical. We want to be in the position of giving the same policy to our non-medical risks as to our medically examined business. For that reason we have the one-year clause in the non-medical also.

DR. C. M. McCULLOCH:

Dr. McCulloch is my name, of the State Life of Indiana. I wish to take advantage of the courtesy extended to the gentlemen of my profession to make such inquiries as occur to us in connection with this very interesting, vital, and imminent subject.

In listening to the discussion and in recalling the thoughts that have arisen in the minds of our own company, there are one or two points on which I should like to seek some light.

We have always believed that we have an anchor, a safety factor, in the M.I.B., and of course we have, but is not that going to

be of diminishing value for the simple reason that since these cases are turned in on a non-medical basis, the source of supply as to impairments will gradually vanish or diminish at any rate?

Secondly, I listened with much interest to the gentleman from Texas relating his experiences with the examiners. I have been informed, however, by some of his colleagues in other companies that they hear a great deal of back-lash and resentment over that. There comes up in my mind the action of the various county societies all over the United States on the fee question of several years ago.

Our company at one time, except when otherwise obliged, was on a $3.00-fee basis, but, as we all know, a prairie fire ran across the country, and county society after county society—made up, of course, as a general thing of the best men in any community—began passing resolutions declassing any doctor who would accept a less fee than the $5.00 fee which they had set as their standard. Is there danger of that kind of a result with the installation of this new method?

Thirdly, I should like to inquire, and this subject has not been touched upon this morning and is probably on the other side of the question, whether we are not insuring ourselves against some ultimate disaster by taking up this particular branch of service? And I refer more specifically to the very evident socialistic trend that is sweeping over the country in matters of legislation. We hear more and more as the legislatures meet in the various states of the question of old-age pensions or state insurance.

In the state of Ohio, for instance, now, there is a state-compensation insurance law, working to the great disadvantage of the companies which deal in that kind of business.

In the state of Indiana, a very formidable and influential organization, the Eagles, tried very diligently, without success to be sure but they are not discouraged, to get an old pension law passed, throwing the burden of this thing upon the taxpayers of the state and in effect remotely affecting the business of all insurance companies, particularly those that carry total-disability clauses.

This, gentlemen, I think is a very important angle on the question that we are discussing. Are we not insuring ourselves against the passage of such legislation in various states by anticipating that need and making available, properly paid for, to be sure, insurance for mass mankind in general.

These are the facts that have occurred to me this morning. I should like to hear answers, if possible, to some of these things, bearing always in mind, as any insurance company should, and paraphrasing Shakespeare, that no company whose blood runs warm within its veins should sit like its grandsire cut in alabaster.

MR. V. R. SMITH:

No criticism should be made of the use of the Medical Impairment Bureau by the companies in handling the non-medical business. We obtain an inspection in every case and upon investigation by one of the companies it was found that the company was giving more information to the Bureau than was obtained from it. In other words, we were obtaining, in many cases, more information through the questionnaire and through the inspection report than we were in the cases of similar policies that had been examined.

I think we have several cases where we have had the man examined subsequent to the non-medical questionnaire, and the information given in the non-medical questionnaire was more exact and fuller than in the medical report.

I think, therefore, that the non-medical business is not a drag on the M.I.B. in any way.

With regard to doctors' fees, I might remind Dr. McCulloch that the match to the magazine that started the non-medical business in Canada was the number of resolutions passed by county and provincial medical associations demanding fees as high as $10.00 and $11.00 for a $1,000.00 application. When the demand reached so high a point, of course the saving in mortality was out of proportion to the fee paid. At that time we were paying a $4.00 or $5.00 fee. We are still paying a $4.00 or $5.00 fee. We have not increased the medical fee in any way in spite of the increase in the amount of the application where the applicant is medically examined.

The question of the medical fee is not any longer a subject of debate in medical societies, nor do you hear it discussed between medical men at all.

MR. C. F. CROSS:

When the American Life adopted non-medical the first of February, the various propositions objecting to or in favor of it were pretty well standardized. Wherever actuaries met, theoretical discussions

were started on the propositions, "Your agency force won't accept it," or "The doctors will object to it," or "Savings will not be sufficient to take care of the extra mortality." Se we jumped over to Toronto and tried to find out what was actually happening along these lines.

Canadian actuaries seemed to feel that the advantages of the system were considerable and that the foregoing objections were not materializing. In our minds, this left the final proposition "that conditions were different in Canada," and I am glad to note this morning that American companies are also reporting favorably on the limited experience they have thus far developed.

We conducted an investigation along a little different line than appears in the literature on this subject. I do not know just how valuable it is as an exact measure of the savings in non-medical business, but it does give an indication of what we may expect, and it is also valuable in showing just what has been found from the medical examinations.

In our company for the year 1924 we took approximately 4,000 cases between the age limits we have adopted, that is, fifteen to forty-five, for amounts of $2,000 and under. An analysis of the rejections and rated cases in this group reveals that 64 cases for $83,000 were declined or rated up because of information submitted by the doctors which would not have been found under a non-medical plan. All the others declined and rated cases, and there was 334 for $952,500, were without these limits, or acted adversely upon because of information obtained from questioning the applicant, from the inspection report, or from the various impairment bureaus such as the M.I.B.

The medical fees paid out in connection with these cases were about $20,000 or, approximately, $240 for each thousand of insurance which was declined.

The question arises as to whether we can say that what has happened under the present plan would happen were we conducting a non-medical business. Probably not, but if these figures are representative for our company and if because of the additional speculative element twice as many unsatisfactory cases crept in, we would still have $120 for each thousand to cover the extra mortality in that group.

Six Michigan Legal Reserve companies compiled data in the

same way. Dr. Rietz said yesterday that the coefficient of correlation when small was valuable when we considered the proposition in mind. A peculiar thing about this investigation was that the companies who seemed to be favoring non-medical business showed the greater savings.

For instance, our company was second with a saving of $240 a thousand. I do not know whether you can call it a saving or not, but the idea is that if that business had been on a non-medical basis and came in just the same way, then it would have been a saving. And the company which seemed to view the matter least favorably showed a much less saving (we will call it that), only $66.00 a thousand.

The combined experience (I take these figures from memory) showed that on three-quarters of a million of declined and rated cases there were paid out medical fees of $116,000.00. This includes 14,000 cases from the Maccabees, who only pay a $2.00 medical fee. This shows approximately $150.00 a thousand assignable to take care of the extra mortality if the same business is received under a non-medical as actually was received under the medical plan.

MR. L. A. ANDERSON:

The statement was made, I believe, that in some cases you get more information from the non-medical business than from the medical. That may be true, if you rely entirely on the doctors' reports.

My company recently added a considerable number of questions that the agent has to fill out; so that we have the doctor's report, the inspection report, the M.I.B., and the additional questions that the agent must answer.

I believe that the doctor's report could be supplemented by a questionnaire, perhaps not quite as elaborate as the one that has been referred to here, but it seems to me we can get more information if we have a medical examination.

MR. H. W. BUTTOLPH

One of Dr. McCulloch's suggestions has a very distinct bearing on the figures given by Mr. Cross.

Mr. Cross stated, I believe, that $83,000 of business was rejected because of information given by the medical examiner which

was not elsewhere disclosed in the application or in the records in the company's office. And he referred to the other rejections being on account of information disclosed by the answers given by the applicant himself or through information secured from the M.I.B.

Now, that is just the point, as I understood Dr. McCulloch, that he was trying to bring out. For Mr. Cross's figures to be reliable it seems to me we should know how much of the business rejected because of impairments discovered in the M.I.B. file was so treated on account of impairments secured through medical examinations.

While we have that information complete, or reasonably complete today, yet we may not have it anything like as complete twenty years from today.

So while Mr. Cross's figures may be regarded as entirely accurate in estimating the condition that maintained or that would have maintained in 1924 had his company been doing a non-medical business that year, they may be very materially altered twenty years from now.

MR. J. G. PARKER:

I think that we are unnecessarily fearful as to the effect in future years of the lessening in the number of reports which we may have on file on account of our companies doing a non-medical business.

I think we can discount almost altogether the few cases of incipient lung affections which might be found by our medical report. Undoubtedly we will miss a few kidney affections and some heart affections. It is to be remembered that the non-medical business, as our companies are practicing it, is for small amounts on lives which ordinarily purchase a small amount of insurance. Consequently, very few of our non-medical cases today have any old reports existing against them. We are therefore doing today a non-medical business knowing that we are getting a certain number of impaired hearts and a certain number of impaired kidneys, and we will be no worse twenty years from now in regard to these cases, providing we do not change the limits as to the amount which we will accept non-medically.

MR. F. B. MEAD:

The limitation to forty-five years would undoubtedly cut out a large percentage of the degenerative diseases. Those are what we are afraid of in regard to the non-medical.

With reference to the incipient tuberculosis, it is probably quite true that some cases of incipient tuberculosis are not found or would not be found. They slip through on you. That is always true both ways so that in a thousand cases we would get about the same average. The M.I.B. is of very little moment because we found just two cases recorded in the M.I.B., and we accepted both of these on the modified basis.

MR J. P. BOWERMAN:

With reference to the incontestable clause to which some of the previous speakers referred, about thirty years ago there was a case which was fought through all the principal courts in Canada on the so-called incontestable clause, and then taken to the Privy Council of England.

The Supreme Court of Canada held that the policy was null and void, having been entered into with the appellant in his own name, for his own benefit, and he having no insurable interest in the life of Pettigrew, that the one year clause was contrary to public law and order and that the respondents were not estopped by the one-year clause or otherwise from disputing the validity of the policy.

The Lordships in the Privy Council concurred in the opinion expressed by the majorities of the Supreme Court and of the Superior Court sitting in review—the rule of the Code appears to them to be one which rests upon general principles of public policy or expediency and which cannot be defeated by the private convention of the parties. Any other view would lead to the sanction of wager policies.

A man named Antoine Pettigrew was insured in the Manufacturers Life of Canada in favor of Joseph Anctil for $2,000.

This case was first referred to in the report of the Superintendent of Insurance for the Dominion of Canada for 1897 on page 103, and the decision of the Privy Council of England is given in the Canadian Report for the year 1899 on page xxxii.

The incontestable clause in the policy referred to above when translated into English reads as follows: "After this policy has been in force for one full year, it shall be incontestable on any grounds whatever, providing the premiums herein mentioned have been paid promptly and the age of the insured has been admitted."

I think there have been decisions of the Supreme Court of the

United States along the same line, that it is against public policy to compel the payment of policies that were obtained by fraud even when the incontestable period provided for in the policy has already expired.

MR. P. H. EVANS:

Mr. McCankie has just passed over a question that I myself should like to get some light on, and the question is, Whether the companies doing non-medical business are requiring that the answers in the blank, the answers of the applicant, shall be in his own handwriting? Have any of the companies doing a non-medical considered that question, and what has been their practice?

MR. CAMERON: We accept the agent's own writing.

MR. JORDAN: We do the same thing. We accept the agent's writing.

MR. COLER: I happen to know that the Guardian Life does, or did, require the answers to be in the applicant's own handwriting, and they reported soon after they started the non-medical that it was one of the very difficult things in which to get the co-operation of their agents as it slowed up the process of getting applications. A busy man would object to taking time, or at least the agent thought he would object to taking the time to fill out this blank.

MISS E. B. KOCH:

I have a question I should like to raise in connection with the non-medical, and that is, Is there any danger in extending the non-medical benefit to the younger ages, where we are issuing standard policies, down to say age ten or fifteen? I should like to hear the expression of any of the members in that connection.

MR. PARKER: My personal opinion would be against extending the limit below age fifteen for non-medical business.

In most of the provinces in Canada an infant of fifteen years or older can apply for insurance and can do with his policy what a person of mature age might do. I think, therefore, that a company would have more success in canceling a policy on the life of an infant of fifteen years or more in the event of the policy having been obtained by means of fraudulent statements than if the policy had been issued on the life of an infant under fifteen.

SALARY DEDUCTION PLAN

A. What premiums should be charged?
B. How should premium defaults be handled?
C. To what extent may usual medical examination requirements be modified?
D. To what extent will this plan affect the American agency system?

MR. C. F. STEIN:

The company with which I am connected has been granting accident and health insurance on the so-called pay-roll deduction plan for over a period of thirty years and has operated it not only successfully but profitably as well. It occurred to me that some of our observations based on that experience might be of interest in so far as they apply to the granting of life insurance on the same basis.

We found that the rate of persistency decreased rather markedly with the decrease in the standing of the employee. In other words, the insurance written on superintendents, managers, and officials of the company showed a very much higher rate of persistency than that written on the industrial grades of risks.

It seems to me, therefore, that we should have some restrictions as to the proportion of pay-roll deduction insurance written among industrial workers compared to the total number of employees aside from the usual extra premium added for occupational hazard.

Then we found that while, of course, it was necessary to secure the approval of the employer or the head of the company, that, after all, the most essential thing was to secure the co-operation and the assistance of the cashier, the paymaster, or whatever title may designate the pay-roll officer. It seems to me, therefore, that in determining the compensation to be allowed to the agent under pay-roll deduction insurance, some provision should be made for a collection fee, if possible, as is done in accident and health, to be given to the cashier.

We found that it was of essential importance to give him some pecuniary consideration. I would suggest a collection fee of 5 per cent the first year and $2\frac{1}{2}$ per cent thereafter, the agent's commission being correspondingly reduced, because it not only secured his assistance in continuing the business already written but it offered a very fertile field in securing new employees to come under the benefits of the plan.

It is a comparatively simple matter for the head of the concern to tell the cashier or the paymaster to deduct these premiums every month, but since it means work to him if he receives no pecuniary compensation, the fewer deductions he has to make the easier his work will be. And so we have found in the accident and health branch that by paying 5 per cent first-year collection fee and $2\frac{1}{2}$ per cent renewal, you secure the full co-operation and assistance of the cashier or the pay clerk, and he was of material assistance to our agents, as I said, not only in continuing the business already written but in securing new prospects.

Then we found that one of the difficulties was, because of the comparatively low monthly premium, especially among the industrial workers, that quite a number would continue for a month or two and then discontinue, because they have the right of revoking the authorization which they give to the employer for deduction from the pay-roll. So we have a provision that it can only be revoked on his request sixty days after written notice has been given to the employer, thereby practically assuring that a quarterly premium will be collected before being discontinued. I know the objection will be raised that that will interfere with the sale of the proposition but I do not think in my connection with the company that we have ever had that complaint arise.

Then I think another important factor to remember is the method of home-office handling, because, after all, the profitableness of the plan is dependent upon the simplicity of handling in the home office at the minimum expense.

I think many of the companies contemplate sending to the employer, who is to make the deductions and remit to the company, a detailed list showing the amount of the monthly premium due from each employee.

Our system varies somewhat in that we have on the authorization card which the employee signs when the application is taken, and which is turned over to the employer, a blank space for noting the date of the notice of cancellation of the authorization and also a space for the date of the termination of service of the employee. The employer forwards this card to the home office, and we send him a statement based on the preceding month's aggregate premiums, showing the increase in premium because of new entrants and the decrease because of those who have discontinued either by revoca-

tion of the authorization or by discontinuance of service, and he remits the resulting gross aggregate premium shown.

Then on each anniversary date, or on some fixed date in the year, we total our entire premiums and balance the total thus obtained with the monthly premium as shown on the authorization card on file with the employer.

I think you will find that as the business grows, as our accident and health business has grown, it becomes quite a laborious task to list every employee with his monthly premium, and it also involves considerable detail in the home office to check the list as it goes to the employer and again when it is returned by the employer to the company, with the discontinuances noted.

I offer these as suggestions only based upon our experience with the pay-roll deduction plan, as we designate it in connection with accident and health insurance, in the hope that they may prove applicable, in a measure at least, to life insurance upon the same plan.

MR. J. R. LARUS:

The Phoenix Mutual is just going into this salary-deduction plan, and while we have had no actual experience with it, yet it may be that some of the thought we have given to it may supplement some of the remarks previously made.

I think Mr. Milligan took care of Section A as to the premium that should be charged when he said it should be based on one-third of the quarterly premium. It hardly seems feasible to charge less than the quarterly rate or there would be a possible discrimination against your regular quarterly policyholders and some difficulty when a man left the pay-roll-deduction plan if he were then asked to change to quarterly at a higher rate.

It hardly seems necessary, with the advantages in group collection that the salary-deduction plan affords, to charge more than the quarterly rate except in so far as legal difficulties may complicate the matter.

Mr. Milligan also answered, in Section B, how premium defaults are handled when the default in premium was on the part of the employer.

The next development is, When the employee leaves the concern, what shall be done with his policy? Shall it be changed to a quarterly basis, or may it still continue on a monthly basis although

the insurance company will no longer have the advantage of collective premiums?

I think the practices of the several companies recently inaugurating this salary-deduction plan are different on this point. The company with which I am connected is requiring that the policy shall then be handled absolutely as an individual policy. It must be changed on its next succeeding anniversary to quarterly or less frequent payments, unless it so happens that the monthly premium is more than the minimum of $10.00, which allows policies to be written individually on the monthly basis.

With relation to Section C, "To what extent may usual medical examination requirements be modified?" our own feeling is that when a company is operating on the non-medical basis for its individual policies, those same requirements, except possibly for slight practical modifications, should be applied to the salary-deduction plan. There seems to be but little difference in the circumstances under which the policy is written or the possible selection against the company between these two methods. Our feeling was that if we went up to a certain limit and had certain age limits on the individual non-medical, we should, in justice to our individual policyholders, apply those same limits of amount and age to the salary-deduction plan. A possible difference might occur in allowing a short form medical examination in salary-deduction cases for amounts beyond the limits of the regular individual non-medical. That I do not believe could be criticized by anyone in the interest of the individual policyholder.

"To what extent will this plan affect the American agency system?" Just as the non-medical individual policies were introduced largely to conserve the field man's time, this new step enables him to get his prospects under one roof and conserves his time. In that way I think it is a decided advantage to the American agency system.

Our own figures show that we have twenty-three interviews for one sale. We feel that under this salary-deduction plan we can cut down the number of interviews per sale, even taking into consideration that the average policy will probably be smaller.

In talking in one of our agencies recently the question came up as to the possibility of a competing company going to the employer and saying that he, the employer, would have the right to

appoint one of his own force an agent of the insurance company, and that that employee would in return for the collection of the premiums and the handling and selling of the insurance receive the commissions as regular agent of that company. I do not know that that is true. Probably it is not. I hope it is not, because it seems to me that would be a return to some of the unfortunate agency practices that all of us faced some years ago.

The advantage of this salary-deduction plan is that it broadens out the scope of our own field men to write more risks in less time. If we are going to take it away from them and give it to a new man with no connection with our home-office force or with our agency force, let him receive the benefit of it, let him corral that group of employees and use it to draw his future business from to the almost certain exclusion of our other agents in the writing of that group, I think we are working against our own interests as life insurance companies, not only against other companies' interests but against our own field force. I hope that development is not being considered, and I certainly feel that it would be a most unwise one from every point of view.

There is one problem, not covered in this questionnaire, that we must meet. It is really an agency problem, and possibly a question for each individual agency or company to decide for itself.

Most agencies have some regulations as to the rights of a particular agent over his own policyholders. This point comes up very acutely in relation to this salary-deduction plan.

For instance, one agent has written Mr. Jones in the Smith employ. Another agent writes the Smith Company on the salary-deduction plan. Can he then include Jones in this salary-deduction plan and receive the commissions from it? Or again, Jones may not be a policyholder but he may be a prospect. Can the agent writing the firm take the prospect away from the old agent?

That I do not think is a question for us to decide here. It is really not in our province. It may not be in the home office's province. But it is just a caution that in my mind should be kept before us when we put this up to our individual agencies to make sure they realize that possible barrier and to make sure that they predetermine what their own course will be if that comes up.

I have just been talking in several of our own agencies and the reaction on this point was very dissimilar in several of them. I

think we must keep it in mind and bring it before our own agents so that it will be settled ahead of time rather than cause individual disputes and possibly break up an agency's morale by having each agent running after the others' policyholders without a predetermined program decided upon.

THE NEW GAIN AND LOSS EXHIBIT REQUIRED FOR 1925

MR. E. G. FASSEL:

The new Gain and Loss Exhibit contains substantially everything in the former exhibit and in addition requires a new exhibit showing the changes in surplus according to classes or lines of business. Moreover, an instruction sheet is furnished covering many difficult features of the Gain and Loss Exhibit and the companies are required to follow the instructions but are not required to show the working figures. Considerable light is thrown on the new blank in the *Report* of the Committee Appointed by the National Convention of Insurance Commissioners on the Revision of the Gain and Loss Exhibit for Life Insurance Companies, February 1, 1923, and the *Report* of the Special Committee of the National Convention of Insurance Commissioners *in re* Revision of Gain and Loss Exhibit for Life Companies submitted to the National Convention of Insurance Commissioners at New York, December 4, 1923.

The Gain and Loss Exhibit proper covers in fifty-one items all the ground covered in one hundred and ten items in the former exhibit. The instruction sheet relates to this exhibit. The exhibit of changes in surplus by classes or lines of business has very little relation to the regular exhibit. This is a separate job and practically every item, including the increase in surplus, is different from the corresponding items in the exhibit proper. The items are to be carried to even dollars instead of cents as formerly.

The purpose of my remarks is to review the various items, making such comments as may be of help to a person with experience in completing the old exhibit and now confronted with filling out the new blank. The new exhibit items are numbered consecutively from 1 to 91 and for convenience I have designated these *E*. The instruction items run from 1 to 82 and I have designated them *I*. It will be more satisfactory to consider the instructions first and then proceed with the exhibit.

INSTRUCTIONS

I:2.—This is the amount of net insurance premiums corresponding to the gross in *E*:1, i.e., cash adjusted for in advance. There are two methods of calculation. We may calculate *E*:2 directly by breaking up *E*:1 into its elements of disability, accidental death, annuity, dividends applied to purchase additions, cost of change, etc., and regular premiums, the object being to segregate the gross premiums according to the groups in which it is easiest to apply loading percentages; deducting the loading with allowance for additional loading where premiums are payable oftener than once a year results in the total net premium corresponding to *E*:1. In this method the loading factors must be specially worked up. Then, having also calculated *I*:31, *I*:53, and *I*:75, we have $I{:}2 = E{:}2 - I{:}31 - I{:}53 - I{:}75$.

The second method is to work up *I*:2 directly as in the case of companies which determine net premiums and cost of insurance at the same time as determining mean reserves. The only change now required is that *I*:2 corresponds with cash gross premiums adjusted for in advance, whereas in the old exhibit the net premiums were on the revenue basis. *I*:2 will be correctly obtained by performing the same work as before to secure insurance net premiums on the revenue basis and then subtracting *I*:4 and adding *I*:8.

It does not seem to have occurred to those drafting the instructions that net deferred and uncollected premiums contain an element of disability and accidental death so that this non-ledger asset should be distributed into the disability and accidental-death benefits portion of the instructions as well as into Items 4, 8, 23, and 33. The foregoing two methods yield *I*:2 in conformity with the instructions and produce the correct amounts of required interest and expected mortality, but *I*:2 taken alone is less than the true insurance net premiums by the amount of increase in net deferred and uncollected disability and accidental-death premiums. No real harm is caused by this error because elsewhere *I*:2 occurs only in combination with cancellation of the error.

I:4.—This is the non-ledger asset item minus *I*:33. Similarly, *I*:8 is the non-ledger asset item minus *I*:23.

It must not be forgotten that the blank as printed is intended to agree with the regular Annual Statement items. If an individual company has special items in its statement, they must be accounted

for and may require insertion of special items when making up the exhibit. For example, maturing endowments paid in advance would entail insert Items $I:5(a)$ and $I:10(a)$.

I:12.—This item has an intimate relation with the surrenders, lapses, and changes section of the exhibit. Inasmuch as that section shows greater detail than $I:12$, it will be of advantage to first calculate $E:21$–26. There will probably be different ideas on these items, but I shall give one method that will be found satisfactory.

$E:21$=Terminal reserve released by policies surrendered and by policies changed to extended or paid-up insurance+reserve released on annuities surrendered.

$E:22$=Initial reserve on extended and paid-up insurance granted+disbursement item: surrender values paid in cash or applied in liquidation of loans on notes+disbursement item: surrender values applied to pay new or renewal premiums+this year liability item: liability on policies canceled and not included in "net reserve" upon which a surrender may be demanded−similar liability item last year+indebtedness restored on policies changed back from extended or paid-up insurance or otherwise reinstated.

$E:23=E:21-E:22$.

$E:24$=Reserve after change or restoration−reserve before change or restoration−indebtedness restored on policies changed back from extended or paid-up insurance or otherwise reinstated.

$E:25$=Terminal reserve released on lapsed policies.

Having determined the foregoing items we have $I:12=E:21$ (excluding annuity element)−reserve element of $E:22$−reserve element of $E:24+E:25$.

I:14, 15, 16.—If reserves are at various rates of interest it is satisfactory to use a single average rate of interest derived from the average mean insurance reserves at beginning and end of the year. See *Transactions of Actuarial Society of America*, XIII, 28.

I:31, 32.—In addition to regular premium income under annuities the intention of these items is to bring into this section net premiums to cover all contracts regarded as annuities this year for the first time. For such contracts disbursement is made from this section in $I:24$ and reserves are set up in $I:22$. If, therefore, any items are overlooked in $I:31$ and 32, an error in $I:43$ will result. It will probably pay to inspect the annuity valuation cards at the end of the year to be sure that net premiums are not overlooked by

failure to carry journal entries into the books to cover the transfer of cards in the Actuary's Department.

I:53, 75.—These are the mathematical net premiums corresponding to the basis on which reserves are held. A method of calculation is to apply a percentage to the cash disability and accidental-death premiums received, with further application of a ratio. To transform from the cash to the incurred basis a suitable ratio has as denominator the total cash premium income of the company and as numerator the total cash premium of the company adjusted for in advance and for uncollected and deferred.

I:54, 70, 58, 61.—These are new items. They are determined by the basis adopted when calculating the liability item: Present value of amounts incurred but not yet due for total- and permanent-disability benefits. This liability item is the mean reserve based on disabled life mortality. *I*:58 and 61 are the corresponding initial reserve transferred into the disability section to support disbursements during the current year and to set up the liability at the end of the year. *I*:54 and 70 are the terminal reserve at time of recovery released out of the disability section. This represents the portion of the disabled life reserve unused for disability disbursements because of recovery.

I:55, 64, 76.—These are not net interest at the valuation rate but they are the shares of *E*:12 assignable to disability and accidental-death benefits. The *Report* of February 1, 1923, earlier referred to states on page 12: "Net income from investments may be prorated according to the reserves held in the respective classes of business."

EXHIBIT PROPER

E:2=*I*:2+*I*:31+*I*:53+*I*:75. Comments on *E*:2 were made under *I*:2.

E:4, 11.—The blank contains instructions for obtaining these items but in most cases the resulting figures will be the same as in the old exhibit, except for omission of loading on uncollected and deferred premiums.

E:5.—This differs from the old exhibit by now including disability and accidental-death loading gain.

E:10.—Same as old exhibit.

E:12.—Same as old exhibit unless change was made in *E*:11.

E:13=*I*:55+*I*:64+*I*:76.

E:15=I:18+I:40+I:51. This item differs from the old exhibit by now omitting interest required for disability and accidental death.

E:16.—This item differs from the old exhibit by omitting interest gain for disability and accidental death.

E:17=I:21. It may be of interest to say that for one company this item came out as $30,230,114, whereas in its published statement of the old exhibit the corresponding item is $30,205,981.70.

E:18.—Same as old exhibit.

E:21-25.—Comments on these items were made when discussing I:12.

E:26.—Same as old exhibit.

E:27-40.—The items carried into the final columns as gain or loss in surplus are the same as in the old exhibit.

E:41, 42.—These items differ from the old exhibit in the subdivision that is required and in now excluding loading gain but now including interest gain.

INTERROGATORIES REGARDING BUSINESS OF FIRST POLICY YEAR

E:52.—The language calls for expected death losses which differs from E:17, calling for expected mortality on net amount at risk, and agrees with E:53, which calls for death losses not deducting reserves. Hence it is inferred that this item is the expected face amount of claims. A convenient method of calculation is to apply a persistency factor and a mortality factor. In one company the distribution of new business by applications between annual, semiannual, and quarterly was found to be 50:30:20, and the first-year lapse rate, .075. It may be considered that the first-year lapse rate applies to the second instalment of semiannual and quarterly premiums, and we may assume for the third and fourth quarterly instalments the rate of .02. The number of months of first-year exposure of 100 typical policies will therefore be $50\times12+30(6+.925\times6)+20\,(3+.925\times3+.925\times.98\times3+.925\times.98\times.98\times3)=1169.6922$. Dividing by 1,200 months possible exposure produces the factor .975. The average age at issue by amounts for new business was found to be thirty-six. By applying q_x to a typical distribution, it was found that weighting with q_x results in an increase of the average age to thirty-eight. Inasmuch as the mortality table used for

computing premiums was the American Experience, we have $\frac{1}{2} \times q_{38} = .00471$. To derive $E:52$ we may apply $.975 \times .00471$ to the total in this year's policy exhibit shown as issued during the year omitting dividend additions plus the corresponding amount from the policy exhibit of the preceding year.

E:54.—This is not clear as to reserves released by death but probably excludes such amounts.

E:55.—Premiums paid in advance on first-year business are here required but are not usually available. If it is not convenient to pick out the actual first-year cases, an approximation can be made. The figures given for a particular company under $E:52$ show that approximately 30 per cent of the first year's premium income was for semiannual business, and 20 per cent for quarterly. Therefore, $\frac{1}{2} \times 30$ per cent $+ \frac{3}{4} \times 20$ per cent $= 30$ per cent of the premium income may be considered the amount collected as second or subsequent instalments during the first year. The liability item, premiums paid in advance, arises only in connection with premiums subsequent to the initial premium. We may therefore apply to this liability item a factor of which the numerator is 30 per cent of the income for first year's premiums on original policies and the denominator is the numerator plus the income from renewal premiums.

EXHIBIT OF THE CHANGES IN SURPLUS ACCORDING TO THE CLASSES OR LINES OF BUSINESS

Attention is called to the report already referred to that was submitted December 4, 1923, which on page 21 states:

> Companies which have only odds and ends of various classes of business and which are not issuing business in such classes are not required to separate these odds and ends but can group them with ordinary in making their returns.

The usual procedure will be first to complete the total column, then to complete the required columns for classes other than ordinary, and finally to obtain the ordinary column by subtraction.

This exhibit differs from the exhibit proper in that Items 70–85 are on a strictly cash basis. In preparing this exhibit very little use can be made of the work for the exhibit proper, but the items are derived largely from the pages of the Annual Statement of which this exhibit is really a summary. I shall comment only on the total items. The items for the other columns require a great deal

of study, and this exhibit might properly be the subject of further discussion by the Institute.

E:70=Income: total premium income.

E:71=Income: total interest and rents, gross profit on sale or maturity of ledger assets, and gross increase by adjustment in book value of ledger assets, after subtracting from these three items the investment expenses paid, being the first amount entered under *E*:11.

E:72.—This will be the balance of total income after omitting income items included in *E*:70 and 71.

E:74=Disbursements for death claims, additions, and additional accidental-death benefits.

E:75=Disability premiums waived and payments made.

E:76=Disbursements for matured endowments and additions.

E:77=Surrender values paid in cash or applied in liquidation of loans or notes or applied to pay new or renewal premiums.

E:78=Disbursements for dividends paid, applied, or left at interest.

E:79.—Normally=disbursements for annuities.

The total of *E*:74–79 must agree with total paid policyholders in the disbursement page of the statement.

E:80=Disbursements to stockholders for dividends.

E:81=First amount entered under *E*:4.

E:82=Disbursements for taxes except taxes included as investment expenses paid in the first amount entered under *E*:11.

E:83.—This normally equals payments on supplementary contracts, dividends and interest held on deposit surrendered, and other disbursements not by way of expenses.

E:85.—This must equal the total income minus total disbursements of the statement.

E:86=The increase in this year's gross assets minus ledger assets over the corresponding amount last year.

E:88=Income in this year's assets not admitted over corresponding figure last year.

E:89=Increase in total admitted assets this year over last year.

MR. J. S. ELSTON:

Most features of the new Gain and Loss Exhibit, such as the nature of the changes between the old and the new and the reasons for these changes, together with a discussion of the purposes and

uses of the Gain and Loss Exhibit, are included in the "Report of Special Committee of National Convention of Insurance Commissioners *in re* Revision of Gain and Loss Exhibit for Life Companies, Submitted to the National Convention of Insurance Commissioners, December 4, 1923." That report also includes the joint report of the Committee of the Actuarial Society of America and the Committee of the American Institute of Actuaries, emphasizing the lack of value and the even dangerous possibilities of any Gain and Loss Exhibit essentially like the old one. In many respects it is unfortunate that this material is published neither in the *Transactions of the Actuarial Society of America* nor the *Record of the American Institute of Actuaries*, but the report has received such wide distribution that I scarcely feel like repeating much included therein.

The essential changes between the old and the new forms, well covered in detail by Mr. Fassel, may, I think, be summarized as follows:

1. The principal and fundamental change is the analysis of the change in surplus by classes or lines of business.

2. Perhaps the next most important change is the uniformity to be brought about in the calculation of the amounts of interest required to maintain the reserve and the expected mortality on insurances and on annuities.

3. Various and numerous minor changes clarifying, consolidating, rearranging, and otherwise improving many of the items. The separation of the results on additional indemnity from those on the disability provision, and the further separation of results under the latter between those on active lives and those on disabled lives, might well be considered major changes.

4. The change in the treatment of deferred and uncollected premiums is in some respects a minor change, but in such respects as introducing different accounting principles in the treatment of premiums from what is used elsewhere throughout the Exhibit, may be considered a major change.

One of the best points in the old Gain and Loss Exhibit was that it was, with minor exceptions, on the "incurred" or "revenue." Now the premiums are on a "hybrid" basis which for brevity and practical purposes may be called the "cash" basis, because the cash income is adjusted only by the relatively unimportant item of "Premiums Paid in Advance" with no adjustment for deferred and uncollected premiums. That the latter item is not entirely insignif-

icant may appear from the fact that the difference between the incurred premiums and the hybrid premiums in the case of the Travelers Insurance Company last year would be nearly $2,000,000. Some of the phases of this treatment may be summarized as follows:

1. All items except deferred premiums are now on the incurred basis.

2. The Gain and Loss Exhibit in one way or another directly accounts not only for the income and disbursement items in the year but for all other changes in non-ledger and non-admitted assets and liabilities for the year except possibly for the increase in reserve, the treatment of which is peculiar to the Gain and Loss Exhibit. Not accounting directly for the increase in the largest non-ledger asset items is inconsistent with the rest of the Exhibit.

3. The expenses are on the incurred basis, while the premiums are approximately on the cash basis. This will result in the comparison of incurred expenses with cash premiums not only in the Gain and Loss Exhibit itself but probably in various insurance publications. As indicated above, our premium income last year would have been about 3 per cent less than on the incurred basis which I think is more proper.

4. My recollection is that the revenue basis statements in the Canadian blank and in the British form are properly on the revenue basis. The Life Convention blank is now inconsistent with the Miscellaneous and Fire blanks in which earned premiums are shown.

5. The fundamental relations of computing theoretical items so as to develop the reserve is considerably complicated. In this connection the method I have used in working the Gain and Loss Exhibit may be of some interest as I think it has some advantages in simplifying the work and making sure that there is no "Balance Unaccounted For" or at least the "Balance Unaccounted For" is really a "Balance not Otherwise Accounted For." In the first place, the Gain and Loss Exhibit is developed somewhat along the general accounting principles making the income and disbursement items and changes in non-ledger assets and non-admitted assets and liabilities develop to the change in surplus, during which process the increase in reserve is inserted as one item on the side. Entirely independently of this the reserves are analyzed and developed so that the net changes due to all theoretical items balance to the net change in reserve. This reserve has been analyzed by classes such as Regular $3\frac{1}{2}$ per cent, Regular 3 per cent, Reinsurance, Group, and Annuities so that in each class a development is obtained showing that the mean reserve at the beginning of the year, increased by net premiums and interest required to maintain the reserve and decreased by the cost of insurance and reserves released by death, surrender, lapse, and other methods of termination, with proper allowance for reserves set up on Extended Term, Paid-up,

and other changes, balances to the mean reserve at the end of the year. As this development all balances out to the one item of increase in reserve as first used, the whole Exhibit must necessarily balance. For this purpose our net premiums are obtained on the incurred basis. Under the new Exhibit this formula for the development of the reserve will no longer hold for the items actually inserted in the blank because our net premiums must be reduced by the increase in the net and deferred and uncollected premiums. Instead of using the full increase in the non-ledger assets and the full increase in the liabilities that would be involved in the incurred basis, each of these items is reduced by the same amount without, of course, affecting any change in the surplus. It might be recalled that the loading on deferred and uncollected premiums is not admitted assets, and, therefore, has no effect on the surplus.

In this connection I should like the opportunity of questioning an opinion that has been expressed twice in the *Record:* once in a paper in 1912 on the Gain and Loss Exhibit and once in a discussion in 1920 of the revision of the Gain and Loss Exhibit. It will be clearer to consider this point at first from the standpoint of the old Gain and Loss Exhibit. The opinion has been expressed that in making up the Gain and Loss Exhibit for any given year, instead of using the deferred and uncollected premiums given in the previous statement, that only such premiums should be used as prove collectible during the year.

It is true that in some respects such a treatment would give the premium income for the year, though even in this respect it would be consistent to use at the end of the year only such deferred and uncollected premiums as would prove collectible during the next year. But this seems to be an incomplete view. All changes in non-ledger assets as well as on liabilities necessarily affect the surplus. If a premium is included in one annual statement as a good asset and never collected, it correspondingly reduces the surplus when dropped as an asset. In other words, such a case must be accounted for in some way as affecting the surplus. The only ways that I can see that would be at all possible would be either (1) as affecting the premium income of the year; or (2) as an entirely separate independent item, for instance, as a charge to "loss" similar to the Profit and Loss items; or (3) as being lost somewhere in a balancing item, as, for instance, in the cost of insurance or in the "Balance Unaccounted For." In fact, if such premium items as have proved uncollectible have not been accounted for among incurred premi-

ums, I should expect that they would be lost either in the expected mortality on the net amount at risk or in the "Balance Unaccounted For." Not only do the latter two methods seem illogical and undesirable to me, but the first method seems logical. The incurred premium income actually then obtained is the net increase in premiums for the year after allowing for the loss of all premiums considered in the previous year as good assets but later found to be uncollectible. Another viewpoint is that the addition of these incurred premium incomes for succeeding statements gives as truly as possible a total incurred premium income for any longer period of time, whereas under the method criticized the total premium income over an extended period would be greater than it should be by the amount of all such premiums during that period that proved uncollectible. As deferred and uncollected premiums are eliminated among the gross items under the new blank, this feature is of interest only in connection with the net premiums. If each net premium corresponding to the gross premium is obtained directly by the accounting system, no adjustment of the net premiums will be necessary in this respect; but if the total incurred net premiums are in any way obtained, they should be reduced by the increase in the net deferred and uncollected premiums between the previous statement and the present statement, regardless of whether or not the items in the previous statement proved collectible.

In listing the changes between the old and the new blank, I believe Mr. Fassel omitted the fact that surrender values are now on the incurred basis instead of being treated as two items—one, the cash disbursements; and the other, the change in outstanding surrender values.

As I have attempted to complete the blank on the basis of our last year's figures, some of the points encountered may be of interest. In the first place, it is implied that each company has a division according to each interest rate on which the reserve is carried with respect to the first seventeen items in the instructions; that is, not only must there be a division between the net premiums on the 3 per cent, $3\frac{1}{2}$ per cent, and 4 per cent reserve basis, but there must be a corresponding division of the reserves released, and such a division of deferred and uncollected premiums is also assumed though, inasmuch as the instruction part of the Exhibit will not have to be filed, the latter division can be avoided if the premiums are on the incurred basis in the first place. Incidentally, it might be mentioned

that in computing the "required interest on insurances" that the deferred and uncollected premiums are properly allowed for, though whether premiums paid in advance are properly treated does not appear from the wording of the instructions.

In some respects there is considerable question as to whether the instructions should be followed literally or not. For instance, the expected mortality on insurances is developed as a balancing item by a process proceeding from the mean reserve at the beginning of the year to the mean reserve at the end of the year. In the three following cases if the blank is followed literally the changes will affect the expected mortality, though they should have no such effect:

1. A known error in the reserve as of the beginning of the year.

2. A change in reserve on Reinsurance Ceded to an unauthorized company which has not been deducted from the reserve because not permitted by some Insurance Department, and which change under certain circumstances could be quite appreciable as, for instance, if a non-admitted company is admitted to a state and all reserve on Reinsurance Ceded to that company becomes as admitted at one time.

3. A change in the reserve basis on any block of business as, for instance, from 4 per cent to $3\frac{1}{2}$ per cent.

The separation of the results under active lives and under disabled lives under the disability provision has not seemed desirable to me, as whether a gain or a loss is thrown into one or other classes depends entirely upon the present value of disability claims incurred during the current year. This item, in turn, is apparently dependent upon the reserve basis adopted. I assume that in the company with which I am connected this would be Hunter's Permanent Total Disability Table, which, regardless of its undoubted service in the past and its present value, seems to me unsuitable for such a purpose. In fact, I think this separation is out of line with what I have considered the present development of the permanent total disability provision, the actuarial development of which until recently has been involved in joining and combining actuarially mortality and disability rates among active lives, and separately, mortality rates among disabled lives. More recent development has been more along the lines of accident and health insurance as employed in the Manchester Unity Table where the tables are practically inseparable. From Mr. Hezlett's paper in Volume XXIV of the *Transactions* you cannot obtain the proportion of lives becoming

disabled, nor can you obtain other separations at the foundation of the old theory. The present development considers the amount of sickness. I am wondering what should be done to obtain this separation between active and disabled lives in the case of the company with which I am connected which has examined death claims and in many cases approved disability claims after death even where no application had been in any way made by the insured for the disability benefits. It seems to me that if the instructions are followed, that we should set up as a loss to disability the reserve that we would have set up if the insured had made the claim, and inasmuch as our regular reserve is on the basis of Hunter's Table, that that table should be used as a basis of this calculation. In other words, while I have very little idea what the results in our case would be, it would not be impossible, I believe, to show a loss of $1,000,000 on active lives and a gain of $1,000,000 on disabled lives. In this connection it may be of interest to note that a new liability item has been inserted in the Convention addition to the annual statement to provide for "reserve on unreported disability claims." If the experience of other companies is anything like our company's, these claims are reported usually months after inception of disability. In fact, the average period in our case in the last two years has been something like ten months, most of which has been before any notice at all has been received by the company. I assume that the full reserve the first year that this is instituted and that subsequent increases will be charged against active lives instead of against disabled lives.

Most of the calculation of the gain and loss is a matter of following instructions, which in most cases will not be very difficult. Possibly the greatest need for judgment will be in the separation of expenses and similar items in calculating the gains or losses by class of business in the second part of the Exhibit in cases where companies issue different classes. This subject is perhaps worthy of a separate discussion, but possibly my paper in Volume XXIII, page 83, of the *Transactions*, on "Expense Investigations and Some Other Statistical Problems Dependent Thereon," may be of some assistance. Possibly the point in the whole Exhibit that will the most repay consideration before the end of the year if our company transacts different classes of business will be this separation, as it involves not only the income and disbursements during the year and the non-ledger and non-admitted assets and liabilities at the

end of the year, but it also involves the non-ledger and non-admitted assets and liabilities at the beginning of the year. It is in obtaining these items, involved in last year's annual statement, that will in such cases cause most trouble if this has not already been done, and it should be completed before the end of the year when all the work connected with the new annual statement is pressing. The separation by classes of the ledger assets and surplus at the beginning of the year is not needed though in general it may be more satisfactory to the actuary to have a division of funds by classes of business from inception of business. This, however, is not necessary to complete this blank because only the increase in surplus, instead of the surplus itself in each class, is required.

MR. H. H. PEIRCE:

The members of this Institute may be interested to know that the company with which I am associated (Massachusetts Mutual) has for some years past employed this method of calculating its expected mortality and required interest under insurances. We have found it satisfactory both as to economy of time and reasonable accuracy. One objection to the method is the fact that it is based on the assumption that the reserves are correct. Accordingly, a company that heretofore has used a method of calculating its cost of insurance to check the reserve will either have to continue the old method along with the new or apply some other independent means of checking the reserve.

I agree with the previous speaker that the "Instructions" must not be followed too literally. For example, line 31 of the "Instructions" reads, "Net Consideration for Annuities per Exhibit." The net premiums for annuities, of course, do not appear in the annual statement. In getting the item for this particular line, the loading will have to be deducted from the gross premiums. That also holds true for the next line—"Consideration for Supplementary Contracts Involving Life Contingencies." The loading thereunder will have to be deducted to get the correct amount of net consideration.

You may also be interested to know that the "Instruction Sheet" states that "Companies are required to follow these instructions but are not required to fill in the figures." It is rather unfortunate that the new Gain and Loss consists of two exhibits that do not show identical results.

BOOK REVIEWS AND NOTICES

Statistical Methods. By FREDERICK C. MILLS, Columbia University. New York: Henry Holt & Co., 1924. Pp. xvi+604.

While this book presents certain general statistical methods applicable to all fields of statistics, the primary emphasis is on the analysis of material drawn from economic and business statistics. In fact, the general methods throughout the book have been made concrete by applications drawn from these particular fields.

Very little knowledge of mathematics is required for the reading of approximately the first half of the book. Indeed, the author starts his treatment of graphical representation by the plotting of both statistical data and certain simple mathematical functions such as $y=ax+b$ and $y=\sin x$. He presents early the plotting of certain curves on logarithmic and semi-logarithmic paper. The second half of the book involves considerable mathematics including least squares, and the mathematics of partial and multiple correlation up to four variables.

The student of economics who acquires a thorough knowledge of the methods presented in this book will have at hand the tools for making important investigations which he could not make without such knowledge.

The special features of the book which appeal to the reviewer are the analysis of time series and the treatment of the problem of estimation. The concrete illustration presented in connection with the problem of estimation deals with the relation of the prices of oats in the United States to the corresponding productions for data based on years 1881-1913. The method really deals with ratios of actual production to production as given by a line of trend of production, and ratios Y of price to price as given by a line of trend of prices. Equations of three types,

$$y=ax^n, \qquad y=ab^x, \qquad 1/y=a+bx,$$

are used to represent the regression curve of y on x. Comparisons are made of results by using the standard error of estimate as a criterion, and by using further the correlation between the values of $\log y$ and $\log x$ in $\log y=\log a+n\log x$ obtained from $y=ax^n$, and between similar linearly related functions obtained from the other equations.

On the questions of statistical induction and the problem of random sampling, the treatment is confined to a brief and simple description of the nature of the problem, and to giving certain well-known formulas.

The book contains an Appendix devoted to a simple explanation of the method of least squares, and to a glossary of symbols.

It is an interesting fact that at least a half-dozen books on statistical methods have appeared in the past year, and others are scheduled to appear within the next few months. The present book will be found especially useful to the student of economics who has only a fair knowledge of collegiate mathematics. The first half of the book will be of service even to the student with no knowledge of collegiate mathematics.

H. L. R.

Elements of Statistics. By Frederick C. Kent. New York: McGraw-Hill Book Co., Inc., 1924. Pp. xii+178.

This textbook is designed to meet the requirements of students who have not had college training in mathematics. It is practically free from mathematical derivations, all development of formulas being along lines of general reasoning. Wherever non-mathematical demonstration is not feasible, the formula in question is usually stated and explained without proof.

Considerable strength is found in its brevity, simplicity, and freedom from technical phraseology. The ideas which are the basis of statistical procedure are not obscured by a mass of detailed discussion. However, these characteristics are occasionally a source of weakness. For instance, the chapter on "Index Numbers" appears to be condensed to a point where the treatment given is hopelessly inadequate. The computation of these important functions is so described as to leave the distinct impression that such are derived from tabulated data in as definite and invariable a manner as the square root of a number is extracted. The geometric mean of ratios is certainly not the only method of arriving at an index number to represent the trend of the ratios; it may even be the least used in practice of the great variety of computation methods available.

The examples given illustrate the arithmetical operations satisfactorily; however, in general they fail to go much beyond that point. These could have been chosen so as to give the student a much clearer idea of the scope and uses of statistical method. In fact, the criticism may be made of this and most other textbooks on statistics that the student is hardly given any distinct idea as to the purpose behind the development of statistical method, or as to the practical applications of the method as developed.

This textbook would appear to have a field of usefulness quite distinctly its own, arising from its very simple treatment of the subject.

E. A. P.

The Essence of Life Insurance. By WILLIAM BREIBY. New York: Spectator Co., 1924. Pp. xiv+165.

This volume, as the title implies, is an exposition of the fundamental definitions, principles, and practice of life insurance. The treatment is non-technical throughout and is not beyond the comprehension of anyone having a working knowledge of arithmetic.

The book is clearly written and is well adapted to the needs of the life insurance agent. The only exceptions which the writer would make to this statement appear on pages 92 and 93. On page 92 the author states that "The One Year Preliminary Term method is now prohibited in most states, ..." It seems to the writer that this statement should have been modified so as to exclude the 20-payment life and lower premium forms of insurance, since all states now permit such forms to be valued on the full preliminary term basis. Also, on page 93 the author does not make it entirely clear that the extra reserve under the modified preliminary term systems to provide the net single premium at the end of the premium-paying period is accumulated throughout the premium-paying period including the first year. The writer believes that the author could have improved upon his language at these points.

Life insurance companies would do well to place this volume in the hands of all new agents because through it a new man may gain a knowledge of the business which may otherwise take years to acquire. In fact, there are few experienced life insurance agents who would not profit by a thorough study and mastery of its contents. The chapters on net premiums and reserves should surely establish the agent's confidence in the adequacy of the premiums and the soundness of the legal reserve system of life insurance. Any agent who masters this volume will be adequately prepared to answer almost any question pertaining to life insurance which will arise in connection with his solicitation. The author is to be congratulated on having so clearly and completely covered all essentials of the business in such a brief volume.

The book is relatively free from typographical errors and those which appear are contained in the printed errata which accompanies the book.

J. C. R.

Workmen's Compensation Insurance. By G. F. MICHELBACKER AND T. M. NIAL. New York: McGraw-Hill Book Co., Inc., 1925. Pp. xi+503.

This book gives a complete account of the nature and development of Workmen's Compensation Insurance. It is particularly well adapted for the use of the student who wishes to obtain a comprehensive knowledge of the subject.

The authors have, unquestionably, had a great deal of practical experience in connection with the development of the business of compensation insurance during the past decade. They make the reader thoroughly familiar with the history of the problems of the subject. In doing this they have not hesitated to go into details and ramifications which must have required a great deal of work on their part but which have made the book extremely valuable as a practical reference-book for persons concerned with any aspect of the business of Workmen's Compensation Insurance.

Anyone engaged in the business of life insurance will find in this book a great deal of inspiration and much food for thought, because compensation insurance serves society in a great deal the same way as life insurance. Like life insurance, it is concerned with the protection of the disabled and their dependents.

Among the many parts of the book which are of peculiar interest to the life insurance student is the chapter on malingering on the part of claimants for compensation because of temporary or permanent, total or partial disability.

M. G.

An Introduction to Mathematical Probability. By Julian Lowell Coolidge. Oxford: Clarendon Press, 1925. Pp.xii+216.

A neat and conveniently arranged volume covering such topics as ordinary, inverse, and geometrical probability, errors of observation (including errors in many variables), mean values and dispersion, least squares and curve-fitting, with concluding chapters on the statistical theory of gases and the principles of life insurance, would hardly be expected to furnish entertainment, and even an occasional gleeful chortle. But an author who pays tribute in his Preface to the romance-like interest, originality, and brilliance of French treatises on probability, and includes in his helpful Index references to craps and poker, is himself unlikely to prove dull reading. Even Professor Coolidge's most forbidding chapters are enlivened by lucid exposition, refreshing common sense, and an occasional sly jest. A single paragraph may be quoted as an example of the Harvard professor's impatience with idiotic applications of mere rules. In discussing the applying of Bayes's principle to future events, he contends:

> The most absurd consequences have, in the past, been deduced from this formula [$p=(n+1)/(n+2)$]. Putting n equal to the number of times the sun has risen, it has been used to estimate the probability that it will rise the next day. Nothing could be more grotesque. The rising of the sun is not a statistical event whose cause is obscure, but a mechanical necessity which will continue as long as present astronomical conditions do, and will then cease. To use [this] formula

we should have to assume that all possible cosmogonies were equally likely. What such a phrase may mean is utterly beyond our comprehension: it undoubtedly means nothing whatsoever.

This same common sense combined with human interest pervades the whole book. Thus, a definition of probability is sought from the statistical point of view, while the reader is never permitted to forget the merely approximate nature of the normal curve. The attitude is fairly summarized in the Preface, "It seems to me that, in the last analysis, probability is a statistical, that is to say, an experimental science, and the mathematical problem is to establish rules which yield correct and valuable results."

The final chapter on life insurance will prove more interesting to the mathematician unversed in actuarial lore than to the actuary who is already familiar with the principles presented. The carelessness of even Oxford proofreaders in allowing the obvious misprint in the formula for A_x (p. 199) to escape detection may be noted, and a few printer's lapses elsewhere—but in general the book is as well printed as it is admirably written and arranged.

H. H. J.

OTHER RECENT PUBLICATIONS

Elements of Business Statistics. By ROBERT RIEGEL, PH.D., Professor of Insurance and Statistics, University of Pennsylvania. New York: D. Appleton & Co., 1924. Pp. xx+549.

Complete Surrender Value Ready Reckoner. By FACKLER, FACKLER, AND BREIBY. New York: Spectator Co. (To appear.)

Economics of Business Cycles. By A. B. ADAMS, Dean of School of Business, University of Oklahoma. New York: McGraw-Hill Book Co., 1925. Pp. xvi+268.

Marine Insurance. By WILLIAM D. WINTER, LL.B., Third Vice-President of the Atlantic Mutual Insurance Company of New York. New York: McGraw-Hill Book Co., 1919. Pp. xvii+433.

Mathematical Theory of Life Insurance. By CHESTER H. FORSYTH. New York: John Wiley & Sons, Inc., 1924. Pp. vii+74.

Graphic Statistics in Management. By WILLIAM HENRY SMITH. New York: McGraw-Hill Book Co., 1924. Pp. vi+360.

Calculus of Variations. By GILBERT AMES BLISS, Professor of Mathematics, University of Chicago. Chicago: Open Court Publishing Co., 1924. Pp. xiii+189.

OBITUARY

ARTHUR ENOCH CLEMENT

Born, 1887 Died, 1924

ARTHUR ENOCH CLEMENT

Arthur Enoch Clement, an Associate of the American Institute of Actuaries since 1919, died in Santiago, Chili, in 1924. Mr. Clement was born at La Serena, Chili, January 23, 1887. He graduated from the University of Chili with the degree of Bachelor of Physics and Mathematics. In 1915 he became Actuary of La Mundial Insurance Company, Santiago, Chili, and served in that capacity until his death.

Owing, doubtless, to his residence in a remote part of the world, the members of the Institute did not enjoy opportunities of knowing Mr. Clement personally. His passing in the prime of life is here recorded with regret.

ANNUAL REPORT OF THE TREASURER

of the

AMERICAN INSTITUTE *of* ACTUARIES

FOR THE YEAR ENDING MAY 31, 1925

INCOME

Cash on hand June 1, 1924	$ 586.84	
Dues	5,460.00	
Records	642.50	
Examinations	875.00	
Interest	133.45	$7,697.79

DISBURSEMENTS

Postage	$ 344.30	
Printing	5,437.00	
Library	92.70	
Insurance	54.66	
Salary	200.00	
Express	68.72	
Storage	26.24	
Stenographic expense	347.83	
Traveling expense, board members	218.72	
Examinations	298.60	
Stationery	88.15	
Exchange	20.74	
Entertainment	154.65	
Telegrams	41.90	
Miscellaneous	51.22	$7,445.43
Cash on hand May 31, 1925		252.36
		$7,697.79

Assets:

Cash	$ 252.36	
U.S. 4¼ Third Lib. Loan $3,000, book value	2,970.94	
U.S. 4¾ Treas. Notes $2,000, book value	2,039.34	$5,262.64
Liabilities		None
Surplus		$5,262.64

Audited and found correct.

E. R. CARTER
E. L. MARSHALL } *Auditing Committee*

MINUTES OF THE ANNUAL MEETING

of the

AMERICAN INSTITUTE *of* ACTUARIES

HELD AT THE LA SALLE HOTEL, CHICAGO,
JUNE 11 AND 12, 1925

The Institute was called to order by the President at 10:45 A.M., June 11th and the following Fellows, Associates and Nominees of Contributing Members were present:

FELLOWS

H. W. Allstrom
E. C. Ames
L. A. Anderson
O. J. Arnold
A. E. Babbitt
W. N. Bagley
C. H. Beckett
J. P. Bowerman
H. W. Buttolph
J. C. Cameron
E. E. Cammack
E. R. Carter
Arthur Coburn
R. B. Coit
H. R. Corbett
H. W. Curjel
F. A. Draper
J. S. Elston
P. H. Evans
James Fairlie
E. G. Fassel
George Graham
Marcus Gunn
C. E. Herfurth
J. E. Higdon
R. A. Hohaus
F. M. Hope
H. H. Jackson
W. M. Johnson
R. E. Kennon
J. M. Laird
A. W. Larsen
J. F. Little
R. C. McCankie
E. L. Marshall
H. C. Marvin
F. B. Mead
Henry Moir
P. V. Montgomery
W. O. Morris
J. H. Nitchie
J. G. Parker
G. B. Pattison
S. L. Phelps
W. F. Poorman
E. A. Porter
H. L. Rietz
J. C. Rietz
J. C. Seitz
A. A. Speers
C. F. Stein
B. J. Stookey
W. M. Strong
Helen J. Williams
F. S. Withington

ASSOCIATES

H. H. Buckman
J. A. Budinger
Helen L. Clark
W. P. Coler
C. F. Cross
A. C. Good
W. A. Jenkins
L. J. Kalmbach
A. T. Lehman
J. H. Lithgow
A. J. McAndless
W. H. McBride
C. A. McConaghy
W. D. MacKinnon
R. E. Moyer
G. E. Myers
Gertrude L. Pardieck
J. F. Reilly
H. G. Sellman
L. O. Shudde
R. B. Sturtevant
F. D. Strudell
Margaret Walker
H. S. Wiggins

NOMINEES OF CONTRIBUTING MEMBERS

G. C. Adams.....Wisconsin Life Insurance Company
P. P. Andrews.....Royal Union Mutual Life Insurance Company
G. D. Babbitt.....Bankers Reserve Life Company
O. M. Barnes.....Franklin Life Insurance Company
Henry Bauer.....Continental Assurance Company
G. W. Bishop.....Bankers Reserve Life Company
Richard Boissard.....National Guardian Life Insurance Company
P. F. Bourscheidt.....Peoria Life Insurance Company
E. G. Brown.....Southwestern Life Insurance Company
F. R. Brunner.....Security Mutual Life Insurance Company
O. J. Burian.....Missouri State Life Insurance Company
J. A. Campbell.....London Life Insurance Company
L. D. Cavanaugh.....Federal Life Insurance Company
G. A. Clark.....Equitable Life Insurance Company, Washington, D.C.
Elizabeth M. Colliver.....International Life Insurance Company
N. C. Cortright.....Old Line Life Insurance Company of America
L. C. Cox.....National Life & Accident Insurance Company
J. P. Dandy.....Confederation Life Association
R. E. Dowsett.....Manufacturers Life Insurance Company
J. E. Evans.....Ohio National Life Insurance Company
L. K. File.....Canada Life Assurance Company
C. H. Folz.....Public Savings Insurance Company of America
Neil Gilmour.....Provident Mutual Life Insurance Company
T. H. Galbraith.....Central Life Insurance Company of Illinois
S. R. Garrison.....Union Central Life Insurance Company
D. H. Garver.....Maryland Life Insurance Company
I. L. Grimes.....Mutual Trust Life Insurance Company
G. L. Grogan.....Bank Savings Life Insurance Company
M. N. Hanson.....Wisconsin Life Insurance Company
H. J. Hornberger.....Great Northern Life Insurance Company
M. A. Hyde.....Security Mutual Life Insurance Company
W. W. Lane.....LaFayette Life Insurance Company
J. R. Larus.....Phoenix Mutual Life Insurance Company
R. M. Malpas.....Reinsurance Life Company of America
Dr. C. M. McCulloch.....State Life Insurance Company
J. L. Mims.....Southern Union Life Insurance Company
Carl Mitcheltree.....Columbus Mutual Life Insurance Company
W. A. Munster.....Two Republics Life Insurance Company
H. M. Moore.....Cleveland Life Insurance Company
S. Milligan.....Metropolitan Life Insurance Company
H. H. Peirce.....Massachusetts Mutual Life Insurance Company

E. B. Raub.....Indianapolis Life Insurance Company
T. C. Rafferty.....International Life Insurance Company
G. E. Reilly.....Midland Mutual Life Insurance Company
J. B. Russell.....Peoria Life Insurance Company
J. W. Sinton, Jr.....Atlantic Life Insurance Company
F. M. Smith.....Metropolitan Life Insurance Company
V. R. Smith.....Confederation Life Association
R. J. Stoner.....Lincoln National Life Insurance Company
E. M. Thomas.....John Hancock Mutual Life Insurance Company
H. S. Tressel.....International Life and Trust Company
D. N. Warters.....Bankers Life Company
J. L. Wesley.....Lincoln National Life Insurance Company
J. D. Williamson.....Canada Life Assurance Company
Ethel Welch.....National Life and Accident Insurance Company
C. E. West.....Provident Mutual Life Insurance Company
H. R. Wilson.....American Central Life Insurance Company
A. B. Worsham.....Union Central Life Insurance Company

The Report of the Secretary was presented and accepted.

The Secretary announced that the following were admitted to Fellowship without examination:

Edmund Ernest Cammack

and the following were admitted to Associateship without examination:

Arthur Pedoe
James Thompson Phillips

The President also announced that the following had been admitted to Fellowship by examination:

Henry Hollister Jackson
Dorothy May Jamison
Wilbur Moore Johnson
Ralph Eugene Kennon
William Frank Poorman

and that the following had been admitted to Associateship by examination:

David Wright Bell
John Percy Dandy
Malvin Edward Davis
Allen Carlson Eastlack
Leland John Kalmbach
George Kenigson
Will Donald MacKinnon
D. Ralph McClurg
Ross Emerson Moyer
Louis Otto Shudde
Margaret Walker
Harold Graham Walton
Jay Rollin Young

Election of officers was by motion deferred until 3:00 P.M., June 12.

Formal papers were then presented.

The Treasurer submitted his report which was accepted. The morning sessions adjourned at 12:45 P.M.

The afternoon session reconvened at 2:25 P.M. and the presentation of formal papers was continued.

Papers presented at the November meeting were then discussed.

The meeting adjourned at 4:10 P.M.

Friday, June 12, the Institute reconvened at 10:20 A.M., and proceeded with the discussion of informal topics on the program. The meeting adjourned at 12:00 noon.

The President presented diplomas to Henry Hollister Jackson, Wilbur Moore Johnson, Ralph Eugene Kennon, William Frank Poorman, and Miss Dorothy May Jamison (in absence).

The informal discussion was continued until 3:00 P.M. when the special election of officers took place. This resulted as follows:

President.....PERCY H. EVANS
Vice-President.....JOHN GOWANS PARKER
Secretary.....REGINALD CAMPBELL MCCANKIE
Treasurer.....BERT JAY STOOKEY
Governors for three-year term.....ARTHUR COBURN, SINCLAIR EDWARD ALLISON

The President reported that the Board appointed E. R. Carter as librarian and E. G. Fassel as editor of *The Record* for the ensuing year. Upon motion duly seconded, the Institute confirmed these appointments.

The meeting adjourned at 4:00 P.M.

EXAMINATIONS OF THE INSTITUTE

HELD ON APRIL 14 AND 15, 1925

EXAMINATION FOR ADMISSION AS ASSOCIATE

PART 1

(FORMERLY SECTION I—PART I)

1. Find the H.C.F. of $2x^4+2x^3-x^2-2x-1$, x^4+x^3+4x+4, and $2x^3-3x^2-6x-1$.

2. (*a*) Simplify $\sqrt{3}+\frac{5}{\sqrt{7}}+\sqrt{10-2\sqrt{21}}$.

(*b*) Find a factor that will rationalize the denominator of $\frac{1}{\sqrt{2}-\sqrt[3]{3}}$.

3. Solve the equations

(*a*) $x^2-5x+2\sqrt{x^2-5x+3}=12$

(*b*) $\frac{x^2}{y}+\frac{y^2}{x}=\frac{9}{2}$

$x+y=3$

4. Prove that the fraction $\frac{a_1+a_2+\ldots+a_n}{b_1+b_2+\ldots+b_n}$ has a value between the greatest and the least of the unequal fractions $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \ldots, \frac{a_n}{b_n}$, if their denominators are all positive.

NOTE.—In your proof point out where you make use of the fact that the denominators are positive.

5. Prove that a quadratic equation in one unknown quantity has two, but not more than two, roots.

6. A and B agree to do a piece of work in 6 days for \$45. To finish on time they hire C during the last two days, which results in B's total pay being reduced by \$2. If A could have done the work alone in 12 days, what daily wage does each man earn?

7. If the roots of the equation $ax^2+bx+c=0$ are α and β, and those of the equation $Ax^2+Bx+C=0$ are $\alpha+\delta$ and $\beta+\delta$, prove that $\dfrac{b^2-4ac}{a^2}=\dfrac{B^2-4AC}{A^2}$.

8. Prove that the bisector of an angle of a triangle divides the opposite side into segments which are proportional to the other two sides.

9. Construct a circle which shall touch a given straight line, and pass through two given points.

10. Given $ABCD$ a parallelogram. Any straight line AEF is drawn, cutting BC in E and DC produced in F. Show that half the parallelogram is a mean proportional between the triangles ABE and ADF.

PART 2

(FORMERLY SECTION I—PART II)

1. Sum to infinity the series

$$1-(1-b)r+(1-b+b^2)r^2-(1-b+b^2-b^3)r^3+\ \ldots\ ,$$

the values of b and r being restricted so that the sum is finite. State the necessary restrictions.

2. Given $\Sigma n^2=\dfrac{n(n+1)(2n+1)}{6}$ and $\Sigma n^3=\dfrac{n^2(n+1)^2}{4}$

prove that $\Sigma n^4=\dfrac{3n^2+3n-1}{5}\Sigma n^2$.

3. Solve the equations

(*a*) $(x+9)(x-3)(x-7)(x+5)=385$

(*b*) $x^4+1-3(x^3+x)=2x^2$

4. A city has 20 streets running north and south, and 15 east and west. Find the number of ways in which a man can travel from the southwest to the northeast corner of the city, going the shortest possible distance.

5. In a shooting competition a man can score 5, 4, 3, 2, or 0 points for each shot. Find the number of different ways in which he can score 30 in 7 shots.

6. (*a*) Sum to n terms the series whose nth term is $n(2n-1)$.

(*b*) Sum to infinity the series whose nth term is $\frac{3n^2-2n+5}{n!}$.

7. It is known that at a certain place it rains on the average 6 days out of 30. Find the probablity that it will rain on not more than two out of three specified future days.

8. In the recurring series $\frac{5}{4}-\frac{1}{2}x+2x^2+x^3+5x^4+7x^5+\ldots$,

the scale of relation is a quadratic expression. Find the scale of relation, the generating function, and the general term.

9. A makes a bet with B of \$9 to \$6 that in throwing two dice he will throw 7 or 11 before B will throw 4 or 9. Each has a pair of dice and they throw simultaneously until one of them wins. Find B's expectation.

10. In a certain locality the number of deaths and births per annum is one nth and one mth respectively of the population at the beginning of the year. In how many years will the population increase r-fold, neglecting immigration and emigration?

PART 3

(FORMERLY SECTION II—PART I)

1. Two angles of a triangle are 36° and 46°, and the greater of the two opposite sides is 20. Find the lengths of the two remaining sides, and the area of the triangle, given $\sin 36° = .588$, $\cos 46° = .695$, and $\tan 8° = .141$.

2. Prove that (*a*) $\tan A + \cot 2A = \csc 2A$.

(*b*) $(\sec 15° + \csc 15°)^2 = 24$.

3. Derive a formula for the area of a triangle in terms of two angles and the included side.

4. The angles of depression of the top and bottom of a tower observed from the top of a hill 108 feet high are 30° and 60° respectively. Find the height of the tower.

5. Show how to solve an oblique triangle, given the three sides.

6. Reduce the equation $5x+12y-26=0$ to (1) slope-intercept form, (2) intercept form, (3) normal form. Draw the graph, and state the significance of the constants in each form.

7. Prove analytically that the sum of the squares of the medians of a triangle is equal to three-fourths the sum of the squares of the three sides.

8. From one end of a diameter of a circle whose radius is r, chords are drawn and produced their own length. Find the equation of the locus of the ends of these lines.

9. Find the co-ordinates of the center, the length of the semi-major axis, and the length of the semi-minor axis of the ellipse $4x^2-16x+9y^2+36y+16=0$. Show what transformation of co-ordinates will simplify the equation.

10. A point moves so that its distance from the point (2, 3) is twice its distance from the line $x+y-1=0$. Find the equation of the locus of this point, and name the curve.

PART 4

(FORMERLY SECTION II—PART II)

1. Develop the formula

$$f(x)=f(0)+\Delta f(0)x^{(1)}+\frac{\Delta^2 f(0)}{2!}x^{(2)}+\frac{\Delta^3 f(0)}{3!}x^{(3)}+\ \ldots\ .$$

where $x^{(m)}=x(x-1)\ \ldots\ .\ (x-m+1)$.

2. Express x^3+x^2+x+1 as a sum of factorials, and hence find its finite integral.

3. Given the values of $f(0), f(1), f(2), f(3)\ \ldots\ .$, explain fully how to find $f\left(\frac{1}{5}\right), f\left(\frac{2}{5}\right), f\left(\frac{3}{5}\right), \ldots\ .$

4. Prove that $\frac{d^2y}{dx^2}=\Delta^2 y-\Delta^3 y+\frac{11}{12}\Delta^4 y-\frac{5}{6}\Delta^5 y+\ \ldots\ .$

5. Given $f(0)=58842$, $f(1)=55257$, $f(2)=51368$, $f(5)=37977$, find $f(3)$ and $f(4)$.

6. (*a*) Differentiate $e^{3x}(\sin^3 4x - \log \tan x)^2$.

(*b*) Given $(x-a)^2+(y-b)^2=c^2$, find $\frac{d^2y}{dx^2}$.

7. Derive the formula for integration by parts, and apply it to integrate $\frac{\log (x+2)}{(x+1)^2}$.

8. (*a*) Prove the approximate integration formula

$$\int_0^{2n} f(x)dx = \frac{n}{3}[f(0)+4f(n)+f(2n)]$$

(*b*) Hence show that

$$\int_a^b f(x)dx = \frac{b-a}{6}f(0)+\frac{2}{3}\left[bf\left(\frac{b}{2}\right)-af\left(\frac{a}{2}\right)\right]+\frac{1}{6}\left[bf(b)-af(a)\right]$$

9. Find the interval of convergence of the series obtained by expanding cos x into a power series.

10. (*a*) State sufficient conditions for maximum and minimum values of $f(x)$.

(*b*) Test $x^2+\frac{432}{x}$ for maximum and minimum values.

PART 5

(FORMERLY SECTION III—PART I)

(Table of $a_{\overline{n}|}$ for use with this paper is printed on the last page.)

1. Define effective rate of interest, nominal rate of interest, and force of interest; and calculate them for a loan of \$100 on which interest of \$7 is paid at end of each two years.

2. State what functions are represented by the following expressions and derive them:

(*a*) $\dfrac{\left(1+\frac{j}{m}\right)^{mn}-1}{p\left[\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1\right]}$ (*b*) $\dfrac{1-e^{-n\delta}}{e^{\delta}-1}$

3. A $6\frac{1}{4}$ per cent $1,000 bond with half-yearly coupons and maturing for $1,000 8 years hence is to be replaced by a new 5 per cent $1,000 bond with half-yearly coupons and maturing at a premium 12 years hence. At what premium must the new bond mature, if the exchange is to be equitable and money is worth 6 per cent, convertible semiannually?

4. Given only the rate of interest, explain how you would calculate and verify a table of v^n for years 1 to 20.

5. (*a*) If g is the rate of interest paid on the par value of a bond maturing at end of n years, purchased at a premium of k per unit, and if $a'_{\overline{n}|}$ is at a rate i', which is a fairly close approximation to i, the true effective rate earned, then derive a formula in terms of g, k, and $a'_{\overline{n}|}$ giving a close approximation to i.

(*b*) Explain how the result of a first approximation by the formula derived in (*a*) could be used in the same formula to secure a still closer approximation.

6. A bond of $1,000, redeemable December 31, 1934, bears interest up to and including December 31, 1929, at 6 per cent payable half-yearly and thereafter at 5 per cent payable half-yearly. Find the approximate yield to an investor who buys on December 31, 1924, at the price of 104.

7. A life insurance company lends $100,000 on a certain property. It later purchases the property for home-office use, paying $150,000 in cash and assuming all liens, which consist of the mortgage referred to, $3,500 accrued interest on same, and $7,500 taxes.

The company after purchase proceeds to discharge all liens, either by payment or proper book entries. It also expends $12,500 in cleaning, redecorating and in minor alterations. When ready for occupancy, a competent appraiser finds the property to be worth $290,000.

It is decided to include the property in the company's assets at *cost*. At what amount should it appear on the books? Give in detail how all the foregoing transactions should be entered on the books.

8. State the book entries necessary in a life insurance office for the following:

(*a*) Dividend applied to pay renewal premium.

(*b*) Matured endowment paid in cash.

(*c*) Death claim left for settlement under a supplementary contract not involving life-contingencies.

(*d*) Mortgage loan repaid with accrued interest.

TABLE OF $a_{\overline{n}|}$ FOR USE IN CONNECTION WITH THIS PAPER

n	1½ Per Cent	2 Per Cent	2½ Per Cent	3 Per Cent	4 Per Cent	5 Per Cent	6 Per Cent
1	0.9852	0.9804	0.9756	0.9709	0.9615	0.9524	0.9434
2	1.9559	1.9416	1.9274	1.9135	1.8861	1.8594	1.8334
3	2.9122	2.8839	2.8560	2.8286	2.7751	2.7232	2.6730
4	3.8544	3.8077	3.7620	3.7171	3.6299	3.5460	3.4651
5	4.7826	4.7135	4.6458	4.5797	4.4518	4.3295	4.2124
6	5.6972	5.6014	5.5081	5.4172	5.2421	5.0757	4.9173
7	6.5982	6.4720	6.3494	6.2303	6.0021	5.7864	5.5824
8	7.4859	7.3255	7.1701	7.0197	6.7327	6.4632	6.2098
9	8.3605	8.1622	7.9709	7.7861	7.4353	7.1078	6.8017
10	9.2222	8.9826	8.7521	8.5302	8.1109	7.7217	7.3601
11	10.0711	9.7868	9.5142	9.2526	8.7605	8.3064	7.8869
12	10.9075	10.5753	10.2578	9.9540	9.3851	8.8633	8.3838
13	11.7315	11.3484	10.9832	10.6350	9.9856	9.3936	8.8527
14	12.5434	12.1062	11.6909	11.2961	10.5631	9.8986	9.2950
15	13.3432	12.8493	12.3814	11.9379	11.1184	10.3797	9.7122
16	14.1313	13.5777	13.0550	12.5611	11.6523	10.8378	10.1059
17	14.9076	14.2919	13.7122	13.1661	12.1657	11.2741	10.4773
18	15.6726	14.9920	14.3534	13.7535	12.6593	11.6896	10.8276
19	16.4262	15.6785	14.9789	14.3238	13.1339	12.0853	11.1581
20	17.1686	16.3514	15.5892	14.8775	13.5903	12.4622	11.4699
21	17.9001	17.0112	16.1845	15.4150	14.0292	12.8212	11.7641
22	18.6208	17.6580	16.7654	15.9369	14.4511	13.1630	12.0416
23	19.3309	18.2922	17.3321	16.4436	14.8568	13.4886	12.3034
24	20.0304	18.9139	17.8850	16.9355	15.2470	13.7986	12.5504
25	20.7196	19.5235	18.4244	17.4131	15.6221	14.0939	12.7834
26	21.3986	20.1210	18.9506	17.8768	15.9828	14.3752	13.0032
27	22.0676	20.7069	19.4640	18.3270	16.3296	14.6430	13.2105
28	22.7267	21.2813	19.9649	18.7641	16.6631	14.8981	13.4062
29	23.3761	21.8444	20.4535	19.1885	16.9837	15.1411	13.5907
30	24.0158	22.3965	20.9303	19.6004	17.2920	15.3725	13.7648

PART 6

(FORMERLY SECTION III—PART II)

1. A man aged thirty marries a woman aged twenty. Assuming that he does not remarry after death of his wife, express in terms of l_x and d_x the probabilities of the following events:

(*a*) He is a widower at the end of three years.

(*b*) He becomes a widower within three years.

(*c*) The wife dies in the tenth year.

(*d*) The man survives his wife, and himself dies within one year after attaining age sixty or age sixty-five.

2. Assuming $\omega = 96$, prove that for

$$x = 70,\ e_x = 25p_x - 24_1|q_x - 23_2|q_x - 22_3|q_x - \ \ldots\ldots,\ \text{etc.}$$

3. (*a*) Develop by general reasoning the formula

$$A_x = v(1 - ia_x)$$

(*b*) Write expressions for $P_{x:\overline{n}|}$ in terms of: (i) interest and annuity functions; (ii) interest and assurance functions; (iii) commutation symbols.

4. Express in words the meaning of the following symbols, and give formulas in terms of ordinary single and joint-life annuities by which their values could be calculated:

$$a_{\overline{xy}|z}\,, \qquad a_{x|yz}\,, \qquad {}_{n|m}a_{xy}$$

5. An annuity is to begin ten years from the present, provided x be not then living, and to continue so long as either y or z survives. Write the net annual premium for the contract in terms of ordinary annuities and probabilities and explain why the annuity appearing in the denominator is used.

6. Define and distinguish between

$$\mathring{a}^{(m)}_{y|x} \text{ and } \ddot{\mathring{a}}^{(m)}_{y|x}\,.$$

7. A policy with premiums ceasing at age sixty-five or on prior death provides \$10,000 on death of insured before age sixty-five and a guaranteed income of 5 per cent in advance on \$10,000 for lifetime of insured commencing at age sixty-five with payment of \$10,000 on subsequent death of insured. Calculations will be by a specified mortality table and interest at $3\frac{1}{2}$ per cent.

(*a*) Assuming premium and income payments made monthly and the death benefit payable at moment of death, find an expression in commutation symbols for the net true monthly premium.

(*b*) Modify the expression in (*a*) to give the net annual premium if premium and income payments are annual and the death benefit is payable at end of year of death.

8. Give symbols and find expressions in terms of assurances payable on the first death for:

(*a*) An assurance on z if he die third of three lives.

(*b*) An assurance on y if he die second of three, z having died first.

(*c*) An assurance payable on the failure of the joint lives x and y, if such failure occur before the death of z.

(*d*) An assurance on the survivor of x and z if survivor dies before y.

PART 7

(FORMERLY SECTION IV—PART I)

1. Derive general formulas in terms of l_x and L_x for finding the following for a stationary population supported by l_0 annual births:

(*a*) Number of deaths annually among those aged x to $x+n$.

(*b*) Average age of population above age x.

(*c*) Average age at death of a stationary population kept up by births alone.

(*d*) Average age at death of present members of the community.

2. The working staff of a government department is in a stationary condition and 400 new employees are hired each year at age twenty-five. All employees are required to contribute to a pension fund and are given the option of retiring at age sixty or at age sixty-five. Assuming no separations from the service except by death, that one-half of those who reach age sixty retire at once, and that the survivors of the remainder retire at age sixty-five, give expressions for:

(*a*) The total number of present contributors.

(*b*) The total number of present pensioners.

(*c*) The total number of future years' service with which present contributors will be credited.

3. (*a*) You have available for a certain mortality table and rate of interest the commutation columns Nx and Dx. You require also Mx and Rx. Show how you would prepare work sheets for the purpose of obtaining the required functions.

(*b*) Given a table of q_x, write a continuous formula and a check formula for computing the values of a_x at a stated rate of interest. Give reasons for your choice of the particular formulas given.

4. (*a*) In the following table appear values of a_x for certain ages and rates of interest. Derive an approximate value of a_{68} at 5 per cent interest.

Age x	3½ Per Cent	4 Per Cent	4½ Per Cent
60.....	10.032	9.648	9.288
65.....	8.219	7.952	7.699
70.....	6.482	6.307	6.140

(*b*) Indicate an alternative method of calculation by which the result obtained in (*a*) may be verified.

5. Assume for the sake of simplicity a mortality table with a radix of 60,000 at age thirty, and with 1,000 deaths at each age. Taking $v^n = 1 - ni$, find $\bar{a}_{70:\overline{80}\ 70}$ at 4 per cent, using Hardy's formula:

$$\int_0^{6n} u_x dx = n\{.28(u_0+u_{6n})+1.62(u_n+u_{5n})+2.2u_{3n}\}\ .$$

6. (*a*) Express as definite integrals: $\overline{A}^{1}_{x:\overline{n}|}$, T_x, and $\bar{a}_{x|\overline{yz}}$

(*b*) Find expressions for

$$\frac{d}{d_x}\overline{M}_x \text{ and } \frac{d}{d_x}D_x$$

(*c*) What functions are represented by the following integrals:

$$\frac{1}{l_x l_y l_z}\int_0^{\omega-x} v^t l_{x+t}(l_y - l_{y+t}) l_{z+t}\mu_{x+t}dt$$

and

$$\frac{1}{l_x l_y l_z}\int_0^{\omega-x} v^t l_{x+t} l_{y+t} l_{z+t}(\mu_{x+t}+\mu_{y+t})dt\ .$$

7. Assuming third differences to vanish, derive the relation $\frac{df(x)}{dx} = \Delta f(x) - \frac{1}{2}\Delta^2 f(x)$ and show that $\mu_x = \frac{l_{x-1}-l_{x+1}}{2l_x}$(approx.) .

8. By use of the approximate integration formula

$$\frac{1}{m}\sum_{t=0}^{t=\infty} f\left(\frac{t}{m}\right) = \sum_{t=0}^{t=\infty} f(t) - \frac{m-1}{2m}f(0) + \frac{m^2-1}{12m^2}\cdot\frac{df(0)}{dt} \ \ldots\ ,\ \text{etc.},$$

derive an approximation to $a_x^{(m)}$ in terms of a_x, μ_x, δ, and m.

PART 8

(FORMERLY SECTION IV—PART II)

1. (*a*) Under what conditions will the prospective and retrospective methods of valuation yield the same results? Assuming such conditions, prove the identity of the two for the nth year reserve for a twenty-payment life policy in the case where n is less than 20.

(*b*) Prove algebraically that

$$ {}_nV_x = 1-(1-{}_1V_x)(1-{}_1V_{x+1}) \ldots . (1-{}_1V_{x+n-1}) . $$

2. (*a*) Express ${}_nV_x$ in terms of P_x, P_{x+n} and d. Interpret the expression verbally.

(*b*) Develop by general reasoning the formula

$$ {}_nV_x = A_{x+n}\left\{1-\frac{P_x}{P_{x+n}}\right\} . $$

3. Assuming $\mu_x = A+Bc^x$, show algebraically the effect produced on ${}^{(\infty)}_{n}V^{(\infty)}_{x}$ if the value of c becomes 1.

4. For a certain class of substandard business, your company assumes that the mortality rate will be the standard rate plus a constant of .002, i.e., $q'_x = q_x + .002$. Show briefly how the reserves for a whole-life policy in this class will compare with the regular reserves. Will the same relation hold for endowment policies? Give reasons for your answer.

5. A life-annuity deferred for n years is issued with the provision that if death occurs within the deferred period, the gross premiums are to be returned, and if annuitant survives n years, m annuity payments will be guaranteed. The contract is to be paid for by n annual premiums. Assuming the loading to be a percentage plus a constant, derive an expression in commutation symbols for the net premium and for the tth-year terminal reserve.

6. Your company reinsures a company which has policies in force on the twenty-payment life plan with guaranteed reduced paid-up insurance in the same proportion to the original insurance as the number of years' premiums paid is to twenty. Show how you would use tables of net premiums for limited-payment life

policies to investigate whether the reserve liability on the paid-up insurance would be covered by the reserve on the original policy upon the option to take reduced paid-up insurance being exercised at the end of five, ten, or fifteen years after issue.

7. A survivorship annuity is to begin at the death of the insured, aged x, and continue during the lifetime of the beneficiary, aged y. The annuity is to be paid for by annual premiums during the joint lifetime of x and y.

(*a*) Derive an expression for the mth-year terminal reserve if x and y are then living.

(*b*) Show that if $\frac{a_{y+m}}{a_{x+m:y+m}} < \frac{a_y}{a_{x:y}}$ the mth year reserve will be negative.

8. (*a*) A twenty-premium partial-endowment policy provides for the payment of its face amount if death occurs within twenty years and matures at the end of that time for an amount, K, less than the face amount. Under what conditions would it be necessary to modify the full preliminary term net premium and reserves on this policy to conform with the Illinois Standard of Valuation? Give expressions for the mth-year terminal reserve according to the Illinois Standard.

(*b*) Same as (*a*) for the Ohio Standard.

(*c*) State the minimum reserve requirements either of New York or of Canada.

EXAMINATION FOR ADMISSION AS FELLOW

PART 9

(FORMERLY SECTION V—PART I)

1. (*a*) What do you consider to be the two main purposes of a graduation?

(*b*) State the criteria of a good graduation.

(*c*) Is it necessary that a mortality table should be graduated by Makeham's Law in order to make use of the Law of Uniform Seniority in joint-life calculations? Discuss this point.

2. Describe the methods and processes you would use to obtain the mortality experience of your company on standard business for

comparison of the rate of mortality at each age with the American Men Mortality Table.

3. Describe briefly the sources, construction, and graduation of the American Men Select Mortality Table.

4. (*a*) Define with respect to frequency distributions: (1) dispersion, (2) probable deviation, (3) standard deviation, (4) regression curve.

(*b*) Explain in detail what is meant by the coefficient of correlation and how you would calculate this in a specific case if the proper data were available.

5. Describe the numerical system of rating as used in the selection of risks for life insurance. What are its advantages and disadvantages?

6. Discuss the advantages and disadvantages of the various present-day methods of insuring substandard lives.

7. Your company is considering the issue of insurance without medical examination. What points would you take into consideration in giving your opinion as to the advisability of this step? What rules and limitations would you suggest for this type of business?

8. (*a*) Give the formula for the net extra disability premium on a ten-payment twenty-year endowment policy providing premium waiver and a monthly income of $10 during disability in event of disability occurring before age sixty. If Hunter's tables are used in the calculation what, if any, adjustment would be necessary to provide for commencement of disability benefits from inception of disability instead of from approval of disability or from six months thereafter?

(*b*) Describe an approximate method of obtaining the mean reserves on the disability provision on active lives in a company which issues some business without the disability provision, some with the premium waiver feature only, and some with the foregoing benefit.

PART 10

(FORMERLY SECTION V—PART II)

1. (*a*) What is generally meant by a three-factor method of distributing surplus?

(*b*) Starting with the terminal reserve for the end of the nth

policy year develop a contribution formula which takes account of the effect of the three usually recognized factors during the $(n+1)$th year.

(*c*) Break up the formula into expressions for the contribution from each of the three factors.

2. (*a*) For a company too small to have developed a mortality table of its own, discuss the advisability of using the American Men Select Table as a guide in arriving at a mortality factor in its dividend formula. If you were to use this table, what would be your method in apportioning the mortality profit?

(*b*) What is the effect of distributing surplus from mortality as a percentage of the net amount at risk? Would you consider this a practical method, using the same percentage for all ages?

3. (*a*) You have been asked to revise the non-participating rates of a company which has been doing business in a small section of the United States for about twenty years and is about to enter more states. Name the mortality tables which you would take into consideration (giving reasons), the items in the company's past statements which you would study, and mention any other questions which might receive your consideration in making your rates.

(*b*) After you have calculated your gross premiums and before adopting them, how would you proceed to demonstrate their probable ability to earn surplus for the company in, say, the next five years?

4. Discuss the mortality and interest basis that should be used in calculating instalment settlement options to beneficiaries, including those providing settlement of the face amount of insurance as a life-income and those similarly providing a certain period followed by a life-income.

5. Describe a system by which a company could value its policies by the age-attained method. What are the advantages and the disadvantages of this method of valuation?

6. Discuss what method you would consider of valuing the company's policies so as to disclose the true financial position as compared with the position shown by the legal-reserve basis in use.

7. Explain how you would check a proposed set of surrender values to determine whether or not at any age or duration, or on any policy forms, they would cause a drain on surplus.

8. A twenty-year endowment policy has non-forfeiture values beginning at the end of the third year. At the end of the second year, before the third premium is paid, the policyholder wishes to change to an ordinary life policy. Would you allow the policyholder any cash credit on the change? If cash credit is not given, how would you allow a credit on the change, and what method would you adopt in calculating the credit?

PART 11

(FORMERLY SECTION VI—PART I)

1. Explain the differences between the old Gain and Loss Exhibit in the Convention blank of the annual statement and the revised exhibit. Discuss the main purpose of the revision.

2. Assuming that your company has decided to write life insurance without medical examination and wishes to determine separately the development of the surplus in this section of its business, outline a method of providing the desired separation.

3. Before approving the investment of funds of your company in a farm-mortgage loan, what information would you require? Discuss what features of such a security you would consider.

4. Discuss the effect of a prolonged period of increasing prices upon the general prosperity of a life insurance company.

5. (*a*) Give an outline of the Federal Reserve Banking System of the United States.

(*b*) What led to the adoption of this system?

6. (*a*) What is meant by "par of exchange" as between countries?

(*b*) What approximately was the equivalent of the pound sterling, and of one hundred francs, in United States dollars just before 1914?

(*c*) State approximately the rates of exchange between the United States and Great Britain and the United States and France today.

(*d*) State some recent occurrences, financial, political, etc., which have influenced these rates of exchange, and explain why the rate of interest on domestic securities in the United States may respond to these rates of exchange.

7. Discuss the legal aspects of the ownership of a policy of life insurance, when the insured is bankrupt and has reserved the right to change the beneficiary: (*a*) under decisions of the United States courts, (*b*) as affected by state statutes.

8. (*a*) What are the provisions of the Revenue Act of 1924 in the Estate Tax section regarding life insurance?

(*b*) Is this a change from the acts of 1918 or 1921?

(*c*) Give a résumé of any important legal decisions bearing on the application of these provisions.

PART 12

(FORMERLY SECTION VI—PART II)

1. (*a*) Draft an automatic premium-loan clause to be incorporated in a life insurance policy.

(*b*) Mention methods in use in connection with automatic premium-loan provisions for giving the policyholder credit for the remaining cash value after there is not sufficient to advance another entire premium, and state your reasons for preferring the method selected in your clause drawn as above.

2. As a rule, annual premiums for life insurance are calculated on the assumption that premiums are required to be paid to the end of the policy year of death.

(*a*) State the different parts of the policy contract in which reference to this requirement should be made and give an outline of the wording which should be used.

(*b*) Would you enforce this requirement in the case of a surrender value desired at a semiannual or quarterly due date? Give your reasons.

3. Give a brief review of what you consider the most important developments of the life insurance business during the last ten years in the United States.

4. You are asked to investigate a pension fund which has not been carried on an actuarial basis. Outline the steps you would take in making the investigation and give your recommendations for putting the fund in a sound condition.

5. In what ways can a life insurance company be of service to an employer with reference to life insurance and pension plans on his employees?

(*a*) In advising him as to pension plans.

(*b*) In furnishing specific contracts through which pension benefits may be given.

(*c*) In furtherance of thrift plans for the employees.

6. Discuss the characteristics of unemployment insurance with special reference to its applicability to the United States.

7. How would you calculate rates for non-cancellable accident and health insurance?

RESULTS OF EXAMINATIONS

Examinations were held by the Institute on April 14 and 15, 1925, with the following results:

ASSOCIATESHIP

Part 1

One hundred and three candidates presented themselves for examination and the following forty-two passed:

Ambler, A. B.	Frederick, R. A.	Prenner, M. R.
Atrubin, H.	Graeser, A. O.	Raiford, T. E.
Atwater, R. B.	Green, W. C.	Ruby, H. H.
Batho, E. R.	Higginson, G. W.	Rust, (Miss) E.
Bell, W. M.	Jones, W. R.	Sarason, H. M.
Beveridge, J. R.	Longenecker, J. Van S.	Schillig, (Miss) E. M.
Buckner, D. E.	Lyle, (Miss) A. M.	Schroeder, G. D.
Carson, (Miss) C. E.	Menge, W. O.	Skelding, G. A.
Coleman, J. H.	Merkle, J. W.	Tuck, E.
Conrod, S. F.	Murdoch, J. M.	Turner, P. A.
Danzinger, L. J.	Myers, O. J.	Wadleigh, W. L.
Davidson, W. G.	Nelson, C. E.	Wallace, W.
Ellsworth, L. W.	Nicholson, E. H.	Whitbread, R. B. T.
Flesner, (Miss) I. E.	Pall, G.	Woolery, J. M.

PART 2

Ninety-two candidates presented themselves for examination and the following thirty-five passed:

Atrubin, H.
Batho, E. R.
Bell, W. M.
Beveridge, J. R.
Bucknell, E. F.
Buckner, D. E.
Burgar, W. J.
Chen, Ssu Tu
Davidson, W. G.
Ellsworth, L. W.
Flesner, (Miss) I. E.
Green, W. C.
Hondorp, P.
Howard, W. A. M.
Jones, W. R.
Li, Shou Kun
Longenecker, J. Van S.
Louis, Pak Him
Lyle, (Miss) A. M.
Menge, W. O.
Messinger, L. W.
Murdoch, J. M.
Nelson, C. E.
Pall, G.
Prenner, M. R.
Raiford, T. E.
Rust, (Miss) E.
Sarason, H. M.
Schillig, (Miss) E. M.
Skelding, G. A.
Van Fleet, G.
Wadleigh, W. L.
Wallace, W.
Wellman, A. C.
Woolery, J. M.

PART 3

Eighty-two candidates presented themselves for examination and the following forty-two passed:

Atrubin, H.
Atwater, R. B.
Batho, E. R.
Bell, W. M.
Beveridge, J. R.
Bucknell, E. F.
Buckner, D. E.
Burgar, W. J.
Carson, (Miss) C. E.
Danzinger, L. J.
Davidson, W. G.
Easley, K. H.
Flesner, (Miss) I. E.
Graeser, A. O.
Green, W. C.
Hacking, F.
Hogan, F. J.
Hondrop, P.
Jones, W. R.
Kirkpatrick, (Miss) C. C.
Larsen, M. N.
Li, Shou Kun
Lyle, (Miss) A. M.
Menge, W. O.
Merkle, J. W.
Myers, O. J.
Nelson, C. E.
Nicholson, E. H.
Pall, G.
Prenner, M. R.
Raiford, T. E.
Rust, (Miss) E.
Sarason, H. M.
Schillig, (Miss) E. M.
Skelding, G. A.
Tuck, E.
Turnbull, D. M.
Van Fleet, G.
Wadleigh, W. L.
Wallace, W.
Wellman, A. C.
Woolery, J. M.

PART 4

Fifty-four candidates presented themselves for examination and the following twenty-eight passed:

Atrubin, H.
Batho, E. R.
Benner, (Miss) A. L.
Beveridge, J. R.
Buckner, D. E.
Conrod, S. F.
Cox, L. C.
Davidson, W. G.
Estes, E. F.
Evans, J. H.
Flesner, (Miss) I. E.
Green, W. C.
Hogan, F. J.
Hondorp, P.
Jones, W. R.
Kirkpatrick, (Miss) C. C.
Lyle, (Miss) A. M.
Menge, W. O.
Nelson, C. E.
Pall, G.
Parks, G. A.
Prenner, M. R.
Raiford, T. E.
Schillig, (Miss) E. M.
Shepherd, P.
Wallace, W.
Wellman, A. C.
Woolery, J. M.

PART 5

Twenty-three candidates presented themselves for examination and the following sixteen passed:

Bell, D. W.
Benner, (Miss) A. L.
Brahana, H. R.
Davis, M. E.
Eastlack, A. C.
Ede, C. F.
Glover, L. A.
Kenigson, G.
Linder, J.
MacKinnon, W. D.
Preston, R. F.
Richter, O. C.
Smith, L. G.
Walker, (Miss) M.
Walton, H. G.
Young, R.

PART 6

Twenty-two candidates presented themselves for examination and the following sixteen passed:

Anderson, T. L.
Angerer, J. W.
Brahana, H. R.
Davis, M. E.
Eastlack, A. C.
Estes, E. F.
Evans, J. H.
Irwin, P. C.
Kalmbach, L. J.
Kenigson, G.
Linder, J.
Little, J. E.
MacKinnon, W. D.
Richter, O. C.
Smith, L. G.
Young, R.

PART 7

Twenty-one candidates presented themselves for examination and the following twelve passed:

Brahana, H. R.
Davis, M. E.
Eastlack, A. C.
Kalmbach, L. J.
Kenigson, G.
McClurg, D. R.
MacKinnon, W. D.
Moyer, R. E.
Richter, O. C.
Walker, (Miss) M.
Walton, H. G.
Young, R.

PART 8

Twenty-one candidates presented themselves for examination and the following thirteen passed:

Anderson, T. L.	Kalmbach, L. J.	Preston, R. F.
Dandy, J. P.	Kenigson, G.	Shudde, L. O.
Davis, M. E.	McClurg, D. R.	Walker, (Miss) M.
Eastlack, A. C.	MacKinnon, W. D.	Young, R.
Glover, L. A.		

FELLOWSHIP

PART 9

Ten candidates presented themselves for examination and the following eight passed:

Boyd, L. T.	Dubuar, C. C.	Oates, M.
Budinger, J. A.	Jackson, H. H.	Poorman, W. F.
Coler, W. P.	Johnson, W. M.	

PART 10

Ten candidates presented themselves for examination and the following eight passed:

Bjorn, W.	Good, A. C.	Oates, M.
Boyd, L. T.	Jackson, H. H.	Poorman, W. F.
Coler, W. P.	McAndless, A. J.	

PART 11

Ten candidates presented themselves for examination and the following seven passed:

Bjorn, W.	Jamison, (Miss) D. M.	Koch, (Miss) E. B.
Goodrich, C. R.	Kennon, R. E.	Poorman, W. F.
Jackson, H. H.		

PART 12

Seven candidates presented themselves for examination and the following four passed:

Jackson, H. H.	Johnson, W. M.	Poorman, W. F.
Jamison, (Miss) D. M.		

AMERICAN INSTITUTE *of* ACTUARIES

OFFICERS

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Vice-President.....John Gowans Parker
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Editor of *The Record*.....Elgin George Fassel

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Elgin George Fassel, Editor of *The Record*
George Graham, Ex-President
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John Gowans Parker, Vice-President
Thomas Ashley Phillips, Elected to June, 1926
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COMMITTEES

(Appointments expire at meeting of June, 1926, unless otherwise specified)

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EXAMINATION COMMITTEE

(Appointments expire at completion of examinations of 1926)

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COMMITTEE ON REVIEWS

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COMMITTEE ON STATUTORY VALUATION

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OSWALD J. ARNOLD — JOHN M. LAIRD

LAWRENCE M. CATHLES — HENRY MOIR

ROBERTSON G. HUNTER — EDWARD B. MORRIS

PAST PRESIDENTS

	Term of Office	
Lucius McAdam	1909–10	Died 1913
Henry Wright Buttolph	1910–12	
Oswald James Arnold	1912–14	
Joseph Howard Nitchie	1914–16	
Albert Garfield Portch	1916–18	Died 1918
Charles Harrison Beckett	1918–20	
George Graham	1920–22	
Lawrence M. Cathles	1922–24	

SCHEDULE OF MEMBERSHIP—AUGUST 1, 1925

Particulars	Fellows	Associates	Company Members	Total
Membership—December 15, 1924	112	72	121	305
Increase:				
By examination	5	13	.....	18*
By election	2	2	1	5
	119	87	122	328
Decrease:				
By death	.....	1	.....	1
By other withdrawals	.....	1	3	4
Transfers from Associate to Fellow by examination	.....	5	.....	5
Membership—August 1, 1925	119	80	119	318

* Includes transfers from Associate to Fellow.

LIST *of* MEMBERS *and* STUDENTS

August 1, 1925

(KEY: † Charter Member at Organization in 1909; * Member by Examination)

FELLOWS

ADMITTED

1915 ALLISON, SINCLAIR EDWARD, A.A.S., Actuary, Pan-American Life Ins. Co., New Orleans, La.

1914 ALLSTROM, HENRY WILLARD, A.A.S., Secretary and Actuary, Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1915 AMES, ERNEST CAPRON, A.B., LL.B., Vice-President and Actuary, Bankers Life Ins. Co. of Nebraska, Lincoln, Neb.

† ANDERSON, LEWIS ALBERT, B.L., Actuary, Central Life Assurance Society, Des Moines, Iowa.

1921 ANGELL, CHARLES HART, A.B., A.A.S., Actuary, Massachusetts Mutual Life Ins. Co., Springfield, Mass.

† APPLEBY, TROY WILSON, A.M., President, The Ohio National Life Ins. Co., Cincinnati, Ohio.

† ARNOLD, OSWALD JAMES, B.S., Secretary and Actuary, Illinois Life Ins. Co., 1212 Lake Shore Drive, Chicago, Ill.

1924 *BABBITT, ALBERT E., A.M., Actuary, Lamar Life Ins. Co., Jackson, Miss.

1920 BAGLEY, W. NELSON, B.S., F.A.S., Assistant Actuary, The Travelers Ins. Co., Hartford, Conn.

† BARNARD, WILLIAM FRANCIS, 1402 Midland Ave., Syracuse, N.Y.

† BARNETT, SAMUEL, A.B., C.E., Insurance Lawyer and Consulting Actuary, 502 Forsyth Building, Atlanta, Ga.

† BECKETT, CHARLES HARRISON, A.B., MEM. A.M.S., Actuary, State Life Ins. Co., Indianapolis, Ind.

1912 BOWERMAN, JUDAH PHILIP, Actuary, George Washington Life Ins. Co., Charleston, W.Va.

1917 BUCK, GEORGE BURTON, LL.B., Consulting Actuary, 25 Spruce St., New York, N.Y.

ADMITTED

1913 Burton, Russell Conklin, A.B., Secretary and Actuary, New World Life Ins. Co., Spokane, Wash.

† Buttolph, Henry Wright, A.B., A.M., A.A.S., Actuary, American Central Life Ins. Co., Indianapolis, Ind.

1914 Cameron, John Campbell, M.A., F.F.A., A.A.S., Vice-President and Actuary, Great Southern Life Ins. Co., Houston, Tex.

1925 Cammack, Edmund Ernest, A.I.A., F.A.S., Vice-President and Actuary, Aetna Life Ins. Co., Hartford, Conn.

† Campbell, Dr. Donald Francis, A.M., Ph.D., Mem. A.M.S., Consulting Actuary, Suite 1832, 160 N. La Salle St., Chicago, Ill.

1920 Cann, Francis Edgar, M.A., F.A.S., Associate Actuary, Jefferson Standard Life Ins. Co., Greensboro, N.C.

† Carter, Edwin Richmond, Actuary, National Life Ins. Co. of the U.S.A., 29 S. La Salle St., Chicago, Ill.

1913 Cathles, Lawrence Maclagan, F.F.A., President, North American Reassurance Co., 250 Park Ave., New York, N.Y.

1921 *Coates, Barrett Nelson, B.S., A.A.S., Consulting Actuary, 354 Pine St., San Francisco, Cal.

1920 Coburn, Arthur, F.A.S., A.I.A., F.F.A., Vice-President, North American Reassurance Co., 250 Park Ave., New York, N.Y.

1914 Coit, Ralph Bolles, Actuary, Jefferson Standard Life Ins. Co., Greensboro, N.C.

† Corbett, Henry Richmond, B.Sc., Ph.M., Consulting Actuary, 175 West Jackson Blvd., Chicago, Ill.

1918 Craig, James Douglas, F.A.S., F.C.A.S., Actuary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

1918 Curjel, Harald Worthington, M.A., F.I.A., A.A.S., Assistant Actuary, Illinois Life Ins. Co., 1212 Lake Shore Drive, Chicago, Ill.

† Draper, Frederick Adams, Actuary, National Life Association, Des Moines, Iowa.

1911 Dunlap, Earl Oglebay, F.C.A.S., Assistant Secretary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

ADMITTED

1924 *Ehrenclou, Orvar A., F.A.S., Actuary, Northern Life Ins. Co., Seattle, Wash.

1918 *Elston, James Strode, A.B., F.A.S., A.A. Suisse, Cor. Mem. A.A. Belg., Mem. A.M.S., Assistant Actuary, Life Dept., The Travelers Ins. Co., Hartford, Conn.

1914 Epsteen, Saul Robinson, B.S., Ph.D., Special Deputy State Bank Commissioner, Antonito, Colo.

1919 Evans, Percy H., Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1914 Fairlie, James, M.A., A.A.S., Vice-President and Actuary, Mutual Life of Illinois, Springfield, Ill.

1922 *Fassel, Elgin George, A.I.A., F.A.S., Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1919 Flanigan, James Edward, F.A.S., F.C.A.S., Agency Manager, Bankers Life Co., 220 Broadway, New York, N.Y.

1915 Fondiller, Richard, M.A., LL.B., Consulting Actuary, Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y.

1917 Gibb, James Burnett, F.F.A., A.I.A., F.A.S., Actuary, Penn Mutual Life Ins. Co., Philadelphia, Pa.

† Glover, James Waterman, B.L., A.B., A.M., Ph.D., Professor of Mathematics and Insurance, University of Michigan, 620 Oxford Road, Ann Arbor, Mich.

† Gould, William H., M.A., A.I.A., A.A.S., F.C.A.S., 75 Fulton St., New York, N.Y.

1913 Graham, George, F.F.A., A.A.S., A.I.A., Vice-President, Central States Life Ins. Co., St. Louis, Mo.

1915 Greene, Winfield Wardwell, Ph.B., Actuary, National Council on Compensation Insurance, 151 Fifth Ave., New York, N.Y.

1920 *Gunn, Marcus, A.B., Actuary, Western States Life Ins. Co., San Francisco, Cal.

† Haight, Frank J., 812 Hume Mansur Bldg., Indianapolis, Ind.

† Hann, Alfred Gilbert, Actuary, Pacific Mutual Life Ins. Co., Los Angeles, Cal.

1913 Harvey, Julian Corneille, LL.B., General Manager and Actuary, St. Louis Mutual Life Ins. Co., St. Louis, Mo.

ADMITTED

1920 Hemsing, Frank C., A.A.S., Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1914 *Herfurth, Carl E., Secretary and Actuary, Montana Life Ins. Co., Helena, Mont.

† Higdon, John E., A.B., American Bankers Ins. Co., Jacksonville, Ill.

1924 *Hohaus, Reinhard Arthur, B.A., F.A.S, Metropolitan Life Ins. Co., 1 Madison Ave., New York.

† Homans, I. Smith, A.A.S., Vice-President and Actuary, Commonwealth Life Ins. Co., Louisville, Ky.

1918 Hope, Francis Moffat, F.F.A., A.I.A., A.A.S., Actuary, Occidental Life Ins. Co., Los Angeles, Cal.

1920 Huenergard, Arthur Jacob Conrad, B.A., F.A.S., Actuary, Ontario Equitable Life and Accident Ins. Co., Waterloo, Ont., Canada.

1914 Hughes, Charles, A.A.S., Auditor and Actuary, New York State Insurance Dept., 165 Broadway, New York, N.Y.

1916 Hunter, Arthur, F.A.S., F.F.A., A.I.A., Chief Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

1919 Hunter, Robertson G., F.A.S., A.I.A., Second Vice-President and Actuary, Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1921 Hutcheson, William Anderson, F.I.A., F.F.A., F.A.S., Second Vice-President and Actuary, Mutual Life Ins. Co., 34 Nassau St., New York, N.Y.

† Hyde, Edward Wyllys, B.C.E., C.E., Columbia Life Ins. Co., Cincinnati, Ohio.

1925 *Jackson, Henry Hollister, M.A., F.A.S., Associate Actuary, National Life Ins. Co., Montpelier, Vt.

1925 *Jamison, (Miss) Dorothy May, A.B., George Washington Life Ins. Co., Charleston, W.Va.

1925 *Johnson, Wilbur Moore, A.M., Vice-President and Actuary, Royal Union Life Ins. Co., Des Moines, Iowa.

1925 *Kennon, Ralph Eugene, B.A., M.S., Mem. A.M.S., Secretary and Actuary, Medical Life Ins. Co. of America, Waterloo, Iowa.

1916 King, Walter Irving, A.B., F.A.S., Secretary Group Ins. Dept., Connecticut General Life Ins. Co., Hartford, Conn.

ADMITTED

1918 Laird, John Melvin, B.A., A.I.A., F.A.S., Secretary and Actuary, Connecticut General Life Ins. Co., Hartford, Conn.

1923 *Larsen, Arthur William, M.A., A.A.S., Actuary, Volunteer State Life Ins. Co., Chattanooga, Tenn.

1924 Linton, Morris Albert, B.S., M.A., F.I.A., F.A.S., Vice-President, Provident Mutual Life Ins. Co., Philadelphia, Pa.

1924 Little, James Fulton, F.I.A., F.A.S., F.C.A.S., Associate Actuary, Prudential Ins. Co. of America, Newark, N.J.

1919 Macfarlane, William, F.F.A., F.A.S., Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

1921 Maclean, Alexander T., F.F.A., F.A.S., Associate Actuary, Massachusetts Mutual Life Ins. Co., Springfield, Mass.

1919 *Marshall, Erston Leland, A.B., J.D., 918 Hubbell Bldg., Des Moines, Iowa.

1924 *Marvin, Harry Cooper, A.M., Consulting Actuary, 2105 N. Meridian St., Indianapolis, Ind.

1923 *McCankie, Reginald Campbell, F.A.S., Assistant Actuary, Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1916 McIntosh, James, F.F.A., A.A.S., Associate Actuary, The Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1924 McKechnie, James Baldwin, M.A., F.I.A., F.A.S., General Manager, Manufacturers Life Ins. Co., Toronto, Canada.

1914 Mead, Franklin Bush, A.B., F.A.S., Secretary and Actuary, Lincoln National Life Ins. Co., Fort Wayne, Ind.

1919 Moir, Henry, F.F.A., F.I.A., F.A.S., President, United States Life Ins. Co., 105 Fifth Ave., New York, N.Y.

1913 Montgomery, Paul Vaughan, B.A., Vice-President and Actuary, Southland Life Ins. Co., Dallas, Tex.

1920 Morris, Edward Bontecou, Ph.B., F.A.S., Actuary, Life Dept., The Travelers Ins. Co., Hartford, Conn.

1913 Morris, William Oscar, Actuary, North American Life Ins. Co., North American Bldg., Chicago, Ill.

1913 Napier, William Ewart, Second Vice-President and Secretary, Scranton Life Ins. Co., Scranton, Pa.

† Nitchie, Joseph Howard, A.B., F.A.S., Consulting Actuary, 19 South La Salle St., Chicago, Ill.

ADMITTED

† Nueske, Bernhard Robert, Old Colony Life Ins. Co., Chicago, Ill.

† Orr, Robert Kelly, Lansing, Mich.

1918 Papps, Percy Charles Herbert, A.I.A., F.A.S., Mathematician, The Mutual Benefit Life Ins. Co., Newark, N.J.

1920 Parker, John Gowans, B.A., A.I.A., F.A.S., Actuary, Imperial Life Assurance Co., Toronto, Canada.

1914 Pattison, George Benjamin, A.A.S., Secretary and Actuary, Peoria Life Ins. Co., Peoria, Ill.

1918 Perrin, Oliver Winfred, M.A., F.A.S., Mem. A.M.S., Assistant Actuary, Penn Mutual Life Ins. Co., Philadelphia, Pa.

1916 *Phelps, Shaffner Leonard, B.S., Secretary, Volunteer State Life Ins. Co., Chattanooga, Tenn.

1915 Phillips, Thomas Ashley, B.A., A.I.A., A.A.S., Vice-President, Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1918 Pipe, Sidney Herbert, F.A.S., A.I.A., Actuary, Temple Bldg., Toronto, Canada.

1925 *Poorman, William Frank, M.A., A.A.S., Actuary, Farmers National Life Ins. Co., 3401 Michigan Ave., Chicago, Ill.

1924 *Porter, Elder Alexander, A.B., F.A.S., Actuary, The Indianapolis Life Ins. Co., Indianapolis, Ind.

† Reynolds, Joseph B., President, Kansas City Life Ins. Co., Kansas City, Mo.

1920 Rhodes, Edward Everett, F.A.S., Vice-President, Mutual Benefit Life Ins. Co., Newark, N.J.

† Rietz, Professor Henry Lewis, Ph.D., F.A.A.S., F.R.S.S., University of Iowa, Iowa City, Iowa.

1916 *Rietz, J. Charles, A.B., Mem. A.M.S., Actuary, Midland Mutual Life Ins. Co., Columbus, Ohio.

1910 Roche, John Francis, F.A.S., Vice-President, Manhattan Life Ins. Co., 66 Broadway, New York, N.Y.

1912 Roslington, George, F.C.A., Vice-President, Occidental Life Ins. Co., Albuquerque, N.M.

1916 Sanders, George White, A.B., F.A.S., Actuary, Michigan Mutual Life Ins. Co., Detroit, Mich.

† Seitz, Jacob Charles, A.B., Secretary and Actuary, Security Life Ins. Co. of America, 209 South La Salle St., Chicago, Ill.

ADMITTED

1915 SIGTENHORST, ANDREW, Texas Life Ins. Co., Waco, Tex.

1915 SPEERS, ALEXANDER ALBERT, A.B., F.A.S., Actuary, Michigan Insurance Dept., Lansing, Mich.

1915 STEIN, CALVERT FREDERICK, Secretary-Actuary, Life Dept., Provident Life and Accident Ins. Co., Chattanooga, Tenn.

† STOOKEY, BERT JAY, Assistant Secretary, Illinois Life Ins. Co., 1212 Lake Shore Drive, Chicago, Ill.

1921 STRONG, WENDELL MELVILLE, B.A., M.A., PH.D., LL.B., F.A.S., Associate Actuary, Mutual Life Ins. Co., 34 Nassau St., New York, N.Y.

1913 THOMSON, GORDON WILLIAM, F.F.A., A.I.A., A.A.S., Vice-President and Actuary, West Coast Life Ins. Co., San Francisco, Cal.

1924 TORREY, MORRIS WHITTEMORE, F.A.S., Assistant Secretary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

† VAIL, CARLETON M., A.B., Room 1005, 110 South Dearborn St., Chicago, Ill.

1916 WASHBURN, JAMES HERMAN, A.A.S., Consulting Actuary, 165 Broadway, New York, N.Y.

1916 WATT, WILLIAM ARTHUR, M.A., A.I.A., F.A.S., Secretary and Actuary, Pilot Life Ins. Co., Greensboro, N.C.

† WEBB, ROBERT MONTAGUE, Kansas City Life Ins. Co., Kansas City, Mo.

1921 WELCH, ARCHIBALD ASHLEY, A.B., A.M., F.A.S., President, Phoenix Mutual Life Ins. Co., Hartford, Conn.

1916 WILLIAMS, FREDERICK ALFRED, F.A.S., A.I.A., F.S.S., Manager and Actuary, "La Nacional" Compañia de Seguros sobre la Vida, S.A., Apartado 1420, Mexico City, Mexico.

1923 *WILLIAMS, (MISS) HELEN JACKSON, A.B., M.S., Bureau of Statistics and Research, 510 Electric Bldg., Cleveland, O.

† WITHINGTON, FREDERIC SCHERER, F.A.I.A., Actuary, Midland Life Ins. Co., Kansas City, Mo.

1924 WOOD, WILLIAM ARCHIBALD PORTER, B.A., A.I.A., F.A.S., Actuary, Canada Life Assurance Co., Toronto, Canada.

1915 WOODWARD, JOSEPH HOOKER, PH.B., F.A.S., Consulting Actuary, Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y.

ADMITTED

1925 YOUNG, WILLIAM, F.F.A., F.A.S., Actuary, New York Life Ins. Co., 346 Broadway, N. Y.

ASSOCIATES

(Numerals indicate what parts of the Fellowship examinations have been passed.)

ADMITTED

1915 *BEACH, (MISS) HENRICKA BRYANT, M.A., Assistant Secretary and Actuary, Providence Life Ins. Co., Bismarck, N.D.

1913 BEECHER, BENJAMIN SANFORD, M.A., Vice-President and Secretary, National Guardian Life Ins. Co., Madison, Wis.

1925 *BELL, DAVID WRIGHT, B.A., Imperial Life Assurance Co., Toronto, Canada.

1923 BIRKENSHAW, JOHN HENRY, B.A., F.A.S., Assistant Actuary, Confederation Life Association, Toronto, Canada.

† BISHOP, GEORGE WALTER, The Bankers Reserve Life Co., Omaha, Neb.

1923 *BJORN, WALTER, B.S., A.A.S., Connecticut General Life Ins. Co., Hartford, Conn. (*10*, *11*)

1915 *BOWERMAN, WALTER GREGORY, A.M., F.A.S., New York Life Ins. Co., 346 Broadway, New York, N.Y.

1922 *BOYD, LORNE THOMAS, B.A., Secretary and Actuary, Commonwealth Life Ins. Co., Hamilton, Ont., Canada. (*9*, *10*)

† BRIMSTIN, WILLIAM EDGAR, Secretary, Federal Life Ins. Co., Chicago, Ill.

1919 BROOKS, CHARLES EDWARD, PH.D., 613 Beavers Insurance Bldg., Madison, Wis.

1921 BROWN, EVERETT GARRABRANT, F.A.S., Actuary, Southwestern Life Ins. Co., Dallas, Tex.

1917 BROWN, (MRS.) HELEN E., 702 S. Sixth St., Ironton, Ohio.

1919 BRUNNQUELL, HELMUTH G., A.C.A.S., Actuary, Wisconsin Insurance Dept., Madison, Wis.

1911 *BUCKMAN, HAROLD HIBBS, Actuary, California State Life Ins. Co., Sacramento, Cal.

1924 *BUDINGER, JOSEPH ALOYSIUS, A.A.S., Assistant Actuary, Franklin Life Ins. Co., Springfield, Ill. (*9*)

1914 BURKART, CHARLES J., Kansas City Life Ins. Co., Kansas City, Mo.

ADMITTED

1919 *Clark, (Miss) Helen Lucy, A.B., Assistant Actuary, American Central Life Ins. Co., Indianapolis, Ind.

1914 *Coler, Wendell Phillips, A.B., Actuary, The Maccabees, 5065 Woodward Ave., Detroit, Mich. (*9, 10*)

† Copeland, John Alexander, A.B., Consulting Actuary, 407 Southeastern Trust Bldg., Atlanta, Ga.

1924 *Cross, Cecil Frank, A.B., Actuary, American Life Ins. Co., Detroit, Mich.

1925 *Dandy, John Percy, B.A., A.A.S., Confederation Life Association, Toronto, Canada.

1925 *Davis, Malvin Edward, B.A., A.C.A.S., Metropolitan Life Ins. Co., 1 Madison Ave., New York, N. Y.

1924 *Dubuar, Charles Carroll, A.B., Assistant Actuary, New York Insurance Dept., Albany, N.Y. (*9*)

1925 *Eastlack, Allen Carlson, A.M., Assistant Actuary, American National Ins. Co., Galveston, Tex.

1912 *Fetsch, Harry Christian, A.B., A.A.S., Actuary, Ohio State Life Ins. Co., Columbus, Ohio.

1924 *Folz, Clifford H., A.B., Actuary, Public Savings Ins. Co., Indianapolis, Ind.

1922 *Good, Alexander Caldwell, 1416 Chemical Bldg., St. Louis, Mo. (*9, 10*)

1921 *Goodrich, Clarence Robertson, Central States Life Ins. Co., St. Louis, Mo. (*9–11*)

† Grimes, Irvin Lorenzo, A.B., Actuary and Assistant Secretary, Mutual Trust Life Ins. Co., 77 W. Washington St., Chicago, Ill.

1916 Griswold, Robert N., B.A., A.A.S., Inter-Mountain Life Ins. Co., Salt Lake City, Utah.

1923 Hale, John Stewart, B.A., Actuary, Northwestern National Life Ins. Co., Minneapolis, Minn.

1919 *Hanna, John Vestal, Actuary and Assistant Secretary, United Life and Accident Ins. Co., Concord, N.H.

1915 *Hiller, William John, A.B., Assistant Actuary, Bankers Life of Nebraska, Lincoln, Neb.

1914 *Hipp, Grady Hayne, A.B., Actuary, New York Insurance Dept., Albany, N.Y.

† Huskinson, George William H., Assistant Superintendent, Illinois Insurance Dept., Springfield, Ill.

ADMITTED

1924 *Jenkins, Wilmer Atkinson, S.B., A.M., Instructor, Harvard University, Cambridge, Mass.

1918 Jordan, F. Russell, B.A., A.A.S., Actuary, Franklin Life Ins. Co., Springfield, Ill.

1925 *Kalmbach, Leland John, A.B., Lincoln National Life Ins. Co., Fort Wayne, Ind.

1924 Kattell, Sherman C., B.S., F.A.S., Assistant Actuary, Lincoln National Life Ins. Co., Fort Wayne, Ind.

1925 *Kenigson, George, 66 Oak St., Jersey City, N. J.

1923 *Koch, (Miss) Eloise Bender, A.B., Central States Life Ins. Co., St. Louis, Mo. (*9–11*)

1923 *Lehman, A. Thomas, A.M., Actuary, Standard Life Ins. Co. of America, Pittsburgh, Pa. (*9, 10*)

1924 Lithgow, James Hector, F.A.S., A.I.A., Actuary, Manufacturers Life Ins. Co., Toronto, Canada

1925 *MacKinnon, Will Donald, B.S., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1916 Maynard, Joseph Duryea, Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1923 *McAndless, A. J., B.A., Assistant Secretary, Lincoln National Life Ins. Co., Fort Wayne, Ind. (*9, 10*)

1923 McBride, William Hugh, Actuary, National Life and Accident Ins. Co., Nashville, Tenn.

1925 *McClurg, D. Ralph, Ph.B., Secretary and Treasurer, National Equity Life Ins. Co., Little Rock, Ark.

1922 McConaghy, Charles Alexander, B.A., A.I.A., F.A.S., Actuary, Continental Life Ins. Co., St. Louis, Mo.

1925 *Moyer, Ross Emerson, A.B., Lincoln National Life Ins. Co., Fort Wayne, Ind.

1921 *Myers, George Elmer, A.A.S., Actuary, International Life Ins. Co., St. Louis, Mo.

1923 *Oates, Merlin, A.B., Secretary and Actuary, Alamo Life Ins. Co., San Antonio, Tex. (*9, 10*)

1922 *Pardieck, (Miss) Gertrude L., A.B., Assistant Actuary, State Life Ins. Co., Indianapolis, Ind.

1925 Pedoe, Arthur, B.Sc. (London), F.I.A., A.A.S., Actuary, Continental Life Ins. Co., Toronto, Canada.

1925 Phillips, James Thompson, B.A., F.A.S., New York Life Ins. Co., 346 Broadway, New York, N. Y.

ADMITTED

1918 *Powell, John Mark, A.B., F.A.S., Actuary, Columbian National Life Ins. Co., Boston, Mass. (*9, 10*)

1924 *Prentice, George Taylor, B.A., A.A.S., Imperial Life Assurance Co., Toronto, Canada.

1920 Reichgott, Henry, Ph.B., A.A.S., Third Vice-President, Missouri State Life Ins. Co., St. Louis, Mo.

1920 *Reilly, Dr. John Franklin, A.M., Ph.D., 212 Physics Bldg., Iowa City, Iowa.

1922 *Robbins, Dr. Rainard Benton, A.M., Ph.D., Assistant Actuary, New York Insurance Dept., 165 Broadway, New York, N.Y.

1916 *Rothschild, Stanford Z., A.B., A.A.S., Third Vice-President and Actuary, Sun Life Ins. Co. of America, Baltimore, Md.

1914 *Scott, Martin R., Actuary, Louisiana State Life Ins. Co., Shreveport, La.

1916 Sellman, Henry G., A.B., Consulting Actuary, 6717 Virginia Ave., St. Louis, Mo. (*10–12*)

1924 *Shepherd, Bruce Edwin, Ph.B., Missouri State Life Ins. Co., St. Louis, Mo.

1916 *Shepherd, Clinton O., F.A.S., Actuary, Missouri State Life Ins. Co., St. Louis, Mo.

1925 *Shudde, Louis Otto, B.A., M.A., Actuary, Merchants Life Ins. Co., Des Moines, Iowa.

1921 Sinclair, Coll Claude, B.A., F.A.S., Assistant Actuary, Great-West Life Assurance Co., Winnipeg, Man., Canada.

† Stayman, Guy L., P.O. Box 86, Indianapolis, Ind.

1913 Strudell, Fred David, Vice-President and Actuary, American Life Reinsurance Co., Dallas, Tex.

1915 *Sturtevant, Robert Brown, A.B., Assistant Secretary, American Central Life Ins. Co., Indianapolis, Ind.

1919 Suzuki, Toshikazu, M.S., Actuary, First Mutual Life Ins. Co., Kyobashi, Tokio, Japan.

1915 Tickner, Verne L., 901 Globe Bldg., St. Paul, Minn.

† Vail, Roger Sherman, A.B., 110 South Dearborn St., Chicago, Ill.

1925 *Walker, (Miss) Margaret, M.A., Lincoln National Life Ins. Co., Fort Wayne, Ind.

ADMITTED

1925 *Walton, Harold Graham, B.S., American Central Life Ins. Co., Indianapolis, Ind.

1923 Warren, Professor Lloyd Arthur Heber, M.A., Ph.D., F.R.A.S., F.C.A.S., A.A.S., University of Manitoba, Winnipeg, Man., Canada.

1917 Werkenthin, Bernard, Texas Insurance Dept. Austin, Tex.

† Wiggins, Horace Scoville, 514–15 Bankers Life Bldg., Lincoln, Neb.

1920 Yort, J. P., 2528 Thirteenth St., Washington, D.C.

1925 *Young, Jay Rollin, Franklin Life Ins. Co., Springfield, Ill.

CONTRIBUTING MEMBERS

1921 Aetna Life Ins. Co., Hartford, Conn.

1915 American Bankers Ins. Co., Chicago, Ill.

1916 American Central Life Ins. Co., Indianapolis, Ind.

1920 American Life Ins. Co., Denver, Colo.

1923 American Life Ins. Co., Detroit, Mich.

1923 American Life Reinsurance Co., Dallas, Tex.

1917 American National Ins. Co., Galveston, Tex.

1921 Amicable Life Ins. Co., Waco, Tex.

1919 Atlantic Life Ins. Co., Richmond, Va.

1921 Bank Savings Life Ins. Co., Topeka, Kan.

1920 Bankers Life Co., Des Moines, Iowa.

† Bankers Reserve Life Co., Omaha, Neb.

1921 Berkshire Life Ins. Co., Pittsfield, Mass.

1922 Canada Life Assurance Co., Toronto, Canada.

1912 Capitol Life Ins. Co., Denver, Colo.

1919 Cedar Rapids Life Ins. Co., Cedar Rapids, Iowa.

1916 Central Life Assurance Society of the U.S., Des Moines, Iowa

† Central Life Ins. Co. of Illinois, Chicago, Ill.

1915 Central States Life Ins. Co., St. Louis, Mo.

† Cleveland Life Ins. Co., Cleveland, Ohio.

1920 Columbus Mutual Life Ins. Co., Columbus, Ohio.

1922 Confederation Life Association, Toronto, Canada.

1921 Connecticut General Life Ins. Co., Hartford, Conn.

1923 Continental Assurance Co., Chicago, Ill.

1922 Continental Life Ins. Co., St. Louis, Mo.

ADMITTED

1920 Continental Life Ins. Co., Wilmington, Del.
1919 Dakota Life Ins. Co., Watertown, S.D.
1922 Detroit Life Ins. Co., Detroit, Mich.
1918 Elkhorn Life & Accident Ins. Co., Norfolk, Neb.
1920 Equitable Life Ins. Co., Washington, D.C.
1920 Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.
1922 Excelsior Life Ins. Co., Toronto, Canada.
1920 Farmers & Bankers Life Ins. Co., Wichita, Kan.
1923 Farmers National Life Ins. Co., Chicago, Ill.
† Federal Life Ins. Co., Chicago, Ill.
1921 Fidelity Mutual Life Ins. Co., Philadelphia, Pa.
1916 Franklin Life Ins. Co., Springfield, Ill.
1914 George Washington Life Ins. Co., Charleston, W.Va.
1923 Great Northern Life Ins. Co., Chicago, Ill.
1916 Great Southern Life Ins. Co., Houston, Tex.
1921 Great-West Life Assurance Co., Winnipeg, Man., Canada.
1920 Guaranty Life Ins. Co., Davenport, Iowa.
1921 Guardian Life Ins. Co. of America, New York, N.Y.
1920 Idaho State Life Ins. Co., Boise, Idaho.
† Illinois Life Ins. Co., Chicago, Ill.
1912 Indianapolis Life Ins. Co., Indianapolis, Ind.
† International Life Ins. Co., St. Louis, Mo.
1923 International Life & Trust Co., Moline, Ill.
1922 Imperial Life Assurance Co., Toronto, Canada.
1915 Jefferson Standard Life Ins. Co., Greensboro, N.C.
1921 John Hancock Mutual Life Ins. Co., Boston, Mass.
† Kansas City Life Ins. Co., Kansas City, Mo.
1913 LaFayette Life Ins. Co., LaFayette, Ind.
† Lincoln National Life Ins. Co., Fort Wayne, Ind.
1922 London Life Ins. Co., London, Ont., Canada.
1923 Manhattan Life Ins. Co., New York, N.Y.
1922 Manufacturers Life Ins. Co., Toronto, Canada.
1921 Maryland Life Ins. Co., Baltimore, Md.
1921 Massachusetts Mutual Life Ins. Co., Springfield, Mass.
1920 Merchants Life Ins. Co., Des Moines, Iowa.
1921 Metropolitan Life Ins. Co., New York, N.Y.
1915 Michigan Mutual Life Ins. Co., Detroit, Mich.
1920 Midland Life Ins. Co., Kansas City, Mo.

ADMITTED

1918 Midland Mutual Life Ins. Co., Columbus, Ohio.
1919 Minnesota Mutual Life Ins. Co., St. Paul, Minn.
1912 Missouri State Life Ins. Co., St. Louis, Mo.
1925 Monarch Life Assurance Co., Winnipeg, Man., Canada.
1920 Montana Life Ins. Co., Helena, Mont.
1919 Mutual Benefit Life Ins. Co., Newark, N.J.
1921 Mutual Life of Illinois, Springfield, Ill.
1919 Mutual Trust Life Ins. Co., Chicago, Ill.
1919 National Life & Accident Ins. Co., Nashville, Tenn.
1921 National Life Ins. Co., Montpelier, Vt.
† National Life Ins. Co. of the U.S.A., Chicago, Ill.
1923 National Guardian Life Ins. Co., Madison, Wis.
1921 New England Mutual Life Ins. Co., Boston, Mass.
1915 North American Life Ins. Co., Chicago, Ill.
1920 Northern Life Ins. Co., Seattle, Wash.
1924 Northern States Life Ins. Co., Hammond, Ind.
1915 Northwestern National Life Ins. Co., Minneapolis, Minn.
1921 Occidental Life Ins. Co., Los Angeles, Cal.
1915 Ohio National Life Ins. Co., Cincinnati, Ohio.
1912 Ohio State Life Ins. Co., Columbus, Ohio.
† Old Colony Life Ins. Co., Chicago, Ill.
1915 Old Line Life Ins. Co. of America, Milwaukee, Wis.
1917 Pacific Mutual Life Ins. Co., Los Angeles, Cal.
1916 Pan-American Life Ins. Co., New Orleans, La.
1923 Penn Mutual Life Ins. Co., Philadelphia, Pa.
1920 Peoples Life Ins. Co., Frankfort, Ind.
1915 Peoples Life Ins. Co. of Illinois, Chicago, Ill.
1915 Peoria Life Ins. Co., Peoria, Ill.
1921 Philadelphia Life Ins. Co., Philadelphia, Pa.
1921 Phoenix Mutual Life Ins. Co., Hartford, Conn.
1921 Provident Mutual Life Ins. Co., Philadelphia, Pa.
1921 Prudential Ins. Co. of America, Newark, N.J.
1913 Public Savings Ins. Co. of America, Indianapolis, Ind.
1916 Register Life Ins. Co., Davenport, Iowa.
1920 Reinsurance Life Co. of America, Des Moines, Iowa
1915 Reliance Life Ins. Co., Pittsburgh, Pa.
† Reserve Loan Life Ins. Co., Indianapolis, Ind.
† Royal Union Mutual Life Ins. Co., Des Moines, Iowa.
1923 San Jacinto Life Ins. Co., Beaumont, Tex.

ADMITTED	
1910	Scranton Life Ins. Co., Scranton, Pa.
†	Security Life Ins. Co. of America, Chicago, Ill.
1922	Security Mutual Life Ins. Co., Binghamton, N.Y.
1923	Shenandoah Life Ins. Co., Roanoke, Va.
1922	Southern States Life Ins. Co., Atlanta, Ga.
1924	Southern Union Life Ins. Co., Fort Worth, Tex.
1912	Southland Life Ins. Co., Dallas, Tex.
1916	Southwestern Life Ins. Co., Dallas, Tex.
†	State Life Ins. Co., Indianapolis, Ind.
1921	Travelers Ins. Co., Hartford, Conn.
1921	Two Republics Life Ins. Co., El Paso, Tex.
1921	Union Central Life Ins. Co., Cincinnati, Ohio.
1920	West Coast Life Ins. Co., San Francisco, Cal.
1912	Western Reserve Life Ins. Co., Muncie, Ind.
1911	Western States Life Ins. Co., San Francisco, Cal.
1917	Western Union Life Ins. Co., Spokane, Wash.
1914	Wisconsin Life Ins. Co., Madison, Wis.

DECEASED FELLOWS

ADMITTED		DIED
1924	Clarence Edmund Cale	1924
†	Carroll Booth Carr	1919
†	Horton W. Cochnower	1913
1914	Isaac Davenport	1916
†	John Maynard Emery	1919
†	Virgil Morrison Kime	1918
1918	Louis Linzmeyer	1918
†	Lucius McAdam	1913
†	Malcolm McNeill	1923
1914	Albert Garfield Portch	1918
†	Henry Sherman Vail	1919
†	Geert Andries Van der Sluis	1917
1913	Harris Elias Vineberg	1920
†	Benjamin A. Welstead	1910
†	Paul Livingston Woolston	1924
†	Wilbur S. Wynn	1913

DECEASED ASSOCIATES

ADMITTED		DIED
†	Nels Nelson	1912
1919	Arturo H. Clement	1924

STUDENTS

THOSE WHO HAVE PASSED ANY PART OF THE EXAMINATIONS DURING THE LAST THREE YEARS AND HAVE NOT BEEN ADMITTED AS ASSOCIATES

(Italic numerals indicate what parts of the examinations have been passed; bold numerals indicate parts waived.)‡

ALDRICH, GLENN P., University of Iowa, Iowa City, Iowa. (*1–4*).
AMBLER, ARTHUR B., Continental Life Ins. Co., St. Louis, Mo. (*1*)
AMOSS, G. BRUCE, Great-West Life Assurance Co., Winnipeg, Man., Canada. (*4*)
ANDERSON, T. LOYAL, care of D. F. Campbell, 160 N. LaSalle St., Chicago, Ill. (*1–6, 8*)
ANGERER, JOHN W., Michigan Insurance Dept., Lansing, Mich. (**1**, *2, 3, 6*)
ARMANTROUT, EVERETT D., Provident Mutual Life Ins. Co., Philadelphia, Pa. (*1, 3*)
ATRUBIN, HARRY, 390 Manitoba Ave., Winnipeg, Man., Canada. (*1–4*)
ATWATER, ROBERT BURDICK, Bankers Life Ins. Co., Des Moines, Iowa. (*1, 2*)
BAILEY, NEIL, 99 Inkerman St., London, Ontario, Canada. (*2*)
BARNES, ORLIN M., Franklin Life Ins. Co., Springfield, Ill. (*1, 2*)
BATHO, ELGIN R., Great West Life Assurance Co., Winnipeg, Man., Canada. (*1–4*)
BELL, WRAY M., 209 Robert St., Toronto, Ont., Canada. (*1–3*)
BENNER, (MISS) AMY L., Peoria Life Ins. Co., Peoria, Ill. (*1–5*)
BEVERIDGE, JAMES R., Manufacturers Life Ins. Co., Toronto, Canada. (*1–4*)
BITTEL, WILLIAM H., 523 Forest Ave., Ann Arbor, Mich. (*1*)
BOISSARD, RICHARD, National Guardian Life Ins. Co., Madison, Wis. (*1–4*)
BOURSCHEIDT, PAUL F., Peoria Life Ins. Co., Peoria, Ill. (*1, 2*)
BOWES, GEORGE H., Great-West Life Insurance Co., Winnipeg, Man., Canada. (*1*)
BRAHANA, HENRY R., 707 S. McCullough Ave., Urbana, Ill. (*1–7*)
BROCKWAY, PARKER W., Bankers Life Co., Des Moines, Iowa. (*1–3*)
BRONSON, DORRANCE C., 615 Monroe St., Ann Arbor, Mich. (*1–4*)
BUCKNELL, EARL F., 604 E. Madison St., Ann Arbor, Mich. (*2, 3*)
BUCKNER, DAVID E., Jefferson Standard Life Ins. Co., Greensboro, N.C. (*1–4*)
BURGAR, WILLIAM J., 54 Murray St., Toronto, Canada. (*1–4*)
CARSON, (MISS) CHRISTIANA E. ,235 Eugene St., Norwood, Man., Canada. (*1, 2*)
CHEN, SSU-TU, 402 Benjamin St., Ann Arbor, Mich. (*3*)
CLEMO, AMBROSE J., Montana Life Ins. Co., Helena, Mont. (*1–4*)
COATES, CLARENCE S., 2019 Los Angeles Ave., Berkeley, Cal. (*1–6*)
COLEMAN, JOHN H., Lafayette Life Ins. Co., Lumberton, N.C. (*1, 2*)
CONROD, STUART F., Great West Life Ass'ce. Co., Winnipeg, Man., Canada. (*1–4*)
COX, LEWIS C., National Life & Accident Ins. Co., Nashville, Tenn. (*1–4*)
CRAIG, CECIL C., 701 S. Division St., Ann Arbor, Mich. (*1–4*)
CRIPPEN, LLOYD K., 1429 Euclid St., N.W., Washington, D.C. (*1–6, 8*)
DANZINGER, LEO J., 24 Hart Ave., Winnipeg, Man., Canada. (*1, 2*)
DAVIDSON, WAID J., Iowa Insurance Dept., Des Moines, Iowa. (*1–4*)
DUNLOP, BURTON E., Rm. 426, 195 Broadway, New York, N. Y. (*2–4*)
DREBIN, HARRY S., 228 S. Warren Ave., Big Rapids, Mich. (*3*)
EASLEY, KNIGHT H., Amicable Life Ins. Co., Waco, Tex. (*1, 2*)
EDE, CECIL F., 1216 Third Ave. N., Great Falls, Mont. (*1–3, 5*)
ELLSWORTH, LEON W., Southwestern Life Ins. Co., Dallas, Tex. (*1, 3*)
ESTES, E. FORREST, Bankers Reserve Life Co., Omaha, Neb. (*1, 3, 4, 6*)
EVANS, JOHN H., Ohio National Ins. Co., Cincinnati, Ohio. (*1–6*)
FERGUSON, ARTHUR A., Western Union Life Ins. Co., Spokane, Wash. (*1–3*)
FLESNER, (MISS) IRENE E., Board of Pensions and Relief, 820 Garland Bldg., Chicago, Ill. (*1–4*)

‡ The parts are shown according to the revised syllabus in which former Part 2 was changed to Part 3 and former Part 3 to Part 2.

Frederick, Russell A., Franklin Life Ins. Co., Springfield, Ill. (*1*)
Glover, Lynn A., 29 S. LaSalle St., Chicago, Ill. (*1–5, 8*)
Graeser, A. Otis, Ohio Nat. Life Ins. Co., Cincinnati, Ohio. (*1, 2*)
Green, Walter C., 2584 Park St., Salt Lake City, Utah. (*1–4*)
Hacking, Frank, 320 Brooklyn St., Winnipeg, Man., Canada. (*2*)
Hall, William D., National Bureau of Casualty and Surety Underwriters, 120 West 42nd St., New York, N.Y. (*1*)
Hammer, H. H., Dept. of Insurance and Banking, Austin, Tex. (*1–7*)
Hartwell, Maurice, 5065 Woodward Ave., Detroit, Mich. (*3*)
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R Louise Roch

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THE RECORD
of the
AMERICAN INSTITUTE *of* ACTUARIES
November, 1925

ADDRESS BY THE PRESIDENT, PERCY H. EVANS

At the meeting of the Institute held in November, 1924, your attention was directed to certain changes in the requirements for admission. Since then a slight change in the order of the examinations has also been made. At a meeting of the Board of Governors last June the revision of the syllabus was referred to the committee working on the subject, consisting of the President, Secretary, and Editor of *The Record*, with power to act. I am pleased to report that the work has been completed, including a laborious revision of the course of reading upon which the examinations will be based. The revised requirements, syllabus, list of the members and students, and other information have been printed in a *Year Book* which we hope will not fail to receive an annual revision.

The work on this revision has suggested some comments which, if somewhat far afield, will not, I hope, be entirely destitute of interest to those among us who are at the threshold of actuarial work and while fortunate in the possession of a thorough technical equipment have the minimum of practical office experience. Students should not fail to realize the limitations of this or any different syllabus. The student who has mastered the prescribed course of reading can pass his examinations, but a syllabus is at best only a bare outline constructed for the guidance of students and examiners. It would fail of its purpose if in its construction an attempt were made to produce anything even remotely approaching a complete list of the material with which the student should be familiar, especially many of the papers printed in *The Record*, the *Transactions*, the *Journal*, and elsewhere.

The revision of our syllabus brings up to date one of the more important functions of the American Institute, that of assisting students to acquire technical proficiency with economy of time and energy. It has seemed to me, however, that after a student has devoted himself to the examination system and has met all the requirements, the obligation of the Institute is rather increased than liquidated, for a time at least. This obligation extends not only into the field of professional ethics but to certain vaguely defined situations which bear on the young actuary's practical usefulness. Let me say in the beginning that I have experienced great difficulty in expressing the thought I have in mind and would have abandoned it but for a belief that, at the risk of being completely misunderstood, I might say something helpful to the young man who is entering or about to enter upon the actual practice of responsible office duties where after some years spent on theory worked out under ideal conditions he is forced to proceed under conditions far from ideal either because he has no theory, or having a private theory has no data, or having both sound theory and adequate data he cannot secure the understanding of others whose opinions must be considered.

Intelligence is the ability to solve problems; knowledge is its accumulated by-product. Some of our knowledge comes to us theory-end first and some of it fact-end first. The philosophers used to talk about deductive reasoning and inductive reasoning, as if one were somehow more respectable, scientific, and even more evidently moral than the other; and as if the two methods were not fundamentally the same thing. Even a science so eminently deductive as mathematics has its inductive aspects. When Sir Isaac Newton began his labors on the problems of the *Principia* he found that he lacked an essential tool. From scattered and fragmentary suggestions he deduced the necessary tool. "In creative thought common sense is a bad master. Its sole criterion for judgment is that the new ideas shall look like the old ones." Newton was not greatly concerned with the logical basis of his method. It was a mere tool constructed for the larger purpose of solving problems. All he said about it during his lifetime was in a letter to Leibnitz, that he had a method of determining maxima and minima, of drawing tangents and performing similar operations. In view of the difficulty lesser men subsequently had in finding the common sense of the calculus,

I venture to believe that even Newton's confidence in his results was founded as much on inductive considerations as upon his deductions from the fundamental principle. Even as lately as my own student days there were mathematical teachers of my acquaintance who would exhibit considerable feeling in discussing the merits of the method of limits as against infinitesimals.

Now I imagine you are wondering what I am driving at in this brief excursion into the realms of philosophy. The fact is I am trying to define some quite vague notions, and have endeavored to give them perspective by sketching in a background. In plain English, I wish to say to the young actuary that if he expects his most carefully considered theories, or the logical consequences of his laboriously compiled data, to secure instant and unqualified acceptance from those with whom his work brings him in contact, he is doomed to disappointment. I suspect that there is no technical employment where adjustments and compromises are so insistent as in that of the actuary, and where there is more need of a delicate sense of the limits beyond which compromise becomes a betrayal of ethics and self-respect.

The young actuary entering upon duties of responsibility soon begins to encounter the human problems of actuarial practice. He will find many such problems for which no answer is to be found in the books and papers in this syllabus. He will sometimes find himself in conflict with the disposition of one type of mind to reason deductively from private doctrines and disregard all discordant facts and with another type of mind to which an isolated fact has all the force of a broad generalization.

One of the most puzzling and annoying things to the youthful intellect is the extent to which different intelligences can disagree and the ease with which the non-technical disputant gains adherents to his views. There is a passage in one of Smollett's novels that every actuary who has occasion to work with legislative committees would do well to paste in his hat. To account for the ease with which the bogus Count Fathom overthrew the old Bristol doctor, Smollett says:

> In all disputes upon physic that happen between a person who really understands the art and an illiterate pretender, the argument of the first will seem obscure and unintelligible to those who are unacquainted with the previous systems on which they are built; while the other's theory,

derived from common notions and superficial observation will be more agreeable because better adapted to the comprehension of the hearers.

My notion of the educational opportunities offered by an association of technical men such as the American Institute of Actuaries goes farther than the setting of subjects, a course of reading, and the machinery of examinations with prospective right to attach certain letters to one's name. The benefits of such association are by no means limited to this system or to the advancement of actuarial knowledge by the publication of original papers that would not otherwise be written at all. The collateral benefits I have in mind are much more elusive and hard to come at, because not directly taught except by experience. I hope it is unnecessary to explain that I am not referring to the vague and superficial commonplace that there is a difference between theory and practice. There can be no difference between sound theory and sound practice except in the sense that the velocity of a body falling in air differs from the theoretical value. Let us therefore say that differences between sound theory and practice occur because of conditions. These conditions are of several kinds. The work of the actuary is conditioned by time and space like everything else, but also by moral and psychological circumstances. We may assume that the young actuary having mastered the contents of our syllabus and passed his examinations is in possession of the sound principle applicable to the particular problem at issue. Whether the practice exactly follows the theory will depend on the force of the conditions, and among these may be mentioned his ability to express his theory intelligibly, to visualize sympathetically the non-technical point of view, avoiding undue pride of intellect and the sin of intolerance. Very often his practical effectiveness will depend upon his ability to work out compromises without at the same time compromising his professional self-respect. He will encounter many such demands for compromise where the temptation to follow the line of least resistance will be very strong.

The technical training of the actuary as reflected in the syllabus is a training in principles, theories, doctrines, in generalizations. The actuary is never so happy as when he can resume a broad group of experience facts in a short generalization upon which he may base his recommendations. He will, however, often be tempted to make recommendations or consent to recommendations

on other grounds. At times an immense amount of pressure will be brought upon him to follow other minds than his own; to cater to popularity by advocating or agreeing to measures that depart from his strict conception of sound theory. He will not find it easy to dispose of these decisions in such a way as to preserve at the same time his effectiveness and his self-respect. It is easy to say that "right is right and wrong is wrong," but in practice the matter is not so simple as that. The martyr-for-principle attitude of mind is admirable and occasionally it is the price of self-respect, but as a daily habit of mind it does not exert much influence. The opportunist mind is vastly dangerous to sound practice precisely because, until exposed for what it is, it exerts more influence in office councils. Fortunately the opportunist mind is not attracted to actuarial studies; but somewhere between these two extremes lies the best solution of a multitude of moral problems which present themselves to the young actuary. The form and substance of his solutions will, to a large extent, determine whether he is regarded by his associates as a mere theorist and his opinions and recommendations neglected; or as a conservative guide whose opinions are intelligible and at the same time practical and safe. Where the young actuary should lay his course to avoid the sand bars of impracticability and the currents of opportunism, while keeping off the reefs of error, is a personal problem in which little specific help can be given, but by his personal contacts at these meetings, through the papers and informal discussions, and generally by the analysis and synthesis of opinions and points of view reflecting the mass judgment of an association of this kind, the individual does enjoy an immense increase in his power of effective service.

It is gratifying to believe—and I think we are justified in the belief—that the actuarial associations in the United States have been extraordinarily successful, not only in their educational function and the enforcement of a professional standard, but also in the creation of a professional atmosphere and background that is of assistance to the younger men by supplying criteria in many situations where unsupported individual judgment might easily go astray for lack of perspective.

The outline of some not very clearly defined notions which I have tried to express on this occasion would be incomplete if, in emphasizing the obligation of the Institute to its students and

successful candidates, I did not also point out the correlative obligation to be ever thoughtful of the standing and welfare of the Institute in its several activities. You are indebted to the Institute for the educational assistance you have had, for much of the improvement in the standing of actuaries as a professional class, and for the personal backing given you by your degrees. You can repay these debts only by your assistance in passing these opportunities on to other deserving persons, by contributing your time and intelligence to the preparation of original papers and discussions, and above all by so conducting your professional work that it shall at all times reflect credit upon the American Institute and upon all actuarial associations.

INTRODUCTION OF MR. ARTHUR B. WOOD, PRESIDENT OF THE ACTUARIAL SOCIETY OF AMERICA

I am pleased to introduce to you this morning a very distinguished guest, Mr. Arthur B. Wood, the president of the Actuarial Society of America, vice-president and actuary of the Sun Life of Canada. I am sure the Institute is more than glad to extend to Mr. Wood the privilege of the floor.

We hope, Mr. Wood, that you will express yourself freely on any topic discussed in which you are interested.

REPLY OF MR. WOOD:

I feel that I should not take up much of your time for I know you are very busy. It is fitting, however, that I should acknowledge the very cordial reception which I have been accorded here this morning. I appreciate it very much indeed.

When I received your kind invitation, Mr. President, to attend this meeting, it looked at first as if I would not be able to accept, but I determined that if I could possibly arrange it I would be here and fortunately for me matters have so regulated themselves as to enable me to attend.

I desired to come for two reasons in particular. First of all, as you will recall, exactly a year ago this week we held a joint meeting of the American Institute and the Actuarial Society at French Lick Springs. The purpose of that meeting, as you all know, the main purpose I may say, was to bring our two organizations into closer touch with each other. We desired that our members should

become better acquainted in order that we might be in a position to co-operate more effectively in any movement which might arise in the future where the general interest of the business of life assurance had to be considered.

That joint meeting, I think, was admitted by everybody to have been a great success, and the purposes we had in mind were satisfactorily carried out.

It is not possible to hold such a meeting every year, but nevertheless I think that our two societies should endeavor to keep in as close touch as possible with each other. Therefore I welcomed this opportunity as President of the Actuarial Society of America to form more acquaintances and to become better acquainted with those of you whom I have already been privileged to meet.

Then there was also a more personal reason. I have heard a great deal about the Institute meetings. You have a very active and enterprising organization. I know that your meetings are exceedingly interesting, and I realized that I would receive much pleasure and satisfaction from being here, and also would derive much benefit from listening to the discussions.

I believe it would be out of place for me to take up more time now. I will therefore simply content myself by expressing my appreciation of the privilege you have accorded me of being here and thanking you for the very kind manner in which you have introduced me. I also take this opportunity to extend to you, one and all, the very cordial greetings of the Actuarial Society of America.

PRESIDENT EVANS:

I am sure that the membership of the American Institute sympathizes very keenly with Mr. Wood's remarks about the relationships that ought to exist between the two societies. I hope that the success of the joint meeting that was held last year will result in other joint meetings in the future.

Mr. Wood says it is impossible to do that sort of thing very often, but perhaps it can be done with reasonable frequency, and we are exceedingly glad to have Mr. Wood here and to have him speak to us. And as I said before, I hope that he will take the privilege of the floor whenever he feels inclined to do so.

RETURNS UNDER DISTRICT-AGENCY CONTRACTS

By M. A. LINTON

Adequate district-agency development is something which many companies desire, but not so many actually possess. The primary producer, the man who carries the ratebook, receives the lion's share of the commission that is paid by the company. In many instances the margin remaining is none too large to compensate the general agent for his own organizing work; and in consequence he frequently is unable, or thinks he is unable, to establish other organizing centers under the control of district agents. Another phase of the same problem is the compensation of supervisors employed to stimulate the production of the agents rather than to build up district-agency centers.

In November, 1924, it was the writer's privilege to present to the Institute a paper, "Returns under Agency Contracts."[1] The subject we shall discuss in this present paper is closely related to that of the former one, and might with propriety have been included in it. However, that paper was sufficiently long in itself to justify holding for subsequent treatment the subject which is now before us.

In arriving at concrete figures, it is necessary, as it was before, to make certain basic assumptions as to agency contracts, production program, average premium and average policy, termination rates and expenses. These will now be taken up in detail.

The contracts.—As in the original paper we shall assume two underlying general agents' contracts: (I) providing renewal commissions of nine $7\frac{1}{2}$'s, five 5's, and a subsequent collection fee of $2\frac{1}{2}$ per cent; and (II) providing renewals of $12\frac{1}{2}$ per cent graded the second year, eight $7\frac{1}{2}$'s, and a subsequent collection fee of $2\frac{1}{2}$ per cent. The general agent's renewal margins under the two contracts (assuming the subagent's renewal commission to be 5 per cent for nine years) are: (I) nine $2\frac{1}{2}$'s, five 5's, and a subsequent collection fee of $2\frac{1}{2}$ per cent; and (II) $7\frac{1}{2}$ per cent graded the second year, followed by $2\frac{1}{2}$ per cent for the duration of the contract. In each

[1] *The Record*, XIII, 283.

case we shall also assume that the general agent has available a first year's margin of 5 per cent.

The district agent's compensation must be paid from the foregoing commissions. We shall assume that any overriding commission which he receives will be confined to the first two policy years. Three combinations of district agent's commissions will be studied: first, $2\frac{1}{2}$ per cent the first year and $2\frac{1}{2}$ per cent the second ($2\frac{1}{2}$–$2\frac{1}{2}$); second, 5 per cent the first year and none the second (5–0); and third, 5 per cent the first year and $2\frac{1}{2}$ per cent the second year (5–$2\frac{1}{2}$).

Production program.—We shall assume for the basic tables a single year's production of $100,000 and a uniform yearly production of $100,000. The figures thus presented enable the building-up of results for any production program that may be assumed.

Average premium and average policy.—The basis of the paper is an assumed average premium of $30 per thousand and an average policy of $3,500. The average premium affects primarily the commission income; the average policy the office expenses, since the accounting work depends upon the number of policy units.

Termination rates.—In the original paper two sets of termination rates, *A* and *B*, were assumed. The *A* rates involved a voluntary withdrawal rate of 10 per cent in the first year, grading down to 2 per cent in the fifteenth year. The *B* rates commenced at 20 per cent and graded down to 4 per cent. Full details are given in the original paper. We shall employ the same rates in the present investigation.

Agency expenses.—This phase of our subject, as before, presents more difficulty than any other, owing to the absence of authoritative data. For example, we should like to know, in addition to the probable expense of handling an old policy, the "office" cost of a new policy during its first year. When a policy is produced in a district agency, about the only direct cost imposed upon the main agency office is that of a clerical or accounting nature. The rent, salary, and most of the other expenses connected with the soliciting agent would be incurred at the district office. These expenses are in the nature of an overhead charge that continues from month to month with but little variation, irrespective, within limits of course, of the actual production.

We shall assume that the yearly expense of maintaining an old

policy at the main agency office is $1.25. Most of this expense is of an accounting or clerical character, being connected with the collection of premiums, the handling of loans, options, assignments, beneficiary changes, surrenders, etc. What is the "office" cost to the main agency of handling a new policy? This figure should be approximated if we are to estimate the general agent's net return from district-agency business. Taking the agency expense data furnished by some of the Provident Mutual general agents for the year 1924, we have attempted to segregate the first year accounting and clerical cost from the cost involved in maintaining the agents located at the main agency office. From this study it would appear that the first year's "office" cost of new business is in the neighborhood of $6.25 per new policy; that is, five times the average old policy cost of $1.25.

There is of course more than one way of regarding district-agency business. The main agency office presumably is well established and producing a substantial volume of business. To maintain this business a certain overhead must be incurred irrespective of any district-agency business. The general agent in contemplating the establishment of a district agency may therefore take the ground that he will charge against the district-agency business only those expenses which are *added* by such business and, within reasonable limits, will not charge against it certain items as rent and clerical expenses that would be incurred anyhow. There is something to be said for this general principle. From this point of view the assumption of new and old policy "office" costs of $6.25 and $1.25, respectively, may be somewhat high. However, it seems better to lean toward conservatism in this respect so that the general agent's probable return will not be overstated.

In computing our basic tables of net returns we shall take into account the expenses just described, but shall not take into account the expenses of the district agency. The reason for this omission is frankly an absence of information which would enable us to relate district-agency expenses to a given volume of district-agency business. The main items of such expenses are rent, salaries, miscellaneous office expenses, and cost of supervision. Take, for example, a district agency producing in the neighborhood of one-half to three-quarters of a million a year, located in a small city. The rent might reasonably be in the neighborhood of $50 per month, the

salary of a clerk $75, miscellaneous office expenses from $25 to $50, and supervising cost of $25, including cost of correspondence and traveling expenses, yielding a total ranging from $175 to $200 per month.

Assuming a given production of business, who should pay these district-agency expenses—the general or the district agent? The

TABLE I

AMOUNTS, PREMIUMS, AND POLICIES IN FORCE; AND "OFFICE" EXPENSES AT MAIN AGENCY

Single Year's Production of $100,000

Average Premium, $30 Average Policy, $3,500

First-Year and Renewal Expenses per Policy: $6.25 and $1.25

YEAR	TERMINATION RATES *A*				TERMINATION RATES *B*			
	Amount in Force	Premiums in Force	Policies in Force	Expenses	Amount in Force	Premiums in Force	Policies in Force	Expenses
1	$100,000	$3,000	28.6	$179	$100,000	$3,000	28.6	$179
2	89,600	2,688	25.6	32	79,600	2,388	22.7	28
3	83,800	2,514	23.9	30	69,700	2,090	19.9	25
4	79,100	2,373	22.6	28	62,300	1,868	17.8	22
5	75,100	2,253	21.5	27	56,400	1,692	16.1	20
6	71,600	2,148	20.5	26	51,500	1,545	14.7	18
7	68,400	2,052	19.5	24	47,400	1,422	13.5	17
8	65,600	1,968	18.7	23	43,900	1,318	12.6	16
9	63,100	1,893	18.0	23	41,000	1,230	11.7	15
10	60,800	1,824	17.4	22	38,400	1,151	11.0	14
11	58,600	1,758	16.7	21	36,000	1,081	10.3	13
12	56,500	1,695	16.1	20	33,900	1,016	9.7	12
13	54,500	1,635	15.6	20	31,900	956	9.1	11
14	52,500	1,575	15.0	19	30,000	900	8.6	11
15	50,600	1,518	14.5	18	28,300	849	8.1	10

answer depends upon several factors, of which one is the contract under which the district agent operates. Obviously, the larger the commission margin he receives on the business his district produces, the smaller the margin left for the general agent and the more able is the district agent to contribute to the expenses. Another factor is a possibly increased company expense allowance which may follow an increase in the production of business through the agency.

Tables I, II, and III.—The first three tables contain most of the basic figures from which the more interesting Tables IV-VII are constructed. They are included primarily for the sake of com-

pleteness; but a brief consideration of them may perhaps make more clear the succeeding tables. Table I corresponds to a single year's production of $100,000, and Table II to a uniform yearly production of the same amount. To some, $100,000 may appear too small a figure to assume. However, it was felt that $1,000,000 would be too large and that any intermediate figure would be less convenient

TABLE II

AMOUNTS, PREMIUMS, AND POLICIES IN FORCE; AND "OFFICE" EXPENSES AT MAIN AGENCY

Yearly Production of $100,000

Average Premium, $30 Average Policy, $3,500

First-Year and Renewal Expenses per Policy: $6.25 and $1.25

YEAR	TERMINATION RATES *A*				TERMINATION RATES *B*			
	Amount in Force	Premiums in Force	Policies in Force	Expenses	Amount in Force	Premiums in Force	Policies in Force	Expenses
1.....	$ 100,000	$ 3,000	29	$179	$100,000	$ 3,000	29	$179
2.....	189,600	5,688	54	211	179,600	5,388	51	207
3.....	273,400	8,202	78	241	249,300	7,478	71	232
4.....	352,500	10,575	101	269	311,500	9,346	89	254
5.....	427,600	12,828	122	296	367,900	11,038	105	274
6.....	499,200	14,976	143	322	419,400	12,583	120	292
7.....	567,600	17,028	162	346	466,800	14,005	133	309
8.....	633,200	18,996	181	369	510,700	15,323	146	325
9.....	696,300	20,889	199	392	551,700	16,552	158	340
10.....	757,100	22,713	216	414	590,100	17,703	169	354
11.....	815,700	24,471	233	435	626,100	18,783	179	367
12.....	872,200	26,166	249	455	660,000	19,799	189	379
13.....	926,700	27,801	265	475	691,800	20,755	198	390
14.....	979,200	29,376	280	494	721,800	21,655	206	401
15.....	1,029,800	30,894	294	512	750,200	22,505	214	411

than $100,000. Results for termination rates *A* and *B* are given. The other basic assumptions are noted in the headings. Table III is devoted to the general agent's gross overriding commission from which the appropriate expense figures from Tables I and II are subtracted to obtain for succeeding tables the net overriding commission which the general agent either divides with his district agent or retains, according to the contract with the district agent.

Tables IV and V.—These tables begin to answer some of the questions in which we are interested. It must continually be borne in mind, however, that the figures take no account of expenses

incurred at the district office. Referring to Table IV, which is based upon termination rates *A*, consider the district agent's contract ($2\frac{1}{2}$–$2\frac{1}{2}$) in conjunction with general agent's contract II.

TABLE III

GENERAL AGENT'S GROSS OVERRIDING COMMISSIONS

Average Premium, $30

YEAR	TERMINATION RATES *A*				TERMINATION RATES *B*			
	Single Year's Production of $100,000		Yearly Production of $100,000		Single Year's Production of $100,000		Yearly Production of $100,000	
	Contract II	Contract I	Contract II	Contract I	Contract II	Contract I	Contract II	Contract I
1	$150	$150	$ 150	$ 150	$150	$150	$ 150	$ 150
2	181	67	331	217	161	60	311	210
3	63	63	394	280	52	52	363	262
4	59	59	454	339	47	47	410	309
5	56	56	510	396	42	42	452	351
6	54	54	564	449	39	39	491	390
7	51	51	615	501	36	36	527	425
8	49	49	664	550	33	33	560	458
9	47	47	711	597	31	31	591	489
10	46	46	757	643	29	29	620	517
11	44	88	801	731	27	54	647	571
12	42	85	843	816	25	51	672	622
13	41	82	884	897	24	48	696	670
14	39	79	924	976	22	45	718	715
15	38	76	962	1,052	21	43	739	758
Total for:								
5 years	510	396	1,839	1,382	452	351	1,686	1,281
10 years	757	643	5,150	4,122	620	517	4,475	3,560
15 years	962	1,052	9,564	8,594	739	758	7,947	6,896
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:								
5 years	477	369	1,635	1,230	426	329	1,501	1,143
10 years	654	545	3,976	3,165	547	449	3,477	2,754
15 years	769	775	6,429	5,639	615	584	5,409	4,604

During the first year, the general agent has a deficit of $104 and the district agent a return of $75. In the second year, the general and district agents receive $82 and $67, respectively. After the second year, the district agent's interest ceases and the general agent receives $33. This decreases year by year with the decrease

in the business in force until it reaches $20 in the fifteenth year. The district agent's total during the two years, for the $100,000 production is $142 or $1.42 per thousand. The general agent's total for five years is 71 cents per thousand of original production; for ten years, $2.00 per thousand; and for fifteen years, $3.06 per thousand. Figures showing discounted values (at 5 per cent) are

TABLE IV

GENERAL AGENT'S AND DISTRICT AGENT'S RETURNS WITHOUT REFERENCE TO DISTRICT-AGENCY EXPENSES

Single Year's Production of $100,000
Termination Rates *A*

Year	District-Agency Contract 2½–2½		District-Agency Contract 5–0		District-Agency Contract 5–2½	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
	General-Agency Contract II					
1	−$104	$ 75	−$179	$150	−$179	$150
2	+ 82	67	+ 149	.....	+ 82	67
3	33	.....	33	.....	33	.....
4	31	.....	31	.....	31	.....
5	29	.....	29	.....	29	.....
6	28	.....	28	.....	28	.....
7	27	.....	27	.....	27	.....
8	26	.....	26	.....	26	.....
9	24	.....	24	.....	24	.....
10	24	.....	24	.....	24	.....
11	23	.....	23	.....	23	.....
12	22	.....	22	.....	22	.....
13	21	.....	21	.....	21	.....
14	20	.....	20	.....	20	.....
15	20	.....	20	.....	20	.....
Totals:						
5 years	71	142	63	150	− 4	217
10 years	200	142	192	150	+ 125	217
15 years	306	142	298	150	231	217
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	57	139	46	150	− 18	214
10 years	149	139	138	150	+ 74	214
15 years	209	139	198	150	134	214

TABLE IV—*Continued*

Year	District-Agency Contract $2\frac{1}{2}$–$2\frac{1}{2}$		District Agency Contract 5–0		District-Agency Contract 5–$2\frac{1}{2}$	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
General-Agency Contract I						
1	−$104	$ 75	−$179	$150	−$179	$150
2	− 32	67	+ 35	.....	− 32	67
3	+ 33	.....	33	.....	+ 33	.....
4	31	.....	31	.....	31	.....
5	29	.....	29	.....	29	.....
6	28	.....	28	.....	28	.....
7	27	.....	27	.....	27	.....
8	26	.....	26	.....	26	.....
9	24	.....	24	.....	24	.....
10	24	.....	24	.....	24	.....
11	67	.....	67	.....	67	.....
12	65	.....	65	.....	65	.....
13	62	.....	62	.....	62	.....
14	60	.....	60	.....	60	.....
15	58	.....	58	.....	58	.....
Totals:						
5 years	− 43	142	− 51	150	− 118	217
10 years	+ 86	142	+ 78	150	+ 11	217
15 years	398	142	390	150	323	217
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	− 53	139	− 64	150	− 128	214
10 years	+ 39	139	+ 28	150	− 36	214
15 years	214	139	203	150	+ 139	214

also included in the table. Turning for a moment to the corresponding section of Table V it is interesting to note the material effect of an adverse lapse rate upon the general agent's return. Instead of $3.06 per thousand for the fifteen-year period it has been reduced to $1.93.

Other examples could be discussed at length, but undue space would be required. The reader may readily draw his own conclusions in connection with any contract included in the tables. We may call attention in passing, however, to the relatively close agreement between the corresponding discounted values of the

general agent's returns for fifteen years contained in the first and second halves of Table IV relating respectively to general agent's contracts II and I. This arises from the fact that on the basis of 5 percent interest and termination rates *A*, a 5 per cent commission the second year is about equivalent in discounted value at the issuance of a policy, to five $2\frac{1}{2}$'s from the eleventh to the fifteenth

TABLE V

General Agent's and District Agent's Returns without Reference to District-Agency Expenses

Single Year's Production of $100,000
Termination Rates *B*

Year	District-Agency Contract $2\frac{1}{2}$–$2\frac{1}{2}$		District-Agency Contract 5–0		District-Agency Contract 5–$2\frac{1}{2}$	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
	General-Agency Contract II					
1	−$104	$ 75	−$179	$150	−$179	$150
2	+ 73	60	+ 133	.....	+ 73	60
3	27	.....	27	.....	27	.....
4	25	.....	25	.....	25	.....
5	22	.....	22	.....	22	.....
6	21	.....	21	.....	21	.....
7	19	.....	19	.....	19	.....
8	17	.....	17	.....	17	.....
9	16	.....	16	.....	16	.....
10	15	.....	15	.....	15	.....
11	14	.....	14	.....	14	.....
12	13	.....	13	.....	13	.....
13	13	.....	13	.....	13	.....
14	11	.....	11	.....	11	.....
15	11	.....	11	.....	11	.....
Totals:						
5 years	43	135	28	150	− 32	210
10 years	131	135	116	150	+ 56	210
15 years	193	135	178	150	118	210
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	30	132	12	150	− 45	207
10 years	93	132	75	150	+ 18	207
15 years	129	132	111	150	54	207

TABLE V—*Continued*

Year	District-Agency Contract 2½–2½		District-Agency Contract 5–0		District-Agency Contract 5–2½	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
	General-Agency Contract I					
1	−$104	$ 75	−$179	$150	−$179	$150
2	− 28	60	+ 32	.....	− 28	60
3	+ 27	.....	27	.....	+ 27	.....
4	25	.....	25	.....	25	.....
5	22	.....	22	.....	22	.....
6	21	.....	21	.....	21	.....
7	19	.....	19	.....	19	.....
8	17	.....	17	.....	17	.....
9	16	.....	16	.....	16	.....
10	15	.....	15	.....	15	.....
11	41	.....	41	.....	41	.....
12	39	.....	39	.....	39	.....
13	37	.....	37	.....	37	.....
14	34	.....	34	.....	34	.....
15	33	.....	33	.....	33	.....
Totals:						
5 years	− 58	135	− 73	150	− 133	210
10 years	+ 30	135	+ 15	150	− 45	210
15 years	214	135	199	150	+ 139	210
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	− 67	132	− 85	150	− 142	207
10 years	− 4	132	− 22	150	− 79	207
15 years	+ 100	132	+ 82	150	+ 25	207

years, inclusive. We would also mention, in passing, that the repetition in the parallel columns of Tables IV and V was permitted in order that the results for any given combination of contracts might be noted with the least chance of confusion.

Tables VI and VII.—These tables are of interest in that they show the result for a uniform yearly production. The amount assumed is $100,000.

Consider again, with termination rates *A*, the same combination of contracts discussed in connection with Tables IV and V. It will be seen that during the first fifteen years, the actual cash returns

to the general and district agents are nearly the same, $2,077 and $2,063, respectively. The general agent, however, has longer to wait for his returns, with the result that the discounted values become $1,242 and $1,481, respectively. Since expenses at the district agency have not been taken into account, it is obvious that there is but little margin for them unless perchance the home office makes

TABLE VI

GENERAL AGENT'S AND DISTRICT AGENT'S RETURNS WITHOUT REFERENCE TO DISTRICT-AGENCY EXPENSES

Yearly Production of $100,000
Termination Rates *A*

Year	District-Agency Contract 2½–2½		District-Agency Contract 5–0		District-Agency Contract 5–2½	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
	General-Agency Contract II					
1	−$104	$ 75	−$179	$150	−$179	$150
2	− 22	142	− 30	150	− 97	217
3	+ 11	142	+ 3	150	− 64	217
4	42	142	34	150	− 33	217
5	71	142	63	150	− 4	217
6	99	142	91	150	+ 24	217
7	126	142	118	150	51	217
8	152	142	144	150	77	217
9	176	142	168	150	101	217
10	200	142	192	150	125	217
11	223	142	215	150	148	217
12	245	142	237	150	170	217
13	266	142	258	150	191	217
14	286	142	278	150	211	217
15	306	142	298	150	231	217
Totals:						
5 years	− 2	643	− 109	750	− 377	1,018
10 years	+ 751	1,353	+ 604	1,500	+ 1	2,103
15 years	2,077	2,063	1,890	2,250	952	3,188
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	− 20	579	− 123	682	− 360	919
10 years	+ 508	1,084	+ 376	1,216	− 100	1,692
15 years	1,242	1,481	1,088	1,635	+ 425	2,298

TABLE VI—*Continued*

Year	District-Agency Contract $2\frac{1}{2}$–$2\frac{1}{2}$		District-Agency Contract 5–0		District-Agency Contract 5–$2\frac{1}{2}$	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
General-Agency Contract I						
1	−$104	$ 75	−$179	$150	−$179	$150
2	− 136	142	− 144	150	− 211	217
3	− 103	142	− 111	150	− 178	217
4	− 72	142	− 80	150	− 147	217
5	− 43	142	− 51	150	− 118	217
6	− 15	142	− 23	150	− 90	217
7	+ 12	142	+ 4	150	− 63	217
8	38	142	30	150	− 37	217
9	62	142	54	150	− 13	217
10	86	142	78	150	+ 11	217
11	153	142	145	150	78	217
12	218	142	210	150	143	217
13	280	142	272	150	205	217
14	340	142	332	150	265	217
15	398	142	390	150	323	217
Totals:						
5 years	− 458	643	− 565	750	− 833	1,018
10 years	− 275	1,353	− 422	1,500	−1,025	2,103
15 years	+1,114	2,063	+ 927	2,250	− 11	3,188
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	− 425	579	− 528	682	− 765	919
10 years	− 303	1,084	− 435	1,216	− 911	1,692
15 years	+ 455	1,481	+ 301	1,635	− 362	2,298

an expense allowance. Termination rates *B* make the general agent's situation still less favorable. The discounted value of his return for the fifteen years is reduced from $1,242 to $731.

Of course, we must not overlook the value of the general agent's future renewal equity in the business which has been built up. This factor was discussed at length in the original paper, of which Tables V, VI, XVII, and XVIII are useful in our present discussion. Assume a renewal equity of the balance of nine renewal commissions of 1 per cent. From Table XVII we find that in nine years a uniform yearly production of $100,000 will build up a renewal equity having

a discounted value of $798 or $563, according as *A* or *B* termination rates are assumed. After the agency is nine years old, these figures represent a future return in addition to the cash that has been realized from operations. They help to ameliorate the situation, but by no means do they solve the problem of providing the general agent with an adequate income.

TABLE VII

GENERAL AGENT'S AND DISTRICT AGENT'S RETURNS WITHOUT REFERENCE TO DISTRICT-AGENCY EXPENSES

Yearly Production of $100,000
Termination Rates *B*

Year	District-Agency Contract 2½–2½		District-Agency Contract 5–0		District-Agency Contract 5–2½	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
	General-Agency Contract II					
1	−$104	$ 75	−$179	$150	−$179	$150
2	− 31	135	− 46	150	− 106	210
3	− 4	135	− 19	150	− 79	210
4	+ 21	135	+ 6	150	− 54	210
5	43	135	28	150	− 32	210
6	64	135	49	150	− 11	210
7	83	135	68	150	+ 8	210
8	100	135	85	150	25	210
9	116	135	101	150	41	210
10	131	135	116	150	56	210
11	145	135	130	150	70	210
12	158	135	143	150	83	210
13	171	135	156	150	96	210
14	182	135	167	150	107	210
15	193	135	178	150	118	210
Totals:						
5 years	− 75	615	− 210	750	− 450	990
10 years	+ 419	1,290	+ 209	1,500	− 331	2,040
15 years	1,268	1,965	983	2,250	+ 143	3,090
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	− 84	554	− 212	682	− 425	895
10 years	+ 262	1,035	+ 81	1,216	− 346	1,643
15 years	731	1,411	507	1,635	− 87	2,229

TABLE VII—*Continued*

Year	District-Agency Contract $2\frac{1}{2}$–$2\frac{1}{2}$		District-Agency Contract 5–0		District-Agency Contract 5–$2\frac{1}{2}$	
	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return	General Agent's Return	District Agent's Return
	General-Agency Contract I					
1	−$104	$ 75	−$179	$150	−$179	$150
2	− 132	135	− 147	150	− 207	210
3	− 105	135	− 120	150	− 180	210
4	− 80	135	− 95	150	− 155	210
5	− 58	135	− 73	150	− 133	210
6	− 37	135	− 52	150	− 112	210
7	− 18	135	− 33	150	− 93	210
8	− 1	135	− 16	150	− 76	210
9	+ 15	135	0	150	− 60	210
10	30	135	+ 15	150	− 45	210
11	71	135	56	150	− 4	210
12	110	135	95	150	+ 35	210
13	147	135	132	150	72	210
14	181	135	166	150	106	210
15	214	135	199	150	139	210
Totals:						
5 years	− 479	615	− 614	750	− 854	990
10 years	− 490	1,290	− 700	1,500	−1,240	2,040
15 years	+ 233	1,965	− 52	2,250	− 892	3,090
Amounts for Given Number of Years Discounted to First Year at 5 Per Cent per Annum:						
5 years	− 442	554	− 570	682	− 783	895
10 years	− 456	1,035	− 637	1,216	−1,064	1,643
15 years	− 61	1,411	− 285	1,635	− 879	2,229

We have been considering the results under district-agency contract ($2\frac{1}{2}$–$2\frac{1}{2}$). Of the three district-agency contracts this is the most favorable to the general agent. Transferring our attention to contract (5–$2\frac{1}{2}$), we find the results under termination rates A in the two right-hand columns of Table VI. From the general agent's point of view they bear a silent testimony to the unprofitable character of the business. There is very little in it for him for the first fifteen years. If the tables had been carried further, more profit would be shown in the years after the fifteenth. However, fifteen years is a rather long time to wait, and a general agent might

reasonably hesitate to enter into an arrangement where the returns were so long delayed. And his hesitation might be increased should he investigate the effect upon his return of a heavier lapse rate. The next to the last column on the right-hand side of Table VII is not very encouraging. Finally, let us repeat again that no provision has been made for the expenses at the district-agency office.

What is the answer? In some instances "no district-agency development." In other instances where there is a greater margin for the general agent, either from company expense allowance or from an additional overriding commission, district-agency development is being carried on successfully. If, for example, each $100,000 of new business could bring to the general agent an expense allowance of say $300, the figures in the general agent's column of Tables VI and VII would yield an amount from which he could provide a reasonable sum for district-agency expenses and at the same time have a margin of profit that would make the business worth while.

Of the three district-agency contracts, much may be said in favor of $(2\frac{1}{2}-2\frac{1}{2})$. It is well that the district agent should have an interest in the conservation of the business for whose production he is responsible. Contract $(2\frac{1}{2}-2\frac{1}{2})$ gives him that interest by basing a part of his compensation upon the premiums paid for the second policy year. Contract $(5-2\frac{1}{2})$ does the same thing to a relatively lesser degree. However, it leaves so little for the general agent that it is hard, in the absence of special circumstances, to see how it can be recommended.

Personal business and unearned renewals.—District agents almost invariably write personal business and thereby augment their earnings. It is also frequently so arranged that they retain for themselves any renewal commissions which their subagents may fail to earn. They may have a rule that the subagents must produce $100,000 in a year to become entitled to nine renewal commissions. The number of renewals may then be graded down so that no renewals at all are paid unless the subagent produces $25,000 or more. In rural districts it frequently happens that the district agent obtains a considerable volume of business with limited renewals or no renewals at all. In determining the added income from unearned renewals and from personal business, the tables on pages 313 and 315 of the original paper[1] may be employed.

[1] *Loc. cit.*

Illustrative example.—It will perhaps help in the consideration of our subject if we investigate the results under an assumed district-agency program commencing with a production of $300,000 in the first year and increasing to $750,000 in the fifth year. The details of the personal business of the district agent, and of the business of the subagents, with the number of renewals, are given in Table VIII.

The net returns, still leaving out of account expenses at the district agency, have been computed for the three district agents' contracts, in conjunction with general agent's contract II; and are set forth in Tables IX, X, and XI. Each table contains two sections —the first one involving termination rates *A* and the second one

TABLE VIII

ILLUSTRATIVE DISTRICT-AGENCY PROGRAM

YEAR	TOTAL PRODUCTION	PERSONAL BUSINESS OF DISTRICT AGENT	SUBAGENTS' PRODUCTION		
			Number of Renewals of 5 Per Cent		
			9	5	0
1	$300,000	$100,000	$100,000	$ 50,000	$ 50,000
2	425,000	125,000	150,000	75,000	75,000
3	550,000	150,000	200,000	100,000	100,000
4	650,000	150,000	250,000	125,000	125,000
5 and thereafter	750,000	150,000	300,000	150,000	150,000

termination rates *B*. These tables have been made up from Tables IV and V of this paper and from Tables XIX and XX of the original paper above referred to. The figures corresponding to each year's production of a given class of business were entered in parallel columns with the figures for the first policy year placed in the proper line representing the agency year. The figures for each agency year were then obtained by horizontal addition.

Table IX.—This table involves the district agents' contract, which is the most favorable from the point of view of the general agent. And yet it is evident that the general agent's return is meager—under termination rates *A*, $12,516, and under termination rates *B*, $7,581, for the entire fifteen-year period. At the end of the period there will still remain the general agents' renewal equity in the business then on the books. Assuming, as before, an equity of

the balance of nine renewal commissions of 1 per cent, it will amount to a figure ranging from $5,985 to $4,223, depending upon the termination rates.

The district agent's return arises from three sources—overriding commissions, unearned renewals from subagents' business, and commissions on personal business. Each year in the given program the commissions from personal business exceeds the sum of the overriding commissions and unearned renewals. As stated at the outset, the primary producer receives the lion's share of the commissions paid by the company. A substantial income from organization profits can only be realized from a relatively large volume of business.

TABLE IX

GENERAL AGENT'S AND DISTRICT AGENT'S RETURNS FROM A PROGRAM OF INCREASING PRODUCTION

General-Agency Contract II — District-Agency Contract $2\frac{1}{2}$–$2\frac{1}{2}$

YEAR	GENERAL AGENT'S NET RETURN	DISTRICT AGENT'S RETURNS			
		Overriding Commission	Unearned Renewals	Personal Business	Total
		Termination Rates *A*			
1	−$ 312	$ 225	.....	$1,200	$1,425
2	− 196	520	$ 67	1,634	2,221
3	− 124	697	164	2,094	2,955
4	+ 8	856	289	2,278	3,423
5	154	998	441	2,452	3,891
6	428	1,065	617	2,617	4,299
7	645	1,065	837	2,776	4,678
8	852	1,065	1,072	2,927	5,064
9	1,048	1,065	1,323	3,073	5,461
10	1,237	1,065	1,590	3,212	5,867
11	1,419	1,065	1,784	3,259	6,108
12	1,594	1,065	1,924	3,282	6,271
13	1,762	1,065	2,018	3,282	6,365
14	1,922	1,065	2,064	3,282	6,411
15	2,079	1,065	2,064	3,282	6,411
Totals:					
5 years	− 470	3,296	961	9,658	13,915
10 years	+ 3,740	8,621	6,400	24,263	39,284
15 years	12,516	13,946	16,254	40,650	70,850

TABLE IX—*Continued*

Year	General Agent's Net Return	District Agent's Returns			
		Overriding Commission	Unearned Renewals	Personal Business	Total
Termination Rates *B*					
1	— $312	$225	.....	$1,200	$1,425
2	— 223	499	$ 60	1,619	2,178
3	— 181	668	143	2,054	2,865
4	— 84	818	245	2,203	3,266
5	+ 16	953	367	2,338	3,658
6	239	1,013	507	2,460	3,980
7	401	1,013	669	2,572	4,254
8	550	1,013	835	2,676	4,524
9	686	1,013	1,006	2,773	4,792
10	813	1,013	1,183	2,863	5,059
11	930	1,013	1,313	2,893	5,219
12	1,039	1,013	1,405	2,907	5,325
13	1,142	1,013	1,465	2,907	5,385
14	1,237	1,013	1,493	2,907	5,413
15	1,328	1,013	1,493	2,907	5,413
Totals:					
5 years	— 784	3,163	815	9,414	13,392
10 years	+ 1,905	8,228	5,015	22,758	36,001
15 years	7,581	13,293	12,184	37,279	62,756

The district agent's returns from the unearned renewals under the given program are quite substantial, rising to $2,064 a year with termination rates *A* and $1,493 with rates *B*. As stated in the original paper, these unearned renewals create a real problem in agency development. If the district agent retains them, he is under the constant temptation to limit his development to smaller men who will not write a sufficient volume to qualify for the maximum number of renewals. If the general agent retains them, there is likely to be continual pressure from the district agent looking toward a reduction in the amount that a subagent must produce to qualify for a given number of renewals. Where the district agent does not receive the unearned renewals, his organization profits are limited to the overriding commissions. In that event he is sure to be in favor of anything that will build up his production without costing him anything. The experience of companies which have satisfacto-

rily solved this problem would be illuminating. We wonder if any of them have experimented with a fifty-fifty subdivision of the unearned renewals between the general and district agents? Or whether in any instances the companies themselves have retained the unearned renewals?

The overriding profit to the district agent rises in the sixth year to a constant return of $1,065 under termination rates *A* and to $1,013 under rates *B*. His stake in the conservation of the business is, therefore, none too large. His total for the fifteen-year period is $13,946 or $13,293, according as termination rates *A* or *B* prevail. Under rates *A* the district and general agents' returns for the fifteen-year period do not differ greatly. Under rates *B* the same cannot be said. The general agent suffers severely from the heavy termination rates—simply another illustration of the truth of the statement that a general agent's salvation is peculiarly linked up with the conservation of his business.

Concentrating attention for a moment upon the two columns showing the net return to the general agent and the overriding commission of the district agent, we may again ask the question, "Who should bear the expenses of the district agency?" If, after the production of $750,000 had been attained, these expenses should run in the neighborhood of $200 per month, it is evident that the returns of the general and district agents taken together would not be large enough to pay them and have anything worth while left over. As stated before, some other source of revenue must be available.

Table X is quite similar to Table IX, but it is slightly more favorable to the district agent and consequently less favorable to the general agent. So far as overriding profits are concerned, the district agent under the contract here assumed has no interest after premiums for the first policy year have been paid. Although the difference in his return under rates *A* and *B* may not be great, it is our belief that the contract which gives the district agent an interest in the second year's premiums is a sound one. It tends to discourage business which is obviously of poor quality.

Table XI presents a situation still less favorable from the viewpoint of the general agent. His net return for the fifteen-year period under termination rates *B* is actually non-existent. District agent's contract ($5–2\frac{1}{2}$) can be used only if the general agent has

a substantial margin from some other source. Even then it is doubtful whether he should be expected to pay any of the district-agency expenses.

Further conclusions from the foregoing illustrative example will be left for the reader to draw for himself. Only in the light of the actual circumstances surrounding any given case can a definite decision be arrived at. Any additional margin as an expense allowance from the home office or an additional overriding commission will have a distinct bearing upon what the general agent can afford to do. Likewise, the disposition of the unearned renewals of sub-agents is a factor to be taken into account. Any arrangement whereby the discounted value of the general agent's net return for

TABLE X

GENERAL AGENT'S AND DISTRICT AGENT'S RETURNS FROM A PROGRAM OF INCREASING PRODUCTION

General-Agency Contract II — District-Agency Contract 5–0

YEAR	GENERAL AGENT'S NET RETURN	DISTRICT AGENT'S RETURNS			
		Overriding Commission	Unearned Renewals	Personal Business	Total
		Termination Rates *A*			
1	−$ 537	$ 450	.....	$1,200	$1,650
2	− 314	638	$ 67	1,634	2,339
3	− 252	825	164	2,094	3,083
4	− 111	975	289	2,278	3,542
5	+ 27	1,125	441	2,452	4,018
6	368	1,125	617	2,617	4,359
7	585	1,125	837	2,776	4,738
8	792	1,125	1,072	2,927	5,124
9	988	1,125	1,323	3,073	5,521
10	1,177	1,125	1,590	3,212	5,927
11	1,359	1,125	1,784	3,259	6,168
12	1,534	1,125	1,924	3,282	6,331
13	1,702	1,125	2,018	3,282	6,425
14	1,862	1,125	2,064	3,282	6,471
15	2,019	1,125	2,064	3,282	6,471
Totals:					
5 years	− 1,187	4,013	961	9,658	14,632
10 years	+ 2,723	9,638	6,400	24,263	40,301
15 years	11,199	15,263	16,254	40,650	72,167

TABLE X—*Continued*

Year	General Agent's Net Return	District Agent's Returns			
		Overriding Commission	Unearned Renewals	Personal Business	Total
Termination Rates *B*					
1	−$ 537	$ 450	.....	$1,200	$1,650
2	− 362	638	$ 60	1,619	2,317
3	− 338	825	143	2,054	3,022
4	− 241	975	245	2,203	3,423
5	− 156	1,125	367	2,338	3,830
6	+ 127	1,125	507	2,460	4,092
7	289	1,125	669	2,572	4,366
8	438	1,125	835	2,676	4,636
9	574	1,125	1,006	2,773	4,904
10	701	1,125	1,183	2,863	5,171
11	818	1,125	1,313	2,893	5,331
12	927	1,125	1,405	2,907	5,437
13	1,030	1,125	1,465	2,907	5,497
14	1,125	1,125	1,493	2,907	5,525
15	1,216	1,125	1,493	2,907	5,525
Totals:					
5 years	− 1,634	4,013	815	9,414	14,242
10 years	+ 495	9,638	5,015	22,758	37,411
15 years	5,611	15,263	12,184	37,279	64,726

the fifteen-year period would be less than $2.00 per thousand would probably be unacceptable.

Supervisors.—Related to the subject of district agents is the subject of supervisors who may be employed at the main office to stimulate the production of the agency force and in general to build up and maintain the agency organization. Sometimes the compensation is a flat salary; sometimes an overriding commission on the business of sub-agents for whom the supervisor is responsible; and sometimes part salary and part commission. Whatever the system of compensation, how much can the general agent afford to pay for the supervising work, taking into account not only the direct compensation of the supervisor but also possible traveling expenses if he is supposed to cover an extensive territory?

The supervisor will probably be entitled to soliciting agents' commissions on the personal business he may produce. The salary

or overriding commission above mentioned will be in addition. Tables IV and V referring to a single year's production of $100,000 are helpful at this juncture. To determine the margin out of which the general agent may compensate the district agent, the figures in the general and district agents' columns under any one of the district agent's contracts should be added together. For example, under general agents' contract II and termination rates *A* (Table IV), the general agent's margin in the first year is −$29, i.e., −$104+$75; in the second year it is $149; in the third year, $33; and so on. For the fifteen-year period it is $448, having a discounted value of $348 ($209+$139), or $3.48 per thousand.

If the supervisor should be responsible for the production of

TABLE XI

General Agent's and District Agent's Returns from a Program of Increasing Production

General-Agency Contract II — District-Agency Contract 5–2½

Year	General Agent's Net Return	District Agent's Returns			
		Overriding Commission	Unearned Renewals	Personal Business	Total
		Termination Rates *A*			
1	−$ 537	$ 450	.....	$1,200	$1,650
2	− 515	839	$ 67	1,634	2,540
3	− 537	1,110	164	2,094	3,368
4	− 480	1,344	289	2,278	3,911
5	− 409	1,561	441	2,452	4,454
6	− 135	1,628	617	2,617	4,862
7	+ 82	1,628	837	2,776	5,241
8	289	1,628	1,072	2,927	5,627
9	485	1,628	1,323	3,073	6,024
10	674	1,628	1,590	3,212	6,430
11	856	1,628	1,784	3,259	6,671
12	1,031	1,628	1,924	3,282	6,834
13	1,199	1,628	2,018	3,282	6,928
14	1,359	1,628	2,064	3,282	6,974
15	1,516	1,628	2,064	3,282	6,974
Totals:					
5 years	− 2,478	5,304	961	9,658	15,923
10 years	− 1,083	13,444	6,400	24,263	44,107
15 years	+ 4,878	21,584	16,254	40,650	78,488

TABLE XI—*Continued*

Year	General Agent's Net Return	District Agent's Returns			
		Overriding Commission	Unearned Renewals	Personal Business	Total
Termination Rates *B*					
1	−$ 537	$ 450	.....	$1,200	$1,650
2	− 542	818	$ 60	1,619	2,497
3	− 593	1,080	143	2,054	3,277
4	− 571	1,305	245	2,203	3,753
5	− 546	1,515	367	2,338	4,220
6	− 323	1,575	507	2,460	4,542
7	− 161	1,575	669	2,572	4,816
8	− 12	1,575	835	2,676	5,086
9	+ 124	1,575	1,006	2,773	5,354
10	251	1,575	1,183	2,863	5,621
11	368	1,575	1,313	2,893	5,781
12	477	1,575	1,405	2,907	5,887
13	580	1,575	1,465	2,907	5,947
14	675	1,575	1,493	2,907	5,975
15	766	1,575	1,493	2,907	5,975
Totals:					
5 years	− 2,789	5,168	815	9,414	15,397
10 years	− 2,910	13,043	5,015	22,758	40,816
15 years	− 44	20,918	12,184	37,279	70,381

$500,000 during a given year, the present value of the general agent's net return on the business during its first fifteen years would be $1,740 (assuming that the subagent receives nine renewal commissions of 5 per cent). If the home office should base its expense allowance on production, there would be an additional margin. If the allowance were $3.00 per thousand or $1,500 for the $500,000 produced, the margin would be increased to $3,240. Out of this margin the general agent would have to pay the supervisor and any attendant expenses. He should, of course, leave something for himself. Unfortunately, this "something for himself" may dwindle if the termination rate is unsatisfactory. From Table V we find that the $1,740 above referred to in connection with rates *A* is reduced to $1,305 (five times the sum of $129 and $132), making the total, with a $1,500 expense allowance, $2,805 instead of $3,240.

The value of a supervisor cannot always be estimated from

the results of a single year. The most important part of his work is the securing of new agents. If he obtains a man who later produces a substantial volume of business year after year, the general agent enjoys a continuing profit that cannot be measured in terms of the supervisor's work during any given year. It is obviously difficult to estimate in advance the value of this part of the supervisor's services, since the future production of new men is an unknown quantity.

It was suggested in the paper "*Returns under Agency Contracts*" that perhaps it would be reasonable to strive toward a basis which would yield returns to the general agent having a discounted value in the year of issue of $5 per thousand of new business. On this basis the man who produced regularly $300,000 a year would be worth $1,500 a year to the general agent. By some such estimate as this the general agent may be helped in his task of finding a proper basis upon which to compensate the supervisor for that part of his work which cannot be measured in terms of immediate production.

In training new men it is sometimes arranged that the supervisor shall share in the commission on the business produced by the joint canvass of the supervisor and the new man. There is much to be said for the justice of this arrangement. Any man entering a new profession may reasonably be expected to contribute something toward the cost of his education in the new line of endeavor. By sharing the commission where he has called upon the supervisor to accompany him to the prospect, he makes the contribution while earning something additional for himself.

If the system is presented to the new man in the proper light, he will see the justice of it. Naturally he will strive to attain a proficiency that will make it unnecessary to request the supervisor to accompany him on his canvass. The plan of furnishing a supervisor without any contribution on the part of the new man may easily lead to abuse. The new man is tempted to remain a "leaner" and to defer the day when he will go out "on his own." This sharing of commissions may help solve the problem of compensation of the supervisor. On a fifty-fifty subdivision of first and renewal commissions, it may mean to the supervisor the equivalent of from $8.00 to $10.00 per thousand of new business thus produced, taking into account the discounted value of the renewals.

The employment of supervisors to increase and organize the business of an agency is highly desirable if done intelligently. General agents quite frequently get the idea that the way to build an agency is to offer new men liberal advance accounts. Almost invariably this course will involve losses. It is much better to reduce or eliminate the advances and build up an organization that will greatly increase the probability of success of every new man who joins the agency force. Training men to be successful life insurance salesmen is a constructive thing to do. Making advances without adequate provision for training the new man is unsound from every angle. He simply goes into debt at a time when his income is at a minimum. He is likely to become discouraged and to quit before he has attained anything worth while. He has harmed himself, harmed the life insurance business in the eyes of the public, and harmed the agency with which he was connected.

It is clear, of course, that the general agent should consider the employment of a supervisor only if the step will increase the production of the agency. Instances have been known where the supervisor was employed in order that the general agent might take things easy and become, in a sense, an absentee landlord looking to his agency for profits, with but little work on his part. General agencies are profitable only if they are properly handled by the general agent. Supervisors should not be employed until the organizing work has become so extensive that the general agent cannot do it alone.

Conclusion.—The subject with which we have been dealing has been made relatively simple by holding to a minimum, the variations in the basic assumptions. In many agencies the average premium will be different from $30, the average policy will be different from $3,500, and the average expenses per policy will be different from $6.25 and $1.25. It is believed, however, that the conditions actually assumed will, in their composite results, produce figures that are applicable in a considerable number of existing agencies. Where the basic conditions are different the results corresponding thereto may be readily obtained through the cooperation of the agency and actuarial departments.

The analysis presented in the paper has indicated the difficulty of establishing profitable district agencies without some additional margin beyond the basic overriding commissions underlying the

tables. Without such additional margin, in the nature of an increased overriding commission or an expense allowance from the home office, district-agency development is not likely to proceed vigorously—perhaps will not be undertaken at all.

Adequate organization is highly desirable in territory of considerable area. It is obviously in the interest of economy to have the premium collections and the other work of a general agency centralized in a relatively large office where the work can be standardized and well supervised. Small general agencies throw too great a burden of expense upon all concerned. Then again, adequate agency development provides an organization from which promotions to general agencies may be made. How many have been the errors committed by companies in appointing to general-agency positions men whose sole qualifications were a pleasing personality and a record of substantial personal production! The day of the small, one-man general agency is passing. It will be forced into the discard by economic pressure. Organization is what is needed—a reasonably large territory covered by soliciting agents, with the business handled through a central office large enough to be run economically. It is for this reason that the subject of district-agency development and the maintenance of agency organization through the use of supervisors is important.

DISCUSSION OF PAPERS READ BUT NOT DISCUSSED AT PREVIOUS MEETING

ON LIDSTONE'S DEMONSTRATION OF THE OSCULATORY INTERPOLATION FORMULA

BY J. F. REILLY

MR. E. G. FASSEL:

Professor Reilly's valuable paper is a sequel to his paper on "Osculatory Interpolation" presented to the Institute in June, 1924. His former paper dealt with the original osculatory interpolation formula which was given by Dr. Sprague at a meeting of the Institute of Actuaries in England in 1880, and was restated by Mr. King in his paper before the same body in 1908. Dr. Reilly gave a general expression for all osculatory curves that result from the conditions proposed by Dr. Sprague if Dr. Sprague's method of analysis is employed. The paper now before us refers to an alternative proof that was given for Dr. Sprague's formula by Mr. Lidstone at the time that Mr. King presented his paper, Mr. Lidstone's method being to employ a different form of mathematical analysis than that used by Dr. Sprague. Dr. Reilly has made the interesting discovery of a fact that has probably not heretofore been suspected which is that Mr. Lidstone's method gives the same results as Dr. Sprague's method only in certain cases, and that in the cases where there is a difference the advantage probably lies with the method of Dr. Sprague.

A great deal of light will be had on these two papers of Dr. Reilly on consulting the original papers in the *Journal* of the Institute. Before reading Dr. Reilly's paper of a year ago it will be wise to study J. I. A., XLII, 238–46, in which Mr. King explains the conditions proposed by Dr. Sprague for osculatory interpolation and shows the details of Dr. Sprague's method of making the mathematical analysis. In similar fashion before reading the present

paper of Dr. Reilly it will be wise to study *ibid.*, pages 394–97, in which Mr. Lidstone makes a very clear statement of his method of analysis to produce curves complying with the conditions proposed by Dr. Sprague.

Osculatory interpolation has to do with a series of given values corresponding with successive values of a variable. Thus we may be given the rate of mortality at quinquennial ages q_{15}, q_{20}, q_{25}, ..., q_{100}, and undertake to find the rates of mortality for the intervening ages. If we doubt the accuracy of the given values, we have no help from osculatory interpolation because this method assumes the given values to be absolutely correct. If we knew the mathematical law connecting the given values, there would be no question of osculatory interpolation. In that case we would trace the curve represented by the known law and thereby obtain correct values for the intervening ages. Osculatory interpolation assumes both that the given values are accurate and that we do not know the law that connects them. In such a case the most obvious supposition is that the correct value at a particular age will be determined chiefly by several given values on either side. Osculatory interpolation puts this assumption into definite form by stipulating the number of values on either side of the required age that shall be regarded as determining the result, and an endless variety of osculatory curves is produced by altering the stipulated conditions. Dr. Sprague's original formula assumes that the value at any age in a particular interval is completely determined by the three given values immediately preceding and the three immediately following. When we come to the next interval we again use six consecutive given values, dropping one of those used before and substituting a new one. The osculatory interpolation formula therefore proceeds by stages, the working constants being different from interval to interval. Within each interval the curve will be perfectly smooth. But as the end of an interval is reached and it is necessary to change constants in order to proceed with the next interval, there will be loss of continuity if precautions are not taken. Osculatory interpolation formulas are so constructed as to maintain a given order of continuity at the junction points, in spite of the change that takes place in the constants, and it is from this property that they derive their name.

Suppose we undertake to interpolate q_{31}, q_{32}, q_{33}, and q_{34}. Dr.

Sprague's original formula assumes that these values are determined by the values given for q_{20}, q_{25}, and q_{30}, on the one side, and for q_{35}, q_{40}, and q_{45}, on the other. If we were to pass a curve through the first five of these six values of q_x by ordinary finite difference interpolation methods and call this ordinary curve u_x, we would be able to read off certain values for q_{31}, q_{32}, etc. On the other hand, if we were to make the second five of these six values of q_x the basis of another ordinary interpolation curve which we may call v_x, we would again be able to read off values for q_{31}, q_{32}, etc., but in general these values would not be the same as those obtained before. In the case of q_{31} we would favor the value from the u_x curve, but in the case of q_{34} we would consider that the curve v_x was more reliable because at this end of the interval the curve v_x is more centrally located than the other.

It is the essence of Dr. Sprague's method that the partial curve u_x is accepted as absolutely determining the osculatory curve at age thirty. As soon as we have entered the interval from age thirty to age thirty-five, however, his curve is determined partly by u_x and partly by v_x, and when age thirty-five is reached the osculatory curve is determined by v_x alone. What Dr. Sprague has done is to form an osculatory curve which at age thirty will have the same actual value, the same first derivative, and the same second derivative as the curve u_x, and at age thirty-five his curve has the corresponding relation to the curve v_x. By tying the osculatory curve up to u_x at the beginning of the interval and to v_x at the end, it should not be possible for it to get very far away from either u_x or v_x within the interval. It can be proved that Dr. Sprague's original osculatory curve will lie somewhere between u_x and v_x throughout the interval, so that his osculatory curve constitutes a blend of the two ordinary curves.

Having settled how the first and second derivatives are to be determined, Dr. Sprague introduced a mathematical analysis that consisted in setting down enough equations to determine as many constants as must be assumed in order to meet his requirements. The result is a formula of the smallest degree that will satisfy the conditions. In extending Dr. Sprague's method in general form Dr. Reilly has in like fashion found in each case the equation of the lowest degree that will satisfy the requirements.

Mr. Lidstone's method accepts the conditions proposed by Dr.

Sprague but he differs from Dr. Sprague in the method of his analysis. He works by differential equations, making an assumption as to the mathematical form of the curve and then proceeding to determine constants that will give a curve of the form assumed by him. What Dr. Reilly has done is to show that in certain cases Mr. Lidstone's method of analysis produces curves of higher order than is necessary, and that as a result his method does not always give a unique curve but rather one of a great many curves, any of which would satisfy the conditions. Mr. Lidstone's is a very clear and direct method of deducing Dr. Sprague's original formula, and he derives the same result as by Dr. Sprague's method in the two cases treated in his paper. As before stated, the work now done by Dr. Reilly is conspicuous for proving that if Lidstone had gone further his method would have shown disagreement with that of Dr. Sprague.

In *T.A.S.A.*, XXII, 190–91, and *R.A.I.A.*, XIII, 22–25, Mr. Henderson brings forward a method of osculatory interpolation in which the degree of the curve does not need to be so great as in Dr. Sprague's method although the same order of continuity is maintained at the pivotal points. Mr. Henderson dispenses with the partial curves u_x and v_x and proceeds directly to the final osculatory curve. He gives a general expression for a curve that could be used in the interval thirty to thirty-five, and finds the values of its first and second derivatives at the beginning and the end of the interval. Thus we arrive at an expression for the first and second derivatives at age thirty-five based on the given values that are used for the calculations in the interval immediately preceding. The next step is to obtain expressions for the same derivatives based on the given values used for the calculations in the interval immediately following. We thus have two values for the first and second derivatives at age thirty-five corresponding to the two sets of constants. It remains only to equate the two values for each derivative to find what the form of the osculatory curve must be. The result is a curve that passes through each of the given values and guarantees against breaks at these pivotal points. This is because the first and second derivatives as the curve approaches the end of an interval merge into the corresponding derivatives as the curve starts out in the next interval because of the equality established at the point of junction.

Referring again to the curves u_x and v_x, we are in a position to compare Dr. Sprague's method with that proposed by Mr. Henderson. It is a property of these two curves u_x and v_x in ordinary cases that they cross each other only when passing through the given pivotal values at the ends of the intervals; throughout one interval one curve will always be below the other, and throughout the next interval it will always be above. The curves u_x and v_x therefore outline a series of crescents, the horns of each crescent being two consecutive given values. If u_x forms the lower edge of one crescent, then the lower edge of the next crescent will be formed by v_x. Dr. Sprague's curve at age thirty coincides with u_x. If u_x forms the lower edge of the crescent from thirty to thirty-five, then when advancing from age thirty Dr. Sprague's curve osculates the lower age of the crescent but when arriving at age thirty-five the curve comes in along the upper edge of the crescent and we know that in the meanwhile the curve has been on the crescent because it is known to have occupied a position somewhere between the upper and lower edges. Mr. Henderson's curve does not make reference to the crescent outlined by the partial curves u_x and v_x. All we know is that his curve passes through the given points and is continuous at those points, but whether in the interval his curve lies on the crescent between u_x and v_x or whether it lies outside these partial curves is not known. From the principles of finite differences we are led to expect in the case of ordinary curves that the best interpolated values will lie somewhere on the crescent between u_x and v_x. Dr. Sprague insures this result by forcing his curve to coincide with the lower edge of the crescent at one horn and with the upper edge at the other.

These considerations suggest an improvement which may be worth investigating. Suppose, for illustration, that in the interval thirty to thirty-five the curve u_x forms the lower edge of the crescent, v_x being the upper edge. Then as we proceed to the next interval thirty-five to forty in the ordinary case we have the same v_x and a new partial curve w_x outlining the next crescent with v_x now appearing as the lower edge. We see here that when Dr. Sprague's conditions stipulate that the osculatory curve shall agree with u_x it is forced to enter the first crescent along the lower edge, and in changing to v_x at the end of the interval it is forced to enter the next crescent also along the lower edge. It would seem that the

osculatory curve has therefore been given an artificial downward bend at the successive pivotal points. This would produce undulating first differences which, as a matter of fact, is a feature of the left-hand table at the bottom of page 283 of *J.I.A.*, XXII (the same as the right-hand table at the bottom of page 381 of *ibid.*, XLII). We would rather have the curve leave age thirty on an intermediate (but not generally midway) course between u_x and v_x and similarly leave age thirty-five intermediately between v_x and w_x. It is suggested that Dr. Sprague's conditions may be susceptible to improvement by fixing the first and second derivatives for the pivotal points at values slightly differing from instead of exactly equal to those of the partial curves.

MR. E. A. PORTER:

My remarks relate to practical application rather than mathematical derivation.

Several years ago I had occasion to make some quite extensive interpolations by osculatory methods. At that time I ran into some of the very things to which Mr. Fassel refers. I found that formulas of great accuracy involving a high order of differences have a very marked tendency to undulate.

Just to illustrate, if we have a series of points, the mathematical conditions of osculatory interpolation may be fulfilled by a curve that simply goes in a cycle through those points and if a large number of degrees of differences are involved, such is likely to take place.

I found the most satisfactory, not only from the point of view of the work involved, but I believe even from the point of view of the results, was a formula that did not involve more than the third differences. I also found that the working of interpolation could be considerably simplified by a little device. The regular osculatory formulas involve working from both ends of an interval. Now, it can be carried out with almost identical results by working continuously through a series of values between which the interpolations are to be made.

For example, suppose that we have a table of the values of a certain function at the end of each year, from one year on up to one hundred, and we wish to interpolate by months or twelfths. We can start in with the first year, make our interpolation by finite difference formulas and use that as the basis of an interpolation for the

next interval. I found that by using a third-degree formula to fit the first interval and then beginning with the first and second differences, at the end of the second interval, and extending the series on by bringing in new values I was able to get almost a continuous working formula that gave very nicely interpolated results and gave very little of this effect of swinging up and down between the points of the curve.

PROFESSOR J. W. GLOVER:

The selection of u_x and v_x is quite arbitrary. In fifth difference osculatory graduation, these curves are each of the fourth degree. As Mr. Fassel implies we may take them to a higher degree and subject the graduated curve to further conditions. But this greatly increases the undulatory tendency of the curve between the two ordinates where the graduation occurs.

The suggestion made by Mr. Fassel can be followed out undoubtedly and a curve be put through the crescent. I doubt, however, whether the final result would be much of an improvement although it is quite well worth investigating.

PROFESSOR H. L. RIETZ:

I think there has been a noticeable tendency for actuaries in their computations, for example, in constructing mortality tables, to make use of fifth differences and neglect differences beyond the fifth.

On the other hand, I am told by men who have worked for the United States Coast and Geodetic Survey that fourth differences are the highest differences used in their work.

This variation in practice has brought up the question in my mind for which I have found no answer at all: Is there any particular reason in connection with the history of the development of actuarial science why we have got in the habit of carrying our interpolations to the fifth difference when another prominent scientific group carry differences quite regularly to fourth differences and think that is satisfactory for their work.

PROFESSOR GLOVER: Isn't it true, Dr. Rietz, that with the osculatory conditions, you would require those fifth differences?

PROFESSOR RIETZ: You are undoubtedly correct, but was it not the habit long before the invention of osculatory interpolation to

use Newton's interpolation formula in subdividing differences to fifth differences without any reference to osculatory interpolation of the kind that Sprague introduced?

PROFESSOR GLOVER: Yes.

DR. J. F. REILLY:

I wish to thank the men who have discussed my paper for their remarks and suggestions, especially Mr. Fassel who has prepared for us a rather complete and very interesting résumé of the whole subject of osculatory interpolation.

TABLE I

Argument	Function	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7	Δ^8
1	35	20	−17	0	69	11	19	−98	307
2	55	3	−17	69	80	30	−79	209	
3	58	−14	52	149	110	−49	130		
4	44	38	201	259	61	81			
5	82	239	460	320	142				
6	321	699	780	462					
7	1,020	1,479	1,242						
8	2,499	2,721							
9	5,220								

The question of what order of differences should be employed has interested me for some time, but I cannot present you any solution of the problem. As I see it no answer can be given, in general, to this question; it depends upon the data.

Take, for example, the natural sines of angles from 25° 40′ to 25° 43′ inclusive at intervals of 20″, carrying fifteen decimal figures.[1] The third differences are practically constant, the slight variation being probably largely due to the neglect of the sixteenth place in the values of the sines. It would seem that for this data differences beyond the third should not be used.

Take, again, the logarithms of the integers from 6,040 to 6,044 to ten decimal figures. The second differences are practically constant, and hence differences beyond the second need not be used.

On the other hand, consider the differences exhibited in Table I. From these nine given values it is impossible to say that differences

[1] Whittaker and Robinson, *The Calculus of Observations* (1924), p. 3.

of any particular order are practically constant, and hence it is impossible to conclude intelligently what order of differences to use.

From the examples cited it is clear that the formation of a difference table will sometimes determine the order of differences to be employed, but that at other times it will fail to do so. One can readily conceive of a difference table in which the eleventh is the first order of differences that are practically constant, and if one desires to use osculatory interpolation, the need for an extension of Dr. Sprague's formula is seen.

EXCESS INTEREST UNDER ANNUITY-CERTAIN

BY E. G. FASSEL

MR. J. R. LARUS:

All of us have probably spent many hours wrestling with the excess-interest problem presented to us whenever the holder of an annuity-certain contract has his instalments payable more frequently than once a year.

The company with which I am connected has for some time been using a formula which, while not identical in form with the one derived by Mr. Fassel, yet produces identical results.

In comparing the two formulas, it does not appear that there is any great difference in the amount of detail necessary to produce the results, and the only reason that I am presenting our own is that it may be derived by general reasoning instead of the detailed mathematical analysis necessary in the case of Mr. Fassel's formula.

We may look at the reserve held at the beginning of any year as being composed of two parts.

(*a*) *The reserve for the current year's payments*,

$$ {}^{(m)}I\left(\frac{1-v}{1-v^{\frac{1}{m}}}\right). $$

As a matter of fact, only ${}^{(m)}I\left(\frac{1-v'}{1-v'^{\frac{1}{m}}}\right)$ has proved necessary, so that at the end of the year the policyholder is entitled to the difference, with a year's interest at the rate i'.

(*b*) *The reserve for subsequent years' payments,*

$$^{(m)}I\left(\frac{1-v}{1-v^{\frac{1}{m}}}\right)a_{\overline{r}|}\,.$$

On this item, excess interest at the rate $(i'-i)$ is due at the end of the year.

This makes the total excess-interest item, if paid at the end of the year,

$$^{(m)}I\left[(1+i')\left(\frac{1-v}{1-v^{\frac{1}{m}}}-\frac{1-v'}{1-v'^{\frac{1}{m}}}\right)+(i'-i)\left(\frac{1-v}{1-v^{\frac{1}{m}}}\right)a_{\overline{r}|}\right]\,.$$

This excess may be distributed over the year by dividing by $\dfrac{i'}{(1+i')^{\frac{1}{m}}-1}$, the $m$thly payment thus obtained being identical with that resulting from the use of Mr. Fassel's formula.

If it is desired to distribute the excess interest with the payments of the following year, the excess interest of the basic formula is divided by $\dfrac{1-v'}{1-v'^{\frac{1}{m}}}$.

The factors

$$(1+i')\left(\frac{1-v}{1-v^{\frac{1}{m}}}-\frac{1-v'}{1-v'^{\frac{1}{m}}}\right) \quad \text{and} \quad (i'-i)\left(\frac{1-v}{1-v^{\frac{1}{m}}}\right),$$

also $\dfrac{i'}{(1+i')^{\frac{1}{m}}-1}$ or $\dfrac{1-v'}{1-v'^{\frac{1}{m}}}$,

if required, may be calculated once for all at the interest rates used by the company, so that any excess-interest item resolves itself into the form $^{(m)}I(\alpha a_{\overline{r}|}+\beta)$, and may be readily obtained as needed with the use of the usual annuity-certain table inasmuch as α and β are constant for all cases where instalments are payable with the same frequency.

MR. J. G. PARKER:

The first thought which is impressed upon one who takes a casual glance at the paper submitted by Mr. Fassel is one of amazement that a formula so large, and apparently so complex, could be evolved in the consideration of a subject ordinarily so simple as an annuity-certain.

The mathematical work involved in the paper is well presented and enables one to follow the work readily. The actual preparation of a table by Mr. Fassel shows that the practical application of the formula to the calculation of the dividends over a period of years is not too complicated for office use. The method outlined by Mr. Fassel has the advantage of enabling a company to arrive at an accurate dividend. It is of particular advantage to a company which has a rate of interest for excess-interest dividends which remains constant from year to year. I know of several companies who have had the same rate of interest for such dividends over a long period. In such a case the rate of interest during the entire payment year is the same. Many companies, however, vary the rate of interest used in the calculation of excess-interest dividends in accordance with the net rate of interest earned by their company during the preceding calendar year. Unless, therefore, the payment year under an annuity-certain synchronized with the calendar year, the tables as brought out by Mr. Fassel would be of little value. In the case of the company with which I am connected it is our practice so to vary the rate of interest used in the calculation of excess-interest dividends.

I was interested in preparing a table which would show the dividends payable under a method more approximate than the method outlined in the paper. For the payment of dividends in our own company we calculate accurately the first dividend due in any year and the last dividend due in the same year and interpolate by first difference for the intermediate payments. Our payments, therefore, decrease slightly month by month, but if we were making equal payments throughout the year such equal payments would be approximately equal to the average of the first and last payments. I have worked out a few cases to compare with the payments made under the table as shown by Mr. Fassel.

In the table where $r = 18$, that is where the number of instal-

ments remaining vary from 227 to 216, the correct figure for the dividend is $24.70. The average is $24.69.

Where $r = 13$, the correct figure is $19.28 as compared with the average of $19.27.

Where $r = 8$, the average and the correct figure are the same, namely, $12.99.

Where $r = 3$, the correct figure is $5.71 as compared with the average of $5.70.

It therefore would appear that the approximate method which we use provides practically the same payments in practice as would be provided by the use of the accurate dividend calculated under the method of Mr. Fassel. Our own method has this advantage for our own company in that the rate of interest which we use in the calculation of those dividends varies from calendar year to calendar year.

At the beginning of each calendar year our Actuarial Department would give a statement to the Accountancy Department, or to whatever department might be responsible for the payment of the instalments, of the twelve monthly payments due during the calendar year. As a separate check is made out each month, very little more work is involved in making the entries in the book for the decreasing payments than for the constant payments.

Mr. Fassel is to be congratulated on the nicety of his mathematical work and in the value which his table will have, particularly to those companies which have a practice of keeping the rate of interest used in the calculation of interest dividends fairly constant from year to year.

I think the fact that I was able to go through the formulae is one of great interest, because if with the length of time which has elapsed since I have been dealing with such formulae I am still able to go through those in this paper I know that our students will be able to follow through the work and that it will prove an interesting problem for them.

MR. C. O. SHEPHERD:

Mr. Fassel's problem is to allot excess-interest dividends under instalment-certain contracts providing for nm payments at equal intervals over n years. Two special conditions are to be satisfied:

first, the excess interest earned within a given contract year according to the rate adopted is to be distributed within that year; and second, the m dividends within the contract year are to be equal.

He begins with a direct expression for the excess interest earned in each of the m intervals accumulated to the end of the year and proceeds through an expansion of annuity functions and re-collection of terms to a final formula stated in terms of $a_{r\rceil}$, i, and i' and three comparatively simple derived functions designated $\theta_{i'}$, θ_i, and $\theta_{i' \& i}$. The formula is convenient for computation purposes as the only variable function is $a_{r\rceil}$. A single computation of constants serves for the construction of a table applying to contracts running for any number of years.

There is another method of solving this problem which avoids the extended algebraic transformations and yields a simpler expression for the value of the dividend. Given a sum A earning interest at i' per annum to reduce it by m equal instalments of $(1+\delta)$ payable at intervals of $1/m$th of a year to B at the end of the year. The problem is to determine δ. In algebraic form taking the guaranteed payment as 1,

$$(1) \quad (1+\delta)ms^{(m)\prime}_{1\rceil} = ma^{(m)}_{r+\frac{m-1}{m}\rceil}(1+i') - ma^{(m)}_{r-1+\frac{m-1}{m}\rceil}.$$

But

$$ms^{(m)}_{1\rceil} = \frac{mi}{j} \quad \text{and} \quad (1+i)ma^{(m)}_{r+1\rceil} = ma^{(m)}_{r\rceil} + \frac{mi}{j}.$$

Therefore

$$(2) \qquad 1+\delta = \left\{\frac{mi}{j} + (i'-i)ma^{(m)}_{r+\frac{m-1}{m}\rceil}\right\}\frac{j'}{mi'}.$$

But

$$ma^{(m)}_{r+\frac{m-1}{m}\rceil} = ma^{(m)}_{r+1\rceil}(1+i)^{\frac{1}{m}} - 1.$$

Therefore

$$(3) \qquad 1+\delta = \left\{\frac{mi}{j} + (i'-i)\left[(1+i)^{\frac{1}{m}}ma^{(m)}_{r+1\rceil} - 1\right]\right\}\frac{j'}{mi'}$$

$$(4) \qquad = \left\{ma_{\overline{r+1}\rceil}(i'-i)(1+i)^{\frac{1}{m}}\frac{i}{j} + \frac{mi}{j} - (i'-i)\right\}\frac{j'}{mi'}$$

$$\delta=\frac{ij'}{ji'}(i'-i)(1+i)\,{}^{1}_{m}a_{\overline{r+1}|}+\frac{ij'}{ji'}-\frac{j'}{mi'}(i'-i)-1 \tag{5}$$

$$=\frac{ij'}{ji'}(i'-i)\left(1+\frac{j}{m}\right)a_{\overline{r+1}|}+\frac{j'i}{ji'}\left(1+\frac{j}{m}\right)-\left(1+\frac{j'}{m}\right). \tag{6}$$

The latter formula is simpler than Mr. Fassel's, involves only well-known interest functions, and is fully as convenient for computation purposes. Formula (1), itself, is a solution of the problem, and the sole purpose of the other subsequent transformations is to derive a formula which will save labor where extended tables are required.

Mr. Fassel's formula can be proved to be algebraically identical with formula (6) but the proof is too involved to be included here.

MR. E. A. PORTER:

We have used for the last two or three years in my own company substantially Mr. Fassel's formulas. However, our practices in the distribution of excess interest are somewhat different from those assumed by him. The excess interest is paid beginning with the second annuity year and in my calculations I have assumed that it is a distribution of the interest earned during the first annuity year.

Now you will find that a formula can very easily be derived meeting those conditions which is in the form of $\alpha\times$annuity factor $+\beta$. The only complication arising is that when you go ahead so calculating a scale of distribution for an annuity-certain and come to the last year, you have not distributed all the excess earnings so that on the last year you have to figure a special additional dividend and add it in so that the entire excess interest for the twenty-year period, for example, has been finally distributed when you get through. But clear through the whole thing, whatever you base it on, your excess interest can be expressed as a function of the form $\alpha\times$annuity value$+\beta$. The annuity value can be the initial reserve of the year before, the terminal reserve of the year before, the initial reserve of the current year, the mean reserve of the current year or the terminal reserve of the current year—almost any annuity value so that there is a constant, definite connection with the particular year's dividends which you are computing, and they will all work out $\alpha\times$constant$+\beta$.

Mr. Fassel's formula incidentally is by all odds the easiest way of making up schedules of excess interest payments that I have ever seen or found. It is very practical.

CUBE ROOT BY ABRIDGED DIVISION AND BY MACHINE

BY E. G. FASSEL

PROFESSOR H. L. RIETZ:

The method of Mr. Fassel for finding the cube root of a number has both a practical and a certain theoretical interest. As indicated in the paper under discussion, the method proposed has been found to be of practical value when more significant figures of the root are to be found than can be obtained by the use of available tables of logarithms. Among the known methods for carrying cube roots of a number N to more significant figures than are given by available tables of logarithms, we may say that Horner's method of solving the equation $x^3-N=O$ for the real root x has been considered a desirable substitute for the laborious method of finding the cube root usually found in textbooks on arithmetic and algebra. It is fairly obvious that the method of Mr. Fassel is a time-saver in comparison with the usual method, but it is not so obvious to me that the new method can ordinarily be carried out in less time than Horner's method after using logarithms to obtain a certain number of significant figures. In this connection, it may also be desirable to direct attention to another process of successive approximations after we have carried the root to a goodly number of significant figures. The process in mind consists simply in applying a correction c to the tentative root x_1 already found. Such a correction is given by $c=\frac{\epsilon}{3x_1^2}$ where ϵ is the difference between the given number N and the cube of the tentative root already found.

Finding the cube root is such an old process that it is not unnatural to expect that everything has long been known about finding the cube root. Hence, it is particularly interesting just now that, in addition to the paper of Mr. Fassel, we are able to refer to another recent paper dealing with the subject by an almost totally different method. The paper to which I refer is "On the Use of the Calculating Machine for Cube and Fifth Roots," by Professor D. N.

Lehmer, of the University of California. The paper appeared in the *American Mathematical Monthly*, August-September, 1925. The method of Dr. Lehmer depends on a scheme for the decomposition of numbers into factors of the type $(1.0)^n p$.

On the theoretical side, the method of Mr. Fassel is interesting because of the form given to the expansion of

$$(a_0+a_1x+a_2x^2+a_3x^3+ \ldots .)^3 ,$$

where each of the numbers a_0, a_1, a_2, a_3, ... , is a digit 0, 1, 2, 3, ... , or 9, and x=.1.

Possibly the representation would seem clearer to some readers if the matter were presented as follows:

In the Arabic notation, any number with n digits in the integral part may be written in the form

$$10^{n-1}(a_0+a_1x+a_2x^2+a_3x^3+ \ldots .) .$$

Let us assume that the cube root of the given number N is thus represented. Then the new method depends on the fact that N may be represented in the form

$$\begin{aligned} N &= 10^{3n-3}(a_0+a_1x+a_2x^2+a_3x^3+ \ldots .)^3 \\ &= 10^{3n-3}(a_0C_0+a_1xC_1+a_2x^2C_2+a_3x^3C_3+ \ldots .) , \end{aligned}$$

where

$$\begin{aligned} C_0 &= a_0^2 \\ C_1-C_0 &= (2a_0+a_1x)\ (a_0+a_1x) \\ C_2-C_1 &= (3a_0+2a_1x+a_2x^2)\ (a_1x+a_2x^2) \\ C_3-C_2 &= (3a_0+3a_1x+2a_2x_2^2+a_3x^3)\ (a_2x^2+a_3x^3) \\ C_4-C_3 &= (3a_0+3a_1x+3a_2x_2+2a_3x^3+a_4x^4)\ (a_3x^3+a_4x^4) . \end{aligned}$$

It would be a matter of interest to me to know how Mr. Fassel came to think of this ingenious representation which is really the basis of his method. The Institute is much indebted to Mr. Fassel for presenting to us a very thoughtful and practical method of carrying cube root to a large number of significant figures.

MR. H. W. CURJEL:

Mr. Fassel's interesting paper is an example of the fact that there is always room for improvement in existing methods of attacking various problems.

Probably the simplest arithmetical method of extracting a cube root is by using Horner's method, which is applicable to any numerical equation, the terms of which are multiples of integral powers of the unknown.

If we compare the example shown by Mr. Fassel on page 38 with the same worked out by Horner's method, we note the following:

Horner's method has columns corresponding to Mr. Fassel's columns A, C, D, and E, the last two being the same in the two methods, and, if a line is not used for one digit to be added to the previous line, and the lines corresponding to the addition of 0 are omitted, the number of lines in the two methods are as shown in Table I.

There are three lines in favor of Mr. Fassel's method, but the difference is really greater as the eight lines in column B never

TABLE I

	Column A	Column B	Column C	Total
Horner	19	0	26	45
Mr. Fassel	15	8	19	42

contain more than two digits. There is a further saving in Mr. Fassel's ingenious application of machines to the calculation.

A good exposition of Horner's method appears in chapter xv of Part I of Chrystal's *Algebra*.

If Horner's method is applied to find the cube root of the number in Mr. Fassel's example and Cotsworth's tables (these are much the same as Crelle's, but in more convenient form) are used, the first twelve figures being calculated exactly and twenty-four more by abridged methods, the calculation can be made on a piece of cross-ruled paper, ruled six lines to the inch, measuring 15 by $4\frac{1}{2}$ inches.

If seven-figure logarithms only are available, the cube root found by them can be used as a first approximation, and there are many methods for finding further figures, most of which will be found in chapter vi of Whittaker and Robinson's *Calculus of Observations*.

However, the problem of extracting the twelfth root to more

places than given by seven-figure logarithms, from which Mr. Fassel's paper arose, can be solved more expeditiously by means of various fairly accessible tables of logarithms.

Namur's twelve-figure tables (unfortunately out of print) will give the twelfth root in a calculation of fourteen lines of figures with an average of twelve figures in a line.

The ten-figure tables at the end of Erskine Scott's five-figure tables will give the twelfth root in a calculation involving an average of about seventy-five figures.

It may be mentioned that the logarithms of the amount of a unit at the end of a year for an extensive range of rates of interest are given to twenty places on page 280 of Professor Glover's *tables*.

MR. A. C. EASTLACK:

I was interested in Mr. Fassel's paper, especially that part which had to do with extracting the cube root by machine, as I have been using a machine to extract the roots of a quadratic equation. It may be of interest to others to know how this is done. The equation is put in the form $X^2+BX=C$, the value of C is set up in the product holes of the carriage, the value of B is then set up on the keyboard. To extract the root, the procedure is the same as in extracting the square root of a number by successive subtractions of the odd numbers, except that the value of B must be considered. By using two machines, the roots of a cubic may be found.

MR. E. G. FASSEL:

The paper presents a method for cube root whereby the root is extracted by a straightforward process just as is the case in the customary arithmetical method. There is no reference to auxiliary tables nor is the method one of refining previous approximations. The distinctive feature of the new process will be understood on examining page 38.

In column C the subtotals, constituting the amounts that are successively divided into D, will be found to be identical with the divisors that are prepared step by step in the customary arithmetical method. The results are therefore identical and the work may be continued as far as may be desired. The customary method requires each successive divisor to be found by independent calculations which quickly become extremely laborious. The new method

prepares each successive divisor by adding an adjustment to the preceding divisor, and it is easy to apply an abridging process to eliminate surplus figures, with the great saving that is evident on comparison with page 36.

I would like to point out that it is not necessary to take advantage of the abridgment feature explained in the paper. For making occasional calculations there is no objection to adhering to the unabridged method exemplified on page 38. In this case I venture to state that the method is easier to memorize than the customary arithmetical method. Also one needs only to make a parallel calculation by the customary method to see how much more easily the new unabridged method yields the same figures.

SUBSTANDARD BUSINESS

BY E. E. CAMMACK

MR. GEORGE GRAHAM:

Mr. Cammack has skilfully assumed a position in the middle ground between the two extremes of a constant extra per thousand and the rate-up-in-age method.

At the present time we know little of the incidence of the extra mortality for different impairments from age to age and from year to year. The combination assumed by Mr. Cammack is as likely to approximate the truth as any other possible assumption.

Arthur Hunter has said that among under-average risks as a whole the ratio of the extra to the normal mortality decreases with the duration of the policy.

Having this condition in mind, the method used in arriving at the amount of extra premium is not a matter of great concern. The rule laid down by Mr. Cammack has the great merit of simplicity, and will doubtless have considerable attraction to the younger and smaller companies which do a limited amount of substandard business, not enough to justify complete tables of extra premiums for all ages and plans for the various substandard classifications.

Of more importance than the method of arriving at the extra premium is the question of the adequacy of the amount of it. Mr. Cammack has proved that his rules will give reasonably adequate

extra premiums for the various substandard groups graded according to percentage increases in the standard mortality rate.

Being satisfied with the adequacy of the extra premium, our next concern should be not to stultify ourselves by tying up a needless amount of the extra premium in reserves and, what is even less desirable, in granting needlessly large surrender values. In my opinion, any thing higher than the standard surrender value is not warranted from our present knowledge of the facts, and can only be justified on the questionable ground of expediency.

I am not the first to say that I would rather have that part of the extra premium not required for current mortality losses in the surplus funds than in the reserve funds, because the surplus funds can be drawn on as occasion demands whereas the extra reserve fund becomes available only in accordance with a predetermined mathematical rule which may or may not conform to the facts and in all probability does not conform to the facts.

Moreover, if in the course of time our investigations show that certain additional reserves are necessary for additional deferred mortality, these can easily be obtained from the accumulations in the surplus funds.

Keeping Mr. Hunter's statement in mind, we know that when the ratio of the extra to the standard mortality diminishes with the duration of the policy, reserves based on either the rated-up age or on a multiple of the standard mortality are needlessly high, and the true reserve may actually be less than the standard reserve at the actual age of entry.

For a long time I have held the view that the safest and therefore the most conservative course is followed by those who maintain the standard reserves on all impaired risks, and while experience may demonstrate that this is not the best method to follow, I know of nothing that has yet been brought to light to disprove the soundness of this method.

A great many years ago George Hardy in considering these problems proceeded on the assumption that the reserves on the substandard groups would be the same as those applicable to standard lives. It is true he was considering the lien method, but the principle need not be changed when considering extra premiums.

If standard reserves, then, are justified there is no need to depart from standard cash surrender values. They have the great

merit of supreme simplicity. They do not require any statement in the policy such as that suggested by Mr. Cammack that non-forfeiture values are based on an age at issue so many years greater than that of the insured. What is simpler for the insured to understand than that because of a certain impairment, doubtless known to him, the company is compelled to charge a stated extra premium?

If by charging such an extra premium the company is actually in a position to allow greater than standard surrender values, then I think it reasonably follows that with standard values a lower extra premium would be sufficient, and I am persuaded that the interests of the policyholder would be best served by keeping the extra premium at a minimum.

I am aware, of course, that if additional reserves are advisable, then in fairness to the insured increased surrender values should be allowed, but not infrequently non-participating premium rates based upon the true facts have to be adjusted upward because of the required legal reserve and the surrender values allowed. We do this of necessity and not from choice in dealing with standard policies. If the necessity does not exist in the case of substandard groups, we surely will not make such adjustment from choice.

In my opinion, Mr. Cammack's method has advantages over the rate-up-in-age and the multiple table methods, but the authorities have not yet disclosed anything that indicates its superiority over standard reserves and standard values. It does, however, offer a simple and useful rule to substitute for elaborate tables of extra premiums, and in this suggestion alone I believe Mr. Cammack has given us something of great practical value and usefulness.

MR. VALENTINE HOWELL:

A possible drawback to this system of combined rating in age and extra premium is the danger of misrepresentation on the part of the agent. That is, in competition with companies assessing the extra mortality by rating up the age, there is a temptation to compare the number of years rated up in each case, which is obviously unfair to the company with the older system. In a similar manner where a company charges an extra premium, the extra annual premium of the combined system may be stressed, ignoring the fact that the policy is also rated up in age.

MR. E. E. CAMMACK:

The main purpose of my paper was to show that there can be used in finding extra premiums for substandard business a combination of rating up in age and of a constant in place of the extra premiums computed upon multiple tables. The extra premiums are about the same under both methods, but the combination of rating up and of a constant has the advantage of extreme simplicity.

With reference to Mr. Graham's remarks, the reserves to be set aside are incidental only to the plan, and the method is equally applicable whether reserves be made for the true age of the insured or for a rated age. On some types of substandard risks reserves undoubtedly should be higher than for standard lives, while on other types it must be admitted that this is not so. Whether on substandard business generally reserves should be higher than for standard business has not been demonstrated, and I am of the opinion that it is better to adopt a conservative course and set up somewhat higher reserves. The reserves that I suggested lie between the reserves required on a strictly rate up in age method and a constant extra method.

I do not believe that the plan invites misrepresentation as suggested by Mr. Howell. The insured should understand that he is being charged a special rate and the policy should so state. The insured does not know how the rate is computed and the question of surrender values is a separate one for, as I have stated, they may be based either upon reserves for a standard life of the actual age of the insured, or of an older age, or, for that matter, upon a multiple table.

FINANCIAL STATEMENTS

BY VALENTINE HOWELL

MR. J. C. SEITZ:

The author's article sets forth in the first paragraph that he enlarges on several points that have arisen in dealing with the conventional blank form of annual statement and with periodical estimates of the financial condition of the company at fractional periods of the year.

The article is one which makes it somewhat difficult for a review,

in that the general nature is of a running comment type. I believe the article will appeal more to the student and not to one who has become familiar with the points referred to, for the reason that it lacks concrete or theoretical new value. It would appear to have some historical benefit to the one interested in statements who is "out looking in" instead of for the one who is "in looking out." It summarizes for the accountant the peculiarities of the life insurance statements. The public accountant is not as well versed in the matter of life insurance statements as he is in commercial statements.

In the author's subdivision dealing with the incorporation of reserves and accruals in books of account, I believe the student, who is always keeping in mind the exact terminology, might be confused by the example, which apparently is for the illustration of accruals. The terminology "accruals" refers more specifically to such items as the provision for the payment of items which actually do accrue. A very good example is that providing for payment of interest on the obligations incurred by the company. The illustration of fire insurance premiums paid in advance, wherein a portion of the disbursement will apply in actual expense of after-periods, has the technical terminology of "deferred charges." The first entry, in reference to the fire insurance premiums, should be the entry which would indicate deferred charges, and then, secondarily, there would occur the entries to indicate each year the amount that is charged off for the current year's expenses.

In the author's subdivision referring to theoretical entries, reference is made to consideration of some of the entries that are made when the entire life insurance statement is arrived at according to formal accounting methods. It is my impression that the student would gain the impression that the life insurance statement can be made theoretically correct by merely having journal entries referring to outstanding premiums and premiums paid in advance. I do not see the advantage of making journal entries, presumably at the end of the year, just in order to place certain non-ledger assets and certain non-ledger liabilities on the books so that the statement may then appear to be on a par with commercial statements. If the idea is to carry at all, it would seem that stress should be laid upon keeping the books throughout the year, on the standard commercial basis.

The author's illustration, in reference to the record of outstand-

ing premiums and the premiums paid in advance, would not appear to give the student the correct viewpoint in order that the statement might be on a revenue basis. My suggestion to stress this point would be that upon premiums becoming due there be entered upon the books the amount of the premiums as a credit to an account entitled "premiums receivable." The corresponding debit for the same amount of premiums would be in an account entitled "premiums due from policyholders."

The next step would be that upon payment of the premium by the policyholder the account "premiums due from policyholders" would be credited, and the cash account would be debited.

If the policyholder paid a premium in advance, the amount of the premium would be a credit to an account entitled "premiums paid in advance." By means of a tickler, on the date that such premium becomes due, the amount of such premium would then be charged to the account entitled "premiums paid in advance," and the amount would be credited to the account entitled "premiums due from policyholders."

The account "premiums receivable" would be the income on a revenue basis. The balance of the account "premiums due from policyholders" would be the outstanding premiums and an asset account. The balance of the account "premiums paid in advance" would be a liability account.

It is suggested by the author that, in keeping a set of books, apparently on the revenue basis, reversing entries would be made in order to begin the following year on the cash basis. Without a specific reference to the author's suggested plan as being a makeshift only, the student might be confused and believe that the plan was a theoretically correct plan for a revenue basis statement.

The greatest difficulty in the matter of a life insurance company maintaining a revenue basis will be due to the proper recording of the policy reserve. I merely make the reference to this point by stating that, if it be attempted, it would be best to use a fund ledger wherein the debit and credit sides in columnar form would correspond very closely to the plus and minus items given in the schedule entitled "check of reserves" on page 84 of the *Record* referring to the statement. The controlling account in the general ledger would be all of the entries combined into debits and credits as a group.

MR. H. W. ALLSTROM:

(Written discussion presented by Mr. James McIntosh)

Mr. Howell's paper is of interest in more respects than one. It leads from a simple statement of bookkeeping principles to a clear illustration of the aspect in which life insurance accounting as practiced in the United States differs from general bookkeeping. The paper leads to the question of the desirability of a revenue statement and a consideration of the gain and loss exhibit, and also covers the question of periodical statements within the calendar year for a company's own use.

The paper in developing the difference between bookkeeping on a revenue and on a cash basis shows that on a revenue basis non-cash items such as due and deferred premiums or premiums paid in advance would be credited or debited, as the case may be, to premium account. This, of course, is the correct principle on the revenue basis. I presume we might develop quite an interesting discussion as to the relative merits of a revenue and cash basis, but so far as our bookkeeping accounts are concerned and our annual statements, there would seem to be little merit in considering this subject at this time for the reason that our practices are fixed by the form of annual statements required. I believe it is a fact, however, that some companies at the close of the year run into their ledgers the items known as non-ledger assets and the liability accounts instead of carrying a general balance account, thus actually showing in the ledger and balance sheet the liability and surplus as well as the asset accounts. When this is done it would seem to me a better practice to open special accounts in the ledger for those accounts designated by the annual statement as "non-ledger accounts" rather than to run such items into the cash receipt accounts; that is, for example, to have a special account for due and deferred premiums rather than to credit them to the premium account.

In covering the subject of gain or loss, Mr. Howell expresses the opinion that a revenue statement should find a place in the blank, and recommends a rearrangement of the present gain or loss exhibit. We are, of course, required this year to file a new form of gain or loss exhibit. While we all have different opinions as to what would be best, it would seem as though after the consideration which the present blank has received that it will be likely to be with us for

some time. As a matter of fact, the exhibit of changes in surplus required by the new blank seems to a considerable extent to fit in with Mr. Howell's suggestion as to a form of revenue statement, except that the items not under a cash basis are covered on a separate line under the item of non-ledger assets. However, the new gain and loss exhibit, proper, as in the past, combines the revenue statement with the reserve analysis. I have found that in working up the gain and loss statement and in checking up the items it is a decided advantage to separate the revenue items from the items which might be called "reserve items," that is, items which effect the reserve fund.

In respect to periodical statements throughout the calendar year, we all doubtless at times have had to make very careful studies in this respect. Anyone doing so will almost inevitably notice how easy it is for wide changes to occur over short periods, owing to fluctuation of expense and also to the fact that the business or the premium income may not be evenly distributed throughout the year, to say nothing of decided variation in the death-rate. It is decidedly important to watch out that a combination of fluctuating results does not give an entirely erroneous impression of the general trend of the year.

MR. E. W. MARSHALL:

Mr. Howell has made a valuable study of problems arising in connection with the preparation of financial statements. It will be particularly helpful to students, although practicing actuaries will also find it useful.

At several points in his paper, Mr. Howell mentions that certain of the methods illustrated might apply only to small companies. It may be interesting to see how they apply to a somewhat larger company such as the Provident Mutual.

For example, the paragraph headed "Accumulation of Reserves" contains a formula for accounting for the increase in reserve during the calendar year. This formula in different form has been given before in actuarial literature, but it is good to have it set forth in the concise and easily understood form presented in Mr. Howell's paper.

Some companies have found it very valuable to compute independently each item for this "accumulation of reserve" formula so that a general test of the accuracy of the work in the Actuarial

Department may be obtained from the size of the "balance unaccounted for." The new Gain and Loss Exhibit does not give this check because the "expected mortality on net amount at risk" will be made the balancing item in the formula and thus the balance is forced. The Provident Mutual will continue to compute the items independently from its valuation records, because it believes that the check is too valuable to lose. It is the custom in the Provident Mutual to apply the formula separately to the reserves for insurance, annuities, paid-up additions, supplementary contracts involving life contingencies, supplementary contracts not involving life contingencies, and dividends left to accumulate. Thus the source of the "balance unaccounted for" may be determined and possible errors more easily located. Mr. Howell indicates that in one company with about $40,000,000 of reserve the average "balance unaccounted for" was about $5,000. In the Provident Mutual, with about $130,000,000 of reserve, the balance unaccounted for is about $12,000. It would be possible to introduce refinements and reduce this balance unaccounted for, but the result does not seem to warrant the additional work.

Our experience has been that this balance unaccounted for arises chiefly from the "running expense" account. In other words, the net premiums are slightly inaccurate. The reason for this condition is that, while the interest required to maintain the reserve and the net cost of insurance can be computed exactly from data obtained from the valuation records, the net premium item contains an element of inaccuracy because it is necessary to use estimates in connection with it.

There are at least three methods in use by different companies in computing the net premium for the Gain and Loss Exhibit, and as I know of no discussion of this subject in actuarial literature, a brief résumé of these methods may be of interest.

The first method is the seriatim method where, as each cash premium is paid during the year, a corresponding net premium is posted. This, of course, is rather laborious and expensive, but is still used by some companies. The Provident Mutual used it many years ago but discarded it later because of the great amount of work involved.

The second method is an estimate based on loading ratios. The total of the net and gross yearly premiums on business in force at

the end of the year is obtained by a seriatim or group valuation, or simultaneously at the time the valuation cards are tabulated, and the ratio of net to gross yearly premiums is computed. This ratio is applied to the cash premiums received during the year, with possibly a slight adjustment to allow for the additional premium charged on semiannual, quarterly, and monthly premiums. The assumption that the ratio of net to gross yearly premiums at the end of the year holds for all cash premiums of the year is somewhat inaccurate and creates a balance unaccounted for in the Gain and Loss Exhibit.

The third method is one which is used by the Provident Mutual and which might be called the "analysis of premium income" method. This method attempts to analyze the sources from which the net premium arises, the formula for item 10 of the 1924 Gain and Loss Exhibit (net premiums of the year) being as follows, all premiums being on a net basis:

Yearly premiums or new single premiums on all business in force December 31,

+premiums paid past current anniversary, less those required to complete payments to current anniversary, on all cancellations and outgoing changed policies of year,

−a similar item on revivals, incoming changed policies of year and new business of last year first paid for in current year,

+initial reserves on paid-up, extended insurance, and paid-up additions,

+premiums on annuities and supplementary contracts involving life contingencies,

+increase during year in net deferred and uncollected, less prepaid premiums, for disability provision,

+excess over yearly net premium of half-yearly and quarterly premiums (estimated closely from addressograph stencils).

The grand total is the net premium item for the 1924 Gain and Loss Exhibit. In case reinsurance is ceded, all of the above items would have to be reduced by any corresponding reinsurance premiums.

As all of the above net premiums on cancellations and changes are required when computing the reserve released for the "Surrenders, Lapses, and Changes" section of the Gain and Loss Exhibit, there is little additional work caused to obtain the net premiums according to the foregoing method. One advantage of the method

is that the deferred premiums of the year required for the Non-Admitted Assets can be obtained by working the above formula backwards, starting with last year's deferred premium, as explained in a paper before the Actuarial Society last year, entitled, "Office Methods of Obtaining Policy Reserve." That estimate comes within very close bounds and has been found to be exceedingly satisfactory in practice.

Mr. Howell also outlines a method of estimating the surplus for the year, based on the experience during the year. Our company has used methods somewhat similar to those of Mr. Howell for some years past. Our method is to obtain from the experience of recent years, a correlation between the various Gain and Loss Exhibit gains and losses and certain cash items easily obtained during the year from our cash bookkeeping totals, and use this correlation to estimate the gains and losses for the current year. While there were some differences between actual and estimated results, by subdivided groups, the totals agreed closely enough for the purpose in mind and the method is satisfactory.

We are all indebted to Mr. Howell for his excellent paper.

MR. FASSEL: I would like to ask Mr. Marshall if he would mind stating again wherein his method differs from the seriatim method.

MR. MARSHALL: From our valuation schedules we obtain at once the total net premiums for the business in force at the end of the year. This total of net premiums is adjusted for the items shown in the foregoing formula. Thus the great bulk of the work is obtained directly from the tabulating machine totals as a by-product of the valuation system.

MR. VALENTINE HOWELL:

In the first place, I wish to thank Mr. Seitz for his correction of my terminology, my use of the word "accruals" being clearly incorrect.

I do not entirely agree with his feeling that unless individual outstanding premiums be entered on the books at the time the liability is incurred, such books could not be considered to be kept on a "theoretically correct" revenue basis. My suggested treatment of the outstanding premiums, for example, would consider such premiums to be equivalent to "deferred assets" (I am again a little

shaky as to terminology) in commercial bookkeeping. However, such treatment, it would appear to me, would still entitle me to claim that the resulting account was on a theoretically correct revenue basis. By "correct," I hasten to add, I do not mean that there would be any difference in dollars and cents, the word referring only to the principle involved. While I specifically stated that I did not advocate keeping life insurance accounts in this manner, yet I think I might claim that it would have the advantage, such as it was, of showing, for example, the premiums of the year, and not the premiums collected in cash during the year, on the books of account; and this again constitutes the advantage of running these items into the cash receipts account in place of setting up separate liability and assets accounts on the company's books as was suggested by Mr. Allstrom.

I should certainly not advocate in view of the additional work necessary the keeping of books as outlined by Mr. Seitz, on the principle of entering all items becoming due in advance of their collection or payment.

I am much interested to learn of the extent of the balance unaccounted for in the "Check of Reserve Statement" for the Provident Mutual. My guess would have been that the figure would be somewhat larger. I agree with Mr. Marshall that the main difference probably lies in the premium account. With regard to the method of obtaining the amount of gross and net premiums for the year as outlined by Mr Marshall, I confess that the method, which in essence estimates the premiums of the year and consequently gets entirely away from the cash premium receipts of the year, fills me with some trepidation. We in the Guardian estimate the net and gross premiums in a manner somewhat similar to that of Mr. Marshall. The resulting gross premiums are checked against a figure obtained by the use of the cash premium receipts and the outstanding premiums (independently computed) and the estimate of net and gross is modified to bring it into agreement with this latter figure. Up to the present, we have not found a particularly close agreement between the two.

AVIATION HAZARD

BY W. F. POORMAN

MR. J. S. ELSTON:

Mr. Poorman has given the most essential data available as to the mortality rate among aviators. This paper is very opportune as this subject will undoubtedly be of increasing interest to life insurance companies. At present, in order to secure any idea of the risk involved, we have to make the best use we can of the available statistics, though my experience has been that these are not as satisfactory as they even appear to be. For instance, in nearly every table available either the number of hours flown or the miles flown is given but no explanation is made as to whether an hour's flight involves both the pilot and passenger or merely the pilot; in other words, the exposure from our standpoint may be much greater than appears from the table.

In the spring of 1919 I went to Washington with Mr. H. H. Hall, who had been an air-service officer, and we analyzed in detail the aviation accidents in the War Department up to that date and tried to assess the relative risk during the war and after the war, and that due to camera-gun work, aerial gunnery, testing engines, trying for altitude, shooting parachutes, silhouette water-target practice, shadow-shooting, and other stunting acrobatics, including figure eights and spirals. We also tabulated the casualties among passengers and similarly analyzed these. We attempted to differentiate between cadets and officers, according to the type of machine and the type of flying, the altitude at which the accident started and the cause of the accident. Information on these points was far from complete. Usually the secondary cause of the accident, or rather a description of the accident, was given instead of the primary cause. Of course the exposure was small and only very general conclusions could be drawn, but I am citing some of the phases which we attempted to consider merely to indicate the difficulty of the problem and to emphasize some of the dangers lurking in tables not originally analyzed or explained from the viewpoint desired.

Mr. Frederick L. Hoffman is studying this problem perhaps more intensively than anybody else. He has given me permission

to use the data included in a report he made July 1. This was, therefore, subsequent to Mr. Poorman's paper but I assume was independent of it. He includes the data in Tables V and VI and also corresponding Tables II, III, and IV, but differing slightly therefrom. His table for the War Department differs sufficiently from Mr. Poorman's so that I am including it though I cannot reconcile the two. In fact, Mr. Hoffman himself includes two tables, the other of which he had been unable to reconcile with this at the time the report was made.

TABLE A

Aviation Hazards in the United States Army Air Service

	Fatal Accidents	Flying Hours	Average Hours Flown per Fatality	Rate per Million Miles	Rate per 1,000 Men on Flying Duty
1921.....	69	70,252	1,018	13.6	50.4
1922.....	48	63,706	1,327	10.0	32.6
1923.....	41	75,771	1,848	7.3	29.1
1924.....	38	129,023	3,395	4.1	25.7
1921–24.....	196	338,752	1,728	7.9	34.2

He comments:

According to the above table the average fatality rate, on the basis of flying hours, has diminished during the last four years from 98 to 30 per 100,000. The fatality rate in proportion to air mileage has diminished from 14 to 4 million miles. The fatality rate, in proportion to the number of flying men on duty, has declined from 50 to 26 per thousand. The fatality rate, in proportion to the total number of flying accidents, has been reduced from 19 to 13 per cent.

In connection with his second table he says:

According to this tabulation the fatality rate per thousand was 28.3 for the four-year period, for enlisted men; 39.2 for cadets; 23.7 for officers and flying surgeons; and 26.5 for the entire air force. For each of the three groups specified, the rate at the end of the period was considerable less than at the beginning.

It is possible, though I see no proof of it, that statistics regarding lighter-than-air machines as well as heavier-than-air machines may be included in one of these tables and partially explain the discrepancies.

Mr. Hoffman's experience in the naval air service includes about

twice the exposure and more than twice the deaths that Mr. Poorman shows though covering the same period.

Mr. Hoffman's table on the air mail service is the same as Mr. Poorman's except that the air mileage in 1924 was 1,853,251 instead of 2,061,073.

TABLE B

Aviation Hazards in the United States Naval Air Service

	Flying Hours	Fatal Accidents	Average Hours Flown per Fatality	Flight Mileage	Rate per Million Miles
1922.....	39,808	14	2,843	2,500,000	5.6
1923.....	52,586	24	2,191	3,681,020	6.5
1924.....	58,907	18	3,273	4,123,490	4.4
	151,301	56	2,702	10,304,510	5.4

Mr. Hoffman also includes the experience of the Laurentide Air Service in Canada, involving 276,000 air mileage for 3,084 air hours in 1922-24 without accidents of any kind. As perhaps only one fatality would be expected on the basis of the other figures, this experience in itself would be inadequate to furnish real information though it is very interesting in connection with the other statistics.

TABLE C

Aviation Accidents in Canadian Air Service
(Air Force in Training)

	Flying Hours	Fatal Accidents	Average Hours Flown per Fatality	Air Miles	Rate per Million Miles	Air Personnel	Rate per 1,000 Men
1921.....	2,620	3	873	153,200	19.6	286	10.5
1922.....	986	.....	.....	59,160	.....	149	.....
1923.....	394	.....	.....	23,640	.....	143	.....
1924.....	1,640	2	820	98,400	20.3	161	12.4
	5,640	5	1,128	334,400	15.0	739	6.8

He also gives the experience in the Canadian air service analyzed between the air force in training and government civil aviation covering forestry, patrol, surveys, etc.

Mr. Hoffman warns us that the figures with relation to commercial aviation companies must be taken with considerable

reserve. This is very apparent from the fact that 8,767,000 air miles with 327,510 flying hours gives a speed of less than 30 miles an hour—which is very hard to explain. Mr. Hoffman believes that the low mortality of aviators from other causes than accidents is "generally overlooked in the rating practices of the American insurance companies." It seems to me that a possible difference in the ordinary premium rate is within the margin of error in the extra rate. Furthermore, while it is probably true that commercial aviators will be a select class physically, it is doubtful whether they will be selected as rigorously as in the Royal Air Force, which shows a very low death-rate from other causes. Furthermore, in

TABLE D

Civil (Government) Aviation of Canada

	Flying Hours	Fatal Accidents	Average Hours Flown per Fatality	Air Miles	Rate per Million Miles	Air Personnel	Rate per 1,000 Men
1921.....	6,547	2	3,273	479,929	4.2	359	5.6
1922.....	5,334	2	2,667	366,720	5.5	296	6.8
1923.....	4,253	2	2,126	278,563	7.2	390	5.1
1924.....	7,861	.....	.....	443,098	.....	385	.....
	23,995	6	3,999	1,568,310	3.8	1,430	4.2

the case of the company with which I am connected, the basic rate is a one-year-term rate on a select basis.

While most of the statistics indicate an improving risk, our rates were originally based on the 1918 experience, which was more favorable than subsequent experience and approximately corresponds to that in 1924. While I believe that insurance companies would like to do their part in promoting aviation, especially companies that would like to be able to provide workmen's compensation, public liability, and property-damage coverage, it does not seem to me that the statistics yet warrant the reduction in premium rates that aviation enthusiasts would like to see.

MR. J. G. PARKER:

I am not sure, Mr. President, whether it would be in order to introduce into this discussion any question as to what companies are doing in regard to the writing of aviation risks.

In Canada our own company has had applications from men

similar to those described by Mr. Elston, that is, men who are engaged in Ottawa in the aviation service, but are not regular aviators although they may have been such in the past. They now may take one or two flights a year just because they are connected with the service in an office position.

It is our own company's practice to accept such men with a clause in the policy to the effect that in the event of death as a direct or indirect result of injuries sustained while flying we would pay 25 per cent of the face value of the policy; (i.e., a 75 per cent lien applicable to injuries received while flying).

Those men who make application to us who are engaged as whole time aviators, men in our Forest Preserve work, men in the Government Aviation service, have not been accepted by us.

MR. P. H. EVANS:

I think Mr. Parker has raised a very interesting and practical question.

My company does not write substandard business at all, and we have a clause which eliminates the aviation hazard during the first year. Like most of you, I presume, we get a grist of applications from men who show an interest in flying. Sometimes they say they would fly once a month or oftener, or they say they would go up if they felt like it. We do not know what to do with those cases.

I had an inquiry from one of our agents recently who said that in a short time there would be an aeroplane service between Chicago and New York and that a certain class of business men would use the aeroplane to save time. He wished to know what we would do with such cases. It would be interesting if members would express the practice of their own companies in cases of that kind at this time. I presume we all feel absolutely in the dark as to what should be done with applications of men who are likely to fly, and there must be some sort of practice developing for such cases.

MR. P. F. BOURSCHEIDT:

I am with the Peoria Life. We have been taking some aviators, the premium depending upon the individual case. It varies from $10.00 to $25.00 per thousand extra. We have been somewhat lenient.

Of course, we do not accept those who do stunt flying while students, but as a rule we accept the experienced aviator by charging an extra premium of $25.00 per thousand. We have taken some at $10.00 to $15.00.

MR. H. BECKETT:

I presume Mr. Parker's rider applies only in the Dominion.

MR. PARKER: Yes.

MR. BECKETT: Of course in Canada the distinction between the limitation of liability and incontestability is very much more clearly defined than here in the States. Sometime ago I was really amazed to find how many companies issue policies on aviators and then limit liability or attempt to do so in exactly the same manner as you suggest.

It did not appear to me that under the laws of most states the attempt to limit liability would have very much standing in court. I know our own Legal Department is of that same opinion.

We think it is an extremely dangerous practice to attach such a rider to a policy as Mr. Parker suggests because it would give us practically no safeguard after the incontestable period in the policy had expired.

MR. W. A. P. WOOD:

On account of the company with which I am connected doing business both in Canada and in the United States, we have endeavored for the most part to have a practice which could be carried on in both countries and for that reason we have not been following the practice that Mr. Parker has spoken of.

In the case of commercial aviators we have been charging extra premiums, but very much larger extra premiums I am afraid than some of the other companies have been charging, judging by what we have just heard.

In the case of the commercial aviators, our minimum extra premium is practically $50.00 per thousand, and it goes from that up to possibly $100.00, according to the estimate we place on the additional risk involved in the particular work in which they are engaged.

There are cases in which persons not commercial aviators would only be flying for a very short time, as far as we could judge from the description of the work in which they were engaged, and

in such cases we would charge smaller extra premiums. They are rarely, however, below $25.00 per thousand. I think possibly in one case it went as low as $15.00 per thousand.

In the case of passengers (Mr. President, you also referred to passengers) it is our practice to allow those who are not the owners of their own machines the privilege of taking flights, and this privilege is really incorporated in the policy contract. We do not allow any person at all within the first two years to engage in aviation, apart from flying as a passenger, without getting permission from the company and paying an extra premium.

MR. A. B. WOOD:

The practice of our company, the Sun Life, is somewhat similar to that which has just been outlined by Mr. W. A. P. Wood. We had the question up particularly in connection with such men as are engaged by the large pulp and paper companies and we adopted a regulation, which is entirely experimental, inasmuch as we have nothing to go on, by which we charge an extra premium to the experienced aviator of $25.00 a thousand and we also limit the amount of assurance to a sum not exceeding five thousand dollars. I do not think we would take any case for a larger sum.

As regards the ordinary individual who says that he may occasionally take a flight, it is not our practice to impose any restriction. If we have inquired into the case and find that he is not going to engage in flying regularly, but it may be that he may desire to do it and may do so occasionally, we have allowed him that privilege without charging any extra premium.

MR. EVANS: I take it that if a line were established between Montreal and New York and you found that your business men were making use of it for ordinary business purposes you would probably take their applications without charging anything extra?

MR. A. B. WOOD: I think in all probability we would. In England, for instance, the same question comes up where men are frequently taking trips from London to Paris and we are not imposing any extra premium at all. We are permitting those trips.

MR. EVANS: No limitation at all?

MR. A. B. WOOD: No.

MR. VALENTINE HOWELL:

In perhaps half a dozen instances in the past the Guardian Life has accepted applicants where there appeared to be some possible suspicion that the aviation hazard existed, on a standard basis, with a clause limiting liability for two years. We limit it for two years because we believe that a clause imposing any longer limitation would not be legal.

In the case of applicants in the reserve aviation corps, or in other situations where it seems probable that they will not fly more than six times a year, we concluded to accept them with an extra premium of $10.00 per thousand for a limit of $5,000.00.

MR. E. A. PORTER:

I was recently very much interested to go over re-insurance papers on a rather celebrated Dutch designer of aeroplanes who came to the United States to head a company. Among the papers submitted was a description of the kinds and amounts and ratings of insurance he had obtained in the United States. One large company had taken him for, I think, a $20,000.00 policy, $5.00 a thousand extra and another company, I think, charged him $25.00 a thousand extra. So that there does not seem to be any great degree of uniformity.

Without any experience and judging only from the information that we have had from time to time in the publications of this Institute and the Actuarial Society, my own judgment would be that a single flight in an aeroplane is not in itself a very great hazard and a company can well afford to assume the risk of a few flights, or of occasional flights, but when the applicant is anything near a continual participant in aviation or is a professional or likely to become so, it appears that the hazard becomes very great indeed. It is doubtful if $25.00 a thousand even really covers it.

MR. F. A. DRAPER:

For those who would like some information and statistics relating to the passenger end of aviation: in the *Saturday Evening Post* about two months ago there was a series of three articles entitled "Europe Takes to the Air." Those articles gave very complete description of passenger aviation in England and on the continent, with tables showing the number of passengers carried and the

number of fatal accidents. The accidents seemed to me to have been remarkably few, almost negligible; comparable only with the possibility of a pedestrian being injured by an automobile in this country when crossing a street. Those who would like information on the passenger hazard will find much of value in those articles.

MR. W. A. P. WOOD:

There is another method of dealing with those cases in which they say, in connection with their occupation, that they perhaps may take two or three flights a year, and you may suspect that they will take more, perhaps ten or twelve. If they think themselves they are only going to take very few flights it is quite fair to say, "All right. Then, if that is the case, we will charge you ——" whatever you agree on, say $25.00 per thousand. "That will cover ten flights or more. If you do not take ten flights we will allow $2.50 off at the end of the year for each flight short of ten that you have taken." In that way if their own estimate in the first place is correct they will have a very small extra premium.

MR. G. B. PATTISON:

As Mr. Bourscheidt told you, our company has made a practice of quoting on aviation risks. The majority of the applications in which there is an aviation hazard are not applications on the lives of professional aviators. They are usually from men of more financial standing than ordinary aviators as, for instance, a man who has become interested in aviation and who takes an occasional flight for pleasure, or an applicant who occasionally uses an aeroplane in his business, averaging from ten to twenty-five flights a year. On this type of risk, we have quoted rates with an annual extra premium varying from $10.00 to $25.00 per thousand.

We have quoted an extra premium of $25.00 per thousand to aviators in the United States Mail Service and, while we receive a great many inquiries for quotations on this character of risk, I believe we have yet to issue a policy to an aviator in the United States Mail Service. All of these applicants complain that the rating is too high and they cannot afford to pay the premium. I do not believe the company is going to be swamped with applications from professional aviators.

MR. W. F. POORMAN:

I am greatly pleased at the amount of information in regard to the practice of the various companies brought out in the discussion.

The comments of Mr. Elston as to the undesirability of basing rates on the experience given are well made. It would appear, however, that any discrepancy would be on the safe side. I had the experience of obtaining two different tables of exposure from the War Department, but as one was based on miles flown and the other on hours flown, I felt that the Department's estimate of 75 miles flown per hour might account for the variation. I have observed that many of the experiences given in various popular articles in regard to aviation have omitted those deaths which probably could have been prevented through proper regulation and inspection. It would seem that the hazard was greatest in taking off and landing, so that those participating in aviation might well be grouped according to the number of flights made with a stated period instead of miles flown.

INFORMAL DISCUSSION

DISABILITY BENEFITS

Are premiums charged for disability benefits adequate?

MR. R. C. McCANKIE:

The Equitable of Iowa did not begin to issue the Disability Benefit until 1916. There was in force December 31, 1924, insurance, including the disability benefit, amounting to over $125,000,000. The Company has not adopted a clause providing that disability shall be presumed to be permanent after it has existed for a period such as ninety days. The only type of benefit which the Company has issued provides that benefits shall accrue upon approval by the Company of the disability claim after a specified time has elapsed from date of commencement of claim.

Originally the benefit provided for a waiver of future premiums and a monthly income payment to commence six months after approval of the proof of disability, and claims could not be filed until such disability had been in effect for sixty days. The benefit has been liberalized on several occasions until it now is in the form of an immediate payment on approval of claim for disability, which must have existed for at least sixty days. This type has been made retroactive to all policies so that the claim experience which we have made up covers practically this type entirely.

We have brought our disability experience down to June 30, 1925, and have incurred claims amounting to $368,000 on business issued on and after July 1, 1919. The claims incurred materially exceed the expected, being 176 per cent thereof. However, the expected mortality on disabled lives is $62,000, whereas the actual mortality is $78,000, and in addition to this there were $88,000 of recoveries. The ratio of actual to expected mortality is 125 per cent, whereas the ratio of mortality plus recovery, that is, the total decrement, to the expected decrement, which is only one of mortality, is 267 per cent.

Distributing the decrement by year, we find that in the first year of disability we have had deaths in the ratio of 142 per cent to

the expected, while we had recoveries which brought the ratio of decrement to expected decrement up to 218 per cent. Recoveries in the first year of disablement were not so material as in later years because, as a rule, no investigation is made of claims until one year has expired. This shows up in our experience, especially in the second year, where we had deaths of 109 per cent of the expected, but when we added recoveries to these deaths we had a ratio of actual decrement to expected decrement of over 425 per cent. The exposed to risk after the second year is so small that no conclusion could be drawn, but recoveries are being made in the third and fifth years of disability.

We also made a distribution by attained age, using three groups of ages, under 35, 35 to 50, and over 50. Though the data is too small to draw conclusions, it was interesting to note that the mortality over 50 was over 200 per cent of the expected, and that there were no recoveries. Under 35, on the other hand, the mortality was just under 75 per cent of the expected, but, adding recoveries, we obtained a ratio of decrement to expected decrement of 212 per cent. Between 35 and 50 the recoveries were material, although the mortality showed the same high ratio as over 50; i.e., over 200 per cent, and total decrement, a ratio of over 375 per cent. Recoveries in tubercular cases probably account for the results at the younger ages.

From the foregoing and the fact that claims incurred have been 176 per cent of the expected, the natural assumption would be that the disability benefit had proved a loss, and in order to determine whether this is the case or not, we keep an account of surplus on disability, charging against the fund the expenses and the reserves on both active and disabled lives, and we found, at December 31, last, that we had a fairly substantial margin of surplus.

However, in one month this year we had an increase in existing claims of almost 50 per cent. This is not the first time we have had such a fluctuation; the same thing happened in the second half of 1923. This is naturally a disturbing factor and raises the question, What should our normal claims be? The personal element enters so largely into this type of business that we hesitate to attempt to answer our own question. Taking our first six months of disability, we would show a loss of surplus of over $80,000 this year, but, if claims continue satisfactorily for the remainder of the year and we

have the same percentage of recoveries as we have had in the past, we will not show nearly as large a loss by the end of the year.

The only conclusion that I can draw is that our exposure is insufficient to warrant definite judgment. We have over $125,000,000 exposed and get wide fluctuations in claims from month to month. It would appear, therefore, that companies with a small exposure would do well to consider carefully the adequacy of their premiums, especially if they are contemplating raising the limit of disability benefit which they carry at their own risk. We look forward to the forthcoming report on disability experience to help us solve this problem!

There is one other point I would like to make. The reserve which we carried on disabled lives was apparently too high, and I have felt that we would probably come nearer a true reserve if we made up a reserve table on the same basis as a select table, where, in the first two years of disability, we would have a comparatively small reserve. This assumption is on the basis that the bulk of our deaths and recoveries will be in the first two years of disability.

After that time our ultimate table might very well graduate into Hunter's Table, since we might assume that our recoveries after that time will be immaterial and our disabled lives will consist of types such as tubercular cases that have lived two years and have not been able to resume active duties, insanity cases, and the like.

MR. F. B. MEAD:

The first disability benefits granted in this country were granted by fraternals, and the first life insurance company began in the later nineties to grant a sort of disability benefit somewhat different from what we are granting now. So far as my information goes, for the benefit granted, the premiums were adequate.

The first of the modern benefits were granted by two companies almost simultaneously in the year 1905. These benefits were merely a waiver of premium, with the option of securing the insurance in advance by instalments, the instalments being deductible from the face.

These companies were followed by many other companies. In those days there was no limiting age, and companies were granting these benefits without any idea as to what they cost. It was not until 1909 that life insurance companies did know really what it was

costing them to grant the waiver at the higher ages. It was estimated at that time that the extra premium would be about fifteen to eighteen dollars per thousand at the higher ages, and then there was a scurrying about to put in the limiting age clause. One small company I know about put up about $50,000 additional liability to cover that feature.

The original fraternal benefits were not even waiver of premium or of dues, as the assessments were continued during disability; they merely advanced $100 per thousand for nine years and $100 was retained for a final death benefit. So you see what they merely granted was the interest on the anticipated payments.

Life insurance companies continued for a number of years to grant merely the waiver of premium, with or without what we now call the instalment benefit.

I recall, ten or twelve years ago, that the Actuarial Society was addressed by the president of a life insurance company who urged that actuaries really find out what disability benefits cost, to learn something about it, and in January following that October meeting the insurance world was startled by the announcement of that particular company introducing what we now call the income disability benefit.

In my opinion the granting of inadequate rates in connection with the income disability benefits is the biggest blot upon the actuarial record in this country. The rates were and are manifestly inadequate.

As you well know, it is a principle of casualty insurance that the rate experienced will depend upon the magnitude of the benefit, and that has been borne out by actual experience, but the companies have ignored that principle and have granted these benefits at very inadequate rates.

I endeavored myself to call attention to that feature a number of times, as is shown by the *Record* of this Institute and by the *Transactions* of the Actuarial Society, but never was able to awaken any enthusiasm on the subject.

Two years ago we were startled by some of the companies calling attention to the fact that they were experiencing, on the income disability benefit, 200 per cent of Hunter's tables at the younger ages. Then there seemed to be some awakening.

I had the privilege last week, and I suppose I might refer to it

just roughly here, of seeing the experience of a large company; and it is the worst I have ever heard of up to the present time, and the selection of this company is very careful. While their waiver of premium experience has been very favorable, well within Hunter's Table, the income disability experience has been disastrous.

It was explained to me that the increased disability claims were due almost entirely to tuberculosis. The business is not old enough to have the most of the causes of disability developed yet. The business is not old enough for the degenerative diseases to commence to show their effect. It seems to me when the time comes when we have a well-balanced experience that the experience will be still heavier. I hope that actuaries will have the courage to charge proper premiums to make provision for the increased claims which are likely to arise from degenerative causes.

This is the first time, I believe, in life-insurance history where the beginning was wrong end foremost. It has always been a principle in life insurance to charge a high premium, to be reduced as experience might justify.

MR. J. S. ELSTON:

It has seemed to me that computing the ratio of actual to expected mortality on disabled lives has perhaps caused more trouble than if it were not computed at all, because if you will examine the original tables from which the aggregate mortality rates were deduced you will find that the rate of mortality in the first year was something like five times what it was in, for instance, the fifth year—that is, the original table is a select table. The aggregate table, however, is used, and perfectly justifiably, probably, by Mr. Hunter in calculating premium rates. In fact, he tested it out in a few cases, as I remember, to show that he got practically the same premium rate by that process as he did by the select table. But, on the other hand, when we show that our ratio of actual to expected mortality on disabled lives is 200 per cent, it may be actually 75 per cent of what would have been originally expected on the basis of the select table, which represents the original data on which Hunter's Table was based. For several years we have computed our experience on disabled lives on the select table, using recoveries and deaths instead of deaths only.

We have recently revised our premium rates, taking into con-

sideration our experience in the accident and health business as well as under the disability provision. Of course, this experience is so entirely within the select period, and the losses are increasing at such a ratio, that we still can't form any final judgment as to it, but the net result of our studies was an increase in disability premiums that may not yet be adequate for the benefits we provide.

MR. J. D. CRAIG:

Mr. Mead intimates that he wishes the actuaries would get their houses in order and charge adequate rates. We would be very glad to have our house in order and charge adequate rates; in fact, are trying to do so.

Of course, the annual statements show that practically all companies are losing money. We are in that respect one of the guilty parties. But the annual statement is misleading to a very large extent. A good part of the loss there is due to bookkeeping, because on group policies all claims paid are losses, with no premium being allowed; also certain benefits have been made retroactive, but no premiums charged therefor. We are one of those guilty companies that have claims in excess of those provided in the table, particularly at the early ages. But it might be of interest to the members here who studied the figures included in the paper presented before the Medical Directors Association last fall to state that our rates, based on the tables of disability there, and including the rates of recovery and death, are ample, and in fact have some latent profit—considerable, in fact—at the older ages.

We are, however, rather pessimistic, as some of our good friends here know, not on what the rate is today, but what it is going to be in the future. All experience shows that health-rates increase as mortality rates decrease. And if the disability rate steadily increases and is combined with an increased liberality on the part of the companies, it is difficult to know just what the rate will be in the future, and consequently what the adequate rate is that we should charge.

MR. E. W. MARSHALL:

In 1922 the Provident Mutual began to issue a "Three Months" disability clause at a gross premium which was about 15 per cent greater than that charged by the majority of companies for the same coverage. The Provident Mutual premiums were based on a

combination of the Manchester Unity Sickness Table and Hunter's Disability Table as presented by Mr. Cammack in the proceedings of the Casualty Actuarial Society in 1921. Mr. Cammack's single premium one-year-term figures, however, were increased at the younger attained ages and diminished at the older attained ages before premiums were computed, and interest of $3\frac{1}{2}$ per cent was employed. As the net premiums on this basis, with suitable loading, were intended to be adequate, it is interesting to see how the Provident Mutual disability claim experience compares with that expected according to the above-mentioned modified table.

For the year 1924 the ratio of actual claims under the disability income provision to expected claims according to the above-mentioned table was 25 per cent. Of course this ratio should be low, because the expected claims are computed from an ultimate table and the policies are all in the select period. Just what ratio for this select business is consistent with an ultimate ratio of 100 per cent after the select period has passed is a question. The Provident Mutual experience does not seem very far out of accord with that which might be expected from a select table merging into the ultimate table on which premiums were based. Certainly there is no indication that the Provident premiums are too large, although they are above the prevailing level of rates of other companies for the same coverage.

Incidentally, according to Hunter's Table, the Provident Mutual 1924 disability experience was 267 per cent of the expected, which shows that Hunter's Table is of little value as a measuring rod for disability claims arising under the "Three Months" clause.

There are several reasons why rates which are now adequate may not be adequate in the future. Mr. Craig mentions the fact that there is a tendency for claims to increase as mortality rates become lower. It might also be added that disability claims increase when policyholders become more familiar with their policy provisions and the benefits which they can obtain. Some companies state that they find a certain class of policyholders, the same class which fire insurance companies often fear, are also causing extra disability claims. Finally, the tendency of policyholders to secure contracts providing for disability income equal to or in excess of their regular earned income creates a very material hazard which will have a bearing on the disability-claim rate. All these factors make

it difficult to state that our present premiums will prove adequate in the future.

The question of reserves is also of current interest. So far as I know, the Provident Mutual is the only company not using the Hunter's Table in computing its reserves for the disability provision on active lives. The Provident Mutual bases these reserves on the same table used for computation of premiums described above. We find that our reserves on active lives are from 50 per cent to 60 per cent greater than they would be on Hunter's Table. To obtain our disability-claim reserve we make a careful estimate of the probable duration of the various claims and compute a corresponding reserve, taking into account the cause of disability. On the average, we find that this claim reserve is only about 15 per cent of that required under Hunter's Disabled Lives Table, the reason for this condition being that the Hunter's reserve is excessive because of the many temporary claims arising under the "Three Months" clause.

The net result of these two conditions, that is, the greater reserves on active lives and the smaller claim reserves, is that the disability reserves on the Provident Mutual basis are about 25 per cent in excess of the aggregate reserves for active and disabled lives under Hunter's Table. Of course, in the Annual Statement, the Provident will set up the usual Hunter's claim reserve, but reduce the reserve on active lives by the amount by which the Hunter's claim reserve exceeds the actual claim reserve needed as described above. Thus the total disability reserve in the Provident will be according to our own basis and not according to Hunter's Table. We believe that this method produces a reserve which follows more closely the claims experience under the "Three Months" clause and, for our company, is thus more scientific.

MR. E. E. CAMMACK:

I was talking the other day to the actuary of an English company which has been writing non-cancelable health insurance for the last forty years, and he told me that his company had underwritten this class of business successfully and that they used the Manchester Unity as a basis for their rates.

It seems to me that it has not been demonstrated that Hunter's Table is inadequate for basing rates for total and permanent disability benefits, but that a mistake was made when only a trifle was

added to the premiums when the disability clause was changed to give what was really a sickness benefit, or, in other words, when the total and permanent disability clause was changed to non-cancelable health insurance with no payments for the first three months of sickness. I cannot help thinking that we should use a sickness table in computing premiums where total disabilities lasting three months are presumed to be permanent.

The Manchester Unity seems to be the only table available, and, roughly, rates based upon the Manchester Unity for a health policy with nothing payable for the first three months of sickness are about equal to Hunter's net rates increased 50 per cent.

So that if you are going to provide for a loading of 25 per cent, you would have to about double Hunter's rates. The going rates are not loaded to anything like this extent.

LIMITS OF RETENTION

A. What points should be considered in fixing them?
B. Should they be less at the older ages?
C. Is a limit of issue in a single year less than the maximum limit of value in practice?
D. How should the substandard limits compare with the standard limits?

MR. J. M. LAIRD:

We have been chiefly interested in the relation between the limit of retention and the limit of issue. On the one hand, we may have a company which refuses to cede reinsurance, a company therefore, in which the limit of issue is the same as the limit of retention. On the other hand, we may have a company which issues substandard business but reinsures the entire amount in another company and therefore retains nothing.

When a company issues more than it retains and passes along the excess to a reinsurance company, it does not mean that the original company expects this reinsurance to be unprofitable. It is expected that the entire amount issued will prove profitable, but the original company cedes a portion to the reinsuring company in order to stabilize its mortality experience by limiting the maximum loss on any one life. If this maximum loss on one life has been fixed at, say, $100,000, then the limit of retention on the most favorable business will presumably be $100,000; for instance, on ordinary life

issued at age 35. At other points, the company is just as able to stand a loss of $100,000, but in practice usually adopts a lower limit of retention at the old ages, as there is more uncertainty about this business. A company which retains $100,000 at age 45 may retain only $10,000 at age 65. If it feels that business issued in excess of $10,000 at age 65 will prove unprofitable, it should make the limit of issue at this point $10,000 and refuse to issue any larger amount. Usually, however, the original company feels that larger amounts can properly be issued at these higher ages, and although it adopts a conservative limit of retention, it issues larger amounts and reinsures the excess.

At the very young ages it is customary to reduce both the limit of issue and the limit of retention. For instance, at age 15 the company may feel that on a boy of this age the total amount issued should not exceed $10,000. As this company retains $100,000 at age 45, then at age 15 it is fully justified in retaining the entire $10,000 issued.

Substandard business, if properly written, should be profitable, but the mortality is not so well established as on standard business. It is therefore customary to adopt a lower limit of issue and of retention on substandard business and to reduce this limit still further as the mortality rating increases. For instance, on a case rated at 200 per cent mortality the limit of retention and of issue will presumably be low, as there is more uncertainty about the accuracy of this rating than about the mortality on standard lives.

For disability benefits and double indemnity it is customary to adopt lower limits of issue and of retention than for standard life-insurance. One additional factor in considering these special benefits is the probability that the disability rate and accidental death-rate will be higher on policies for large amounts.

MR. J. R. LARUS:

It might not be amiss to supplement Mr. Laird's remarks on the reason for reducing the amounts at the older ages by the inclusion of the theoretical point that our expected variation from normal will increase as the probability of loss increases. I believe the formula involves the square root of the number of cases times the product of the probability of happening and the probability of not happening. Now the product of the probability of happen-

ing and the probability of not happening increases as they approach one-half, which means that as the age increases the expected liability to a variance from the normal will increase, so that a company taking $100,000 at age 65 is liable per thousand to have a greater variance from the normal mortality than at age 25.

I think that is one of the points that makes a good many of us reluctant to keep our limits constant as we approach the upper ages.

As Mr. Laird indicated, there are also other reasons that make us feel that the limit at the upper ages should be somewhat more conservative. Medical selection probably is not so sure at the older ages, and we understand that what selection there is wears off more quickly.

Moreover, the basic table that we are using —whether it be the American Experience, or, to a less extent, the American Men— gives us a greater margin at the younger ages than at the older ages, and even in participating companies, where the dividend might be counted on to correct this condition, it seems that the tendency is to charge the younger risks more than they should be charged.

This is brought about by a great many circumstances. It might be well to mention the fact that several large non-participating companies, under the American Experience valuation method, are charging more at the younger ages than they feel they actually need. They would otherwise be faced by the problem of putting up a deficiency reserve.

So that it seems to me at the present time we are not making so much surplus on our policies issued at the upper ages, and for that reason, too, would want to keep down the amount of those policies.

Moreover, the insurable value of a man decreases as he approaches the twilight zone of physical and mental usefulness. Policies issued at younger ages have accumulated reserves, and therefore the risk is not so great when they get to the advanced ages, but a policy issued at the upper ages involves quite a problem in insurable value if issued for any large amount.

The same line of theoretical reasoning would apply to substandard business, because the rate of expected mortality is greater than under standard business. We would therefore expect the divergence from the normal to be greater and it would seem for that reason we should have a lower limit—apart from the fact that our basic knowledge is not so sure, and that from this consideration as well we should deal more conservatively.

MR. F. B. MEAD:

I don't know as I understood Mr. Larus. It is my understanding that it is a well-established principle of the theory of probabilities that the relative amount of deviation decreases as the probability of the event increases. Therefore if we have approximately the same percentage of marginal expected at the higher ages as at the younger we are safer at the higher ages than at the younger when the same number of lives are involved, that is if the margin of tabular expected over the real expected is great enough.

MR. J. G. PARKER:

The divisions of the subject made by our Program Committee do not specifically include the disability benefit and the double-indemnity benefit. I think, however, both of these benefits may be properly included in our discussion of the subject.

First, considering life insurance alone: The point which, in my opinion, should be of the most influence in fixing a limit of retention is the amount of expected mortality in the year. With a small expected mortality and a large limit of risk it is very easy to see how the actual mortality could fluctuate between a very high percentage and a very low percentage of the expected. On account of the mortality being one of the large sources of surplus earning for the year, such fluctuation would affect to too great an extent the continuity of the surplus earnings of the company. It is, therefore, most important that there be some relation between the limit of risk and the amount of the expected mortality.

In order to give some definite figures I will state the limits of retention of our own company one year ago. At this time the expected mortality was, roughly, $1,500,000 and our maximum limit of risk on a single life was $30,000.

The second point which should have a bearing on the limit of retention should be the surplus earning power of a company in any year. I can imagine that a company with very low premiums, earning a low rate of interest and barely earning sufficient surplus to pay its dividends, should be very loath to keep a large limit on any one life. On the other hand, a company with large surplus earnings could afford to risk a greater fluctuation in actual mortality in any one year by keeping a larger limit of risk.

Again, to have some definite figures before us I might state that

our surplus earnings for last year were, roughly, $1,400,000, in which year our limit of risk on one life was $30,000.

The third point which I would bring to your attention is the amount of "free surplus" or contingency reserve which a company carries in its liabilities. If a company has provided large contingency reserves for the purpose of caring for any adverse fluctuation in mortality it could properly carry a larger limit on a single life than a company of the same size with the same surplus earning power but without any contingency reserve.

The fourth point to be considered is the reliance which a company can put on the facts provided in connection with the life to be insured. In this connection there would come under examination the reliability of the company's agency force, the qualifications of the company's staff of examiners, the territory in which the company is operating, and the standard of the risk from an underwriting view-point.

Under the disability benefit very much the same considerations should govern the limit of risk on a single life as under life insurance. If I remember correctly, the maximum loss in any year due to the disablement of a life insured would be, roughly, $1,200 per $1,000 of insurance. A company, therefore, that was keeping $30,000 as a maximum limit for life insurance should not carry more than $25,000 with a monthly-income disability feature of $250 per month. This is having in consideration only the effect of the company's surplus during the year.

Other considerations, however, would induce the company to make a further decrease in this amount, and that is the fear that it is impossible to underwrite disability risks as successfully as life-insurance risks and possibly owing to the danger of the premiums for the disability benefit being inadequate.

Under the double-indemnity benefit it would appear from what little information is available that the companies have been successful in their underwriting. It must be taken into consideration, however, in addition to all of the above points, that the risk involved is very large in comparison with the premium paid. I would, however, be prepared at the present time to recommend that a company which would carry a total on one life of $30,000 carry $15,000 under a life policy granting the double-indemnity benefit.

B) *Should the limits be less at the older ages?*—I think the con-

sideration of this subject involves the point which I mentioned under the first division of the subject, and that is, the reliability of the facts presented in connection with the risk. I think it is generally conceded in underwriting that it is impossible to get as accurate an examination of a risk at the older ages as at the younger ages. This consideration alone should influence a company in keeping a lower limit at the older ages.

I think also it could be argued that a lower limit should be kept at the younger ages, say ages under 20, on account of the uncertainty with regard to the assured's future health and habits. In the case of our own company our limit at age 15 is half of the limit at age 20. The limit is kept constant to age 55, when it decreases with age, being three-quarters of the maximum limit at age 60, less than half at age 65, and one-eighth of the limit at age 70.

C) Is the limit of issue in a single year less than the maximum limit in practice?—In the case of our own company, whose limit of risk is not large, we have never made any distinction between the amount which we would assume in any one year and the total amount which we would carry on a life irrespective of when the insurance was effected.

D) Comparison of substandard lives with standard lives.—In determining the limit of risk under substandard business I think that the information which you have about the particular class of risk in question should be given full consideration. For instance, on account of the definite nature of the statistics obtainable in assessing the value of overweight risks, a company would be much safer in taking large classes of overweights on a substandard basis for their full limit than they would be in taking equally large classes of a more speculative type, such as risks showing a history of suspected cancer, for an equal amount. The general rule might therefore be made that the less information you have about a particular class of risk, the lower should be your limits of retention of this class.

PROFESSOR J. W. GLOVER:

I was interested in what Mr. Parker had to say about various causes which are perhaps not measurable by formulas, which affect the limit of retention.

In regard to the application of pure theory I think Mr. Larus

has given the correct answer to the question. At least that would be the first way to approach it.

Given a sample of n individuals, the probability, p, and the complement, q, the standard deviation would be greater at the older ages because q then increases and p decreases; the maximum value corresponds to $p=q=\frac{1}{2}$.

However, there is another factor entering here which might explain why the experience at the older ages is less reliable.

The probable error of the standard deviation varies inversely as the square root of the number, n, so that if the sample is smaller, as it naturally is, the variation of the standard deviation is less stable.

MR. P. H. EVANS:

I imagine that the actuarial departments do not, as a matter of fact, have a great deal of difficulty in arriving at a satisfactory working answer to these questions. The real difficulty seems to be in framing the argument to satisfy a board of directors. It seems unlikely that the deviation formulas that Mr. Larus and Dr. Glover are talking about will be very convincing to a board or committee of laymen.

MR. J. D. CRAIG:

A formula such as has been mentioned here carried the day in our company and is what our limits are based upon.

MR. E. A. PORTER:

Mr. Larus and I have been continually hitting upon the same ideas. Mr. Larus has referred to practically the same things that I have intended to cover.

I want to emphasize, before I say anything along the theoretical line, that I do not believe that the question of retentions is a theoretical problem in the sense of a problem of mathematical probabilities. There are, after all, too many elements involved in it to make it just a matter of pure theory.

However, there are certain rather enlightening things to be got from the formula for the standard deviation of the binomial series, $\sigma=\sqrt{npq}$. If you allow q to vary and remember that p is equal to unity minus q, you will find that the standard deviation will increase as q increases until q reaches one-half, and then decreases. At age

95, if it is true that death must occur within one year of that age, there would be absolutely no risk in any amount of insurance. The standard deviation would become zero.

If, on the other hand, you take the relative—take the ratio of the standard deviation to the net pure premium, the one-year-term premium received—you will find that that ratio decreases with advancing age, decreases until the end of the table. It is at its maximum value at the lowest possible mortality.

Now, as to this variation of q, the rate of mortality can vary because of increasing age or because of increased hazard due to some other source, because of impairments, or the facts being difficult to ascertain, or something else of that kind.

In either case, in the case of either the old man or the doubtful or substandard risk, the indication of pure theory is to the effect that the limit of retention can properly be decreased.

Now, as to the variations of number, if in the same formula you allow n to vary, you find that the standard deviation varies directly as the square root of the number of lives involved.

That leads to a very interesting and probably erroneous conclusion that we can express something like this:

Suppose we have a company with $9,000,000 of insurance in force—we will assume this volume made up of exactly like policies on like risks—which is able to carry a certain limit, say, $10,000, safely, with a free surplus of $100,000. Now, if we assume that the free surplus is the only test of the risk-bearing possibilities of that company, when that company reaches $100,000,000 in force, if it has not increased its retention, it should, nevertheless, although far larger now, have a surplus of $333,000, an increase of three and a third times, due to the square-root rule.

Now, a conclusion of that kind, it seems to me, can vary a great deal from the mark either way, and is not the real test at all. Mathematics deals with pure premiums. It does not take one very important thing into consideration, which is that, in most of the life-insurance business transacted, presumably there is a margin of profit. This margin of profit is a contribution to the ability of the company to carry risks.

In fact, the problem, both practically and theoretically, in determining retentions can be put something like this:

That every item of business put upon the books adds to the risk

the company must carry and adds to the ability of the company to carry that risk.

If a company is in a stable position, a safe position at any moment, it is only necessary to make sure that the new business added will contribute in equal portions to the risk carried and the ability to carry it. Now that is not a simple mathematical proposition.

During Mr. Gunn's stay in Indianapolis he and I had quite a few prolonged discussions on this subject in which we endeavored to crystallize our ideas, and I don't believe either of us ever succeeded. You have to consider the kind of business, participating and non-participating. You have to consider the surplus, the free surplus, and the various contingency reserves. You have, above everything, also to consider the earning power of the company in the way of recuperation from losses and preparing for possible future losses. As a practical problem I think that the following suggestions can possibly apply:

In the first place, I have inquired of a great many actuaries, or rather I have made a statement which they would not challenge, that I had never known of a life-insurance company which was actually badly hampered by a *purely accidental* fluctuation in mortality. Now, there may be—it only needs one example to change my argument—but my argument is that most of the companies in existence have, as a practical matter, found a safe solution to this matter of retention. Now, if at any particular moment a company does have a safe solution I think we can more or less use these theoretical considerations to determine when it will be safe and how much it will be safe to increase those retentions. We can also use a good deal of judgment on the kinds and classes of business which are expected to be carried, and as to how much additional burden we can put on the company in those particular classes.

It is really, especially to the small company, a very vital problem. Reinsurance is a genuine cost to the company. Every item of business that a company safely can keep at its own retention should be kept. It is up to the actuary of every small company to see, whenever possible, that the cost of reinsurance is reduced just as much as can safely be done.

MR. J. M. POWELL:

Just one point that seems to be important to bring out, and that is that in these limits the continuous instalment benefit should

be considered. Of course this feature is usually considered, but occasionally it is accidentally overlooked through clerical oversight, thereby giving the company a much greater risk than it intended to assume.

THE AMERICAN MEN TABLE AS A BASIS OF VALUATION

PRESIDENT EVANS:

I think it well to state what probably you all know, that while the subject of the use of the American Men table has been more or less under discussion since 1919, the matter has been brought more forcibly to our attention by the Actuary of the Connecticut Department, who presented a very carefully prepared paper at the recent convention of the Insurance Commissioners in San Antonio. As a result of the presentation of this paper the Insurance Commissioners convention appointed a committee.

After some discussion they decided that it was an actuarial question and appointed a committee of actuaries, consisting of the Actuary of the Connecticut Insurance Department, Mr. W. M. Corcoran, Mr. Grady H. Hipp, the actuary of the New York Insurance Department, and Mr. J. F. Williams, the actuary of the Illinois Department. They also appointed Mr. J. D. Craig, as representing the Actuarial Society, and Mr. George Graham, as representing the American Institute. The Board of Governors has confirmed the selection and Mr. Graham is a member of that committee as representing the American Institute of Actuaries, in so far as any individual can represent the American Institute, there being, as you know, in our by-laws, a rule that the Institute cannot consider resolutions expressive of opinion.

The subject is now before us for discussion.

MR. S. E. ALLISON:

Anyone opposing the adoption of the American Men table is leaving himself open to criticism as preventing companies from granting lower rates to younger people. I offer myself as the willing victim of the sacrifice.

I think all of us are more interested in the institution of life insurance than in any company. And if any individual company

wants to grant lower rates to policyholders at the younger ages, it can do so under the American Experience table by putting up a deficiency reserve.

I think there is something more vital, however, in the consideration of this subject, and that is the safety of the younger companies and the problems which the vast majority of the non-participating companies will have to face in this connection.

Theoretically, of course, there is no objection to the adoption of the table, with the possible exception, as I understand it, that the American Men table does not include the experience of any pandemic years, and to that extent is not as safe as the American Experience table.

Those who went through the pandemic with not too great a surplus know how they felt and suffered during that period.

Also, it will be said that the American Men's table causes a smaller reserve to be put up in the earlier years, and therefore it will be argued that it gives the younger companies more leeway in the question of expense.

In the matter of leeway in expenses, those of us who have dealt particularly with the problems of a young company know the necessity of keeping down expenses. We know that all agency contracts have to be made on the basis of a certain profit to be received on the business. If a change were made, and all non-participating companies reduced their rates at the younger ages, it would be necessary to change the agency contracts because, in my opinion, all of us must receive at least the same minimum amount of money in future that we are receiving now in order to conduct our business properly and keep it safe for our policy holders.

I personally would consider the adoption of this table at this time a dangerous thing for the institution of life insurance. Our own company—perhaps because we have a source of surplus on a certain class of our business, namely, the tropical business, which is more real than apparent—would suffer perhaps less than some of the others; but it is a problem that every young company has to face and to which they are giving very serious consideration. It has to be dealt with on a practical basis, and I repeat that I consider the change inadvisable at the present time.

MR. E. E. CAMMACK:

Through the retention of the American Experience table of mortality the stock companies are compelled to charge at the younger ages rates which they know are excessive in comparison with the rates they charge at the older ages. This needs no demonstration, I believe that the net cost under participating policies, both with the big companies and the small companies, is less at the younger ages than the net cost under a non-participating policy, even in the first year of insurance. That shows that the non-participating companies are charging the public an excessive rate.

And from a practical standpoint I think it puts the non-participating companies in an awkward position, and particularly the smaller companies.

All that is necessary to do is to charge an adequate rate, a rate under which you can carry out your obligation to the policyholder and make some profit for the stockholder.

When you are charging at the younger ages so much more than the participating company, the stock company does not get the business. The larger stock companies valuing on the net level-premium basis, use for their gross premiums at the younger ages the net American premiums because they want to avoid putting up a deficiency reserve. The smaller stock company valuing upon a preliminary-term basis, even though it is just as economically managed and can issue insurance just as cheaply, must charge a higher rate than the large stock company with a large surplus, and is therefore in an unfavorable position.

A change in table would enable the small company, if it were well managed (most of them are, and many of them are as economically managed as the big ones), to quote the same rate as the company that was valuing its business on the level-premium basis. I can't think myself of anything that would be more advantageous to the smaller stock company than the optional use of the American Men table.

MR. C. O. SHEPHERD:

Without entering into the question as to whether the American Men table is desirable as a permissive basis of valuation, it seems to me that we can say that the American Experience $3\frac{1}{2}$ per cent table is not a satisfactory basis.

The larger non-profit-sharing companies have felt the pinch of

legal requirements at the younger ages by having their premiums fixed at a point below which they can't go without putting up deficiency reserves. A company valuing on the Illinois Standard basis has high net renewal premiums and it can't reduce its office premiums even to the point reached by the net level non-par companies at the younger ages without running into deficiency reserves. The Illinois Standard companies have felt that restriction already and I believe they are going to feel it to a much greater extent in the future.

I am satisfied of one other fact, and that is that even through the American Men table, with $3\frac{1}{2}$ per cent interest, were made a permissive standard and we were free to issue policies at lower rates at the younger ages, we couldn't do so without modifying our surrender values. Our present surrender values require premiums almost as high as the premiums that are being charged.

In order to take advantage of the American Men table with $3\frac{1}{2}$ per cent interest as a standard we should be privileged to exact a surrender charge greater than $25 a thousand. It is a very undesirable situation to be compelled by law to grant surrender values that are considerably in excess of the amounts that can be accumulated under present-day conditions. We do not need to apologize for the fact that expenses exist. I think our institutions are well managed, and it simply comes down to the point that life-insurance business has taken a certain form under which the expenses run very heavy in the earlier policy years, so that it is impossible to accumulate the legal reserves that are required.

Mr. Cammack showed in his paper in 1919 that under very reasonable conditions it was impossible for a company to accumulate legal reserves in from ten to fifteen years, assuming a rate of interest as high as $4\frac{3}{4}$ per cent. It seems to me that the question of a proper surrender charge is a very important one to be considered if the American Men table is adopted. The American Men table requires higher reserves at certain ages than the American Experience.

I feel that we haven't had nearly the light on this subject that we ought to have.

MR. J. M. LAIRD:

Apparently some companies are afraid that the adoption of the American Men Mortality Table as a legal valuation standard will increase competition. There are, however, various forms of com-

petition. One company may pay a high commission. Another company may feature low net cost to the policyholder. Some of us are operating in New York State, where there is a definite limit on the amount of commission which may be paid. We have now reached the point where, on account of the valuation standard, we are prevented from making a reduction in premiums at the young ages unless we are prepared to set aside a heavy deficiency reserve. At the older ages, where competition is usually most keen, we are today fixing our premiums according to our own best judgment, and this competition has not prevented the smaller non-participating companies from making a splendid development. The adoption of a modern valuation standard might permit some non-participating companies to make a slight revision of rates at the young ages, but the smaller companies would continue to hold their own in competition with these rates, just as they are now doing in competition with the low net cost of some of the mutual companies.

As a body of actuaries, we are chiefly interested in the scientific side of this question. If we had no valuation laws on the books today and were asked to recommend either the older American Experience table or the modern American Men table as a basis of valuation, undoubtedly the recommendation would be in favor of the modern table.

When the American Men table was first published, there was some doubt as to whether such a low mortality would continue, but the experience of the last seven years has been even more favorable. If all the companies represented here today combined their experience on standard business for the last seven years, it would undoubtedly show a mortality lower than the American Men table.

There might be some justification for a valuation basis of 125 per cent of the American Men table, but there is little excuse for forcing us to retain the American Experience table, which gives a mortality of about 200 per cent at age 20 and only 100 per cent at age 60. This means that if the American Experience table really has a margin for extra mortality, it is a margin which is constantly diminishing as the age increases.

It is possible that the present low mortality may be followed by a period of higher mortality, but it is more likely that over a period of years the mortality will show even further improvement. If that is the case, many non-participating companies which today feel that

their present gross premiums are low enough may wish to make a revision downward. Each company should be permitted to base its gross premiums on its own interest earnings, expense rate, and mortality experience. It should not be hampered by a legal standard based on the assumed mortality of more than sixty-five years ago.

MR. W. P. COLER:

I am somewhat in sympathy with Mr. Laird's remarks, particularly when he said that he could understand the desirability of requiring a valuation standard of 125 per cent of the American Men table.

I think that our American Experience table is an absurdity, in view of our experience, but I have been surprised that no one has mentioned the effect of the change on extended insurance.

I believe the requirement of the statute for extended insurance should be either eliminated or that the companies should be allowed to vary the surrender charge on different non-forfeiture values.

MR. C. H. BECKETT:

I have not anything, at this time, to say either for or against the adoption of the American Men table as a permissible basis of valuation. I would like, however, to ask a further explanation of Mr. Cammack's statement that through the retention of the American Experience Table of Mortality the stock companies are compelled to charge at the younger ages rates which they know are excessive in comparison with the rates they charge at the older ages.

If that statement is absolutely true, it will constitute to me one of the strongest possible arguments for the adoption of the American Men table. I have never had occasion to deal with the problems of non-participating stock life-insurance companies, and with participating business we have a fairly equitable method of adjusting these excesses.

However, this movement is likely to be far-reaching since, if new valuation laws are to be enacted, we shall have to deal with our legislatures. If the American Men or any other table should be adopted in some states as a fixed standard instead of a permissible standard, and not adopted at all in other states, the life-insurance business will be in a worse condition than it was some years ago, which, by the way, necessitated twenty-five years of discussion and

educational work before there was anything like a permissible minimum legal standard of valuation.

The prospect of such a situation again arising is not pleasing to me unless the benefits to be derived ultimately fully justify the effort. If it is necessary to make such a change in order to prevent charging excessive premiums at the younger ages, as Mr. Cammack suggests, I am willing to support the movement, but I am not yet persuaded that such a course is necessary. If any company wishes to reduce its premium rates below the net American Experience, why not do so and put up a deficiency reserve? It appears to me that any difficulties that would be met by following such a course could be easily overcome.

MR. CAMMACK: I think that I said that companies were compelled to charge a higher rate than they otherwise would charge, unless they were willing and able to put up the deficiency reserve. If I didn't, I meant to say that.

The deficiency reserve would be so large that there are few companies that could afford to put it up. There are a few, perhaps, but most of the companies could not afford to put up that extra reserve.

I will add one thing, that the statement, even if I had not made that restriction, is true in Pennsylvania, because under the Pennsylvania law you cannot quote less than the net rate, whether you put up a deficiency reserve or not.

MR. H. W. BUTTOLPH:

I don't know that I have anything of particular interest to say, but I cannot resist the opportunity to align myself with those whom Mr. Allison designated as the goats.

I do not know whether the Program Committee had anything particular in mind that led them to frame the title of this discussion just as it was framed. If so, they must be disappointed in the discussion, because it seems to me that we have wandered far, but I will assume that the majority of the speakers who have already addressed themselves to this subject have interpreted it correctly.

The discussion, however, seems to have been more an attack on the American Experience table than a defense of the American Men table as a basis of valuation.

I presume that there are few of us who, from a strictly theoretical, actuarial point of view would severely criticize the American

Men table as a basis for valuation. We certainly would all admit that from that limited point of view it is a table much superior to the American Experience table. But, fortunately or unfortunately, as your point of view may be, the strictly theoretical, actuarial point of view is not the only one to be considered here.

I rather regret that this subject, whenever broached, seems to call forth so much talk about competition. It seems to me that competition is going a little too far in life insurance today. I think that the public generally would be better off if there were not quite so much competition in the business as there now is.

The question of surrender values is a serious one, it seems to me, but I do not think that a change in the basis of valuation would bring about any material change in cash surrender values.

One of the speakers, I believe, said that if the change were made we would need more than the $25 surrender charge that is now allowed us. I am not much of a gambler, but I would like to wager that we would not take the full $25, if we had it, because of competition.

It seems to me that if we could set a limit, a maximum, rather than a minimum, limit, on surrender values we would be getting an improvement in the general situation.

I do not believe that the company which charges the lowest premium rate is the best company by any means.

My experience in the life-insurance business goes back more than thirty years now. And I remember very well two leading exponents of the rival camps in the days when I first entered the business. One of those companies made no pretense whatever of giving cheap insurance. The rates were high. The net results were high, and everybody knew it. The other company scarcely ever issued a piece of advertising literature without calling attention to its low rates of mortality and the resulting low cost.

Now, it may have been superior administration on the part of the company with the high rate of mortality and the consequent high cost in other matters than this one, but it is a fact that while both of those companies are large companies today, and extremely prosperous, in the broad sense of that term, yet the one with the high rate of mortality and the high so-called "net" cost has far outstripped the other.

I like to think that, whatever comparison might be made as to

the skill of the management of those two companies, the success of the one company over and beyond the success of the other has been a recognition by the public of the fact that the one company with the high premium rate and the high rate of mortality was giving to the public a greater service than the company which was seeking, so far as humanly possible, to restrict the selection of risks to the class of those that never die.

So I would say that from the broad economic point of view that company is the best company which has the highest rate or mortality and the consequently high premium rate, just so long as it does not get so high that the average man cannot afford to pay the premium.

In discussing this subject a good deal is said about the injustice done to the young policyholder. It seems to me that we are curiously blind in not saying more about similar injustices, and concentrating solely on this question of a mortality table. I have not prepared myself on this subject to go very far, but it seems to me that the interest rate has a great deal to do with it.

Whatever just criticism may be made of the American Experience table of mortality, it is not more to be criticized than is a 3 or $3\frac{1}{2}$ per cent interest rate today. Whatever may be said of the excess charge forced against the young policyholder as compared with the older policyholder, it is not as great as the excess you are charging your superstandard mortality risks as compared with those who are just inside the line dividing your standard class from your substandard class.

I, of course, need hardly say that I believe in substantial justice to our policyholders, but I do not believe in a very great refinement or segregation into many classes.

I do not think, because you, Mr. Chairman, may be of the class of risk on which the company expects and actually does experience a mortality rate of but 75 per cent of its normal rate, and I may be of the class on which the company expects and does experience 125 per cent of its normal rate, that you are being discriminated against in an unwarranted way and that I am being discriminated in favor of in an unwarranted way because we are being put in the same class and, assuming the same age, we are charged the same premium.

The American Experience table, whatever its technical theoretical defects may be, is, as I think everybody will agree, a safe

table. If the adoption of the American Men table would result in a lower premium charge and, as I think has been proven by others, would in the long run result in the accumulation of a higher reserve, it is of course apparent to everybody that the surplus earning ability of the company will be decreased.

If the influenza epidemic of 1918 taught us any one thing it was that surplus, or a surplus earning-power, is far more valuable than reserve.

The practical difficulties in making a change in the valuation laws of the states of the country would be very, very great. I shudder to think of the slips that might be made by the legislatures of many of our states in putting through any change in the legal standard of valuation, because there are so many things that should be made consistent with that change.

I do not believe the statement which has been made that no company today has a rate of mortality equal to, or in excess of, that shown by the American Men Ultimate Table. I not only do not believe it, but I think I am in a position to prove absolutely the contrary. Of course, I am not speaking of the American Central. Neither will I confine that statement to the one illustration that I am going to give, because that was a very abnormal case. But the question of safety under all conditions seems to me an important one, and I am going to close with just this one illustration.

Down in the state of Georgia is a company which, for various reasons, got into great financial difficulties. It was undoubtedly insolvent, and insolvent to a very considerable extent. It came under the care of the Insurance Commissioner of Georgia, and the Insurance Commissioner of Georgia has for a number of years now been operating that company, not writing any new business, but merely operating it, taking care of its old business and paying its claims, and all of those claims have been paid in full.

The company has ever since that time consistently had a rate of mortality considerably in excess of the ultimate mortality as shown by the American Men table, and there has been no such abnormal decrease in the amount of insurance in force, and consequent selection against the company, as might have been expected, because of the fact that the court, in turning this business over to the Insurance Commissioner, made it practically impossible for a man to get out, ordering that the payment of all cash surrender values and the mak-

ing of any loans for any purpose other than that of paying premiums be discontinued, and consequently there has not been the effect of selection which almost invariably follows an abnormal lapsation. But that company has been restored to solvency, for it is solvent now, not through savings in mortality, which might have been expected under the American Experience table if so terribly out of line for a stationary class of that description, but through excess interest earnings. That has been almost its sole profit.

Therefore I come back to one of the points that I tried to make in the first place. That is, that the American Experience table as a measure of mortality is no more out of line than our legal interest rates are out of line with the rates actually earned by the companies today.

MR. GEORGE GRAHAM:

It may interest you to know that in San Antonio, when this question of a committee was introduced, it was first thought to have a committee of commissioners, perhaps with one or two actuaries on it. But it was finally decided, the question being largely a scientific one, that it would be a good thing to have this committee composed of actuaries. I think it very gratifying to all of us that the commissioners, in reaching that decision, also decided that the two actuarial bodies should be represented on it. I could wish that they had permitted each of the actuarial bodies to name their own men, but I believe they were prompted to take that responsibility on themselves by the desire to have the committee named without delay.

Now, this being an actuarial committee, we are of course expected to approach it from a scientific viewpoint, and it will be our function to try to reach a correct solution on a broad statesman-like basis. Perhaps it will be difficult for some of us to dismiss wholly our personal interests and concerns from consideration.

I think this discussion has been exceedingly interesting and, to me, instructive. I hope it is not finished yet. It has brought out very clearly in my mind this thought, that the solution of the problem is not to be found merely in saying whether the American Experience or the American Men table shall be adopted as the standard. There are other factors than mortality to be considered. I am glad to hear that in this discussion these other factors have been brought out, such as the question of surrender values and extended

insurance. Then there is the question of the interest rate. It seems to me that we have to consider all of these.

We have to reach, as nearly as possible, a correct scientific answer to the problem, but we have also the practical considerations.

No one has questioned the adequacy of the American Experience $3\frac{1}{2}$ per cent table as a standard for legal reserves, so that I see no great urgency that this question be solved immediately. In fact, I believe that if this discussion has demonstrated any one thing it is that we are not in shape yet, at all events, to give an answer to the problem.

The interest rate has been mentioned. May I just leave this thought with you: That there is a uniform assessment bill that has been approved by the Insurance Commissioners, and in that bill 4 per cent is stated as the interest rate. I understand that fraternals use 4 per cent as their interest rate. So that there is good basis for thinking that perhaps the interest rate as well as the mortality rate need not be so fixed and determined as they are at present in the statutes.

MR. J. D. CRAIG:

Possibly there are some people here that might not like to speak out in open meeting, but if anybody would like to take lunch with us, or talk further this afternoon, as the trains for the East do not leave until five or six o'clock, we will be around and glad to get their views. Personally, I am absolutely open-minded on the subject. It does not mean one thing or another to us. Our net cost for the first year that we pay dividends (the third) is about equal to the American Men premium, with a loading of 10 per cent, and I do not know whether or not we would adopt the American Men table if its use were made permissible. We are not in the position of the non-participating companies, nor in the position of the young struggling companies, but we are perfectly willing to do anything that is best for the insurance business at large.

MR. H. R. CORBETT:

(Written discussion submitted after the meeting)

There is one phase of this question that should not be overlooked. We have today substantial uniformity of legal standard throughout our many jurisdictions. This uniformity was not secured

and could not have been secured upon the merits of any table of mortality. The motive for enactment was to regulate life insurance, and not to adopt a particular table. The adoption of a uniform table was merely incidental, since no other table was seriously considered at that time.

This uniformity is no doubt of more importance to life insurance than it is to have a better reflection of the actual experience. If we attempt to get a new table adopted we shall probably find ourselves dropping the substance to seize the shadow, like the dog in the fable.

Consider how difficult it is to secure any uniformity in state laws. Consider the diversity of standards upon other subjects of legislation, even where uniformity is urgently needed—such subjects, for example, as divorce, legal procedure, workmen's compensation, inheritance taxation, etc. That we have the same standard practically throughout this continent is little short of a legislative miracle, and resulted from peculiar conditions which would not hold again. Anyone who thinks the American Men table could attain a like uniformity within any reasonable time must be strangely unfamiliar with the eccentricities of legislation.

Uniformity gives us comparisons and aggregates invaluable to the business. The faults of the American Experience table present some difficulties, but we have become rather skilful in obviating them. Even the American Men table would not remove all the difficulties, nor would it supply a correct standard in all valuations, for example, in the valuation of annuities.

It is evident from this and like discussions in our various bodies that even if the exchange of standards were offered to us complete and uniform, we would not be unanimous in its favor. In attempting to make a change uncertainties would be introduced, much confusion would be inevitable, and it might be decades before we should attain again to the present degree of uniformity in the legal phases of the business.

BOOK REVIEWS AND NOTICES

Foreign Exchange from the Investor's Point of View: A Course of Lectures Delivered at the Institute of Actuaries, of London, England, during the Session 1924–5. By E. H. Lever, F. I. A. London: C. & E. Layton. Pp. 106.

Mr. Lever has in these lectures produced an exceedingly interesting and even fascinating work dealing with an important aspect of the question of money. Especially at this time, when so few countries are on the gold standard and the exchange rates between various countries are changing so rapidly, it is important for all those dealing with investments outside their own country to have such up-to-date information and points of view as Mr. Lever has given in these lectures.

It is perhaps obvious that the American actuary can rarely have the same direct interest in the theory and practice of foreign exchange as his confrère in Great Britain. The power of investment abroad is practically non-existent under the Insurance Statutes of most states, and even in those states which permit life companies to invest funds abroad this sanction is given only in respect of such investments as may be necessary to cover a reserve on policies issued in foreign countries. It happens too that the British actuary is more frequently charged with investment of his company's funds than is common in this country. Furthermore, the lectures were naturally given from the British point of view, but this fact is of no great importance, inasmuch as London is still the world's center in foreign exchange dealings.

However, apart from the immediate question of the practical bearing of this study for the American actuary, the theory of foreign exchange is important in any understanding of monetary affairs, and no actuary who wishes to be conversant with finance in its broader aspects can afford to be without at least a basic knowledge of such kindred questions as foreign exchange, balance of trade, inflation, and a general knowledge of governmental financing, both at home and abroad. All of these matters are lucidly discussed by Mr. Lever.

There is so much loose thinking on the question of inflation that the student may well give his closest attention to a discussion of this subject on pages 18 ff. of these lectures, and it is particularly interesting to observe the devices which may be employed by a government in order to cause inflation, or to reduce it. A reading of certain pages may help the

student to understand why the Federal Reserve Bank has to the date of this review refused to raise its rediscount rate above $3\frac{1}{2}$ per cent, although the Bank of England has recently increased its discount rate to 5 per cent. In this connection, Mr. Lever has also helped us to understand why the dollar exchange in terms of the British pound sterling should have been increased by that action of the Bank of England.

Mr. Lever is evidently a believer in the quantity theory of money, which, as he states it on page 20, in connection with the question of inflation of currency, seems at least to contain a fundamental element of truth.

An especially interesting portion of these lectures is devoted to what are called forward exchange transactions, and in fact such transactions have been only very recently practiced on a large scale in foreign exchange dealings. It may be said that they have virtually arisen from the great fluctuation in exchange rates, resulting from the war, and it is both surprising and interesting to observe the uses of Forward Exchange purchasing which have been made in recent years. Mr. Lever's work, indeed, appears to contain the first serious study ever given to this new development in international financing. While the danger is pointed out of loss which may arise from dealings in forward exchange, it would seem that we are told in these lectures a little too much about the successes which can be and have been obtained by this method. It seems to the layman that the use of forward exchange to make a gain is almost as complicated and dangerous as attempting to make marginal profits on stock exchange dealings. Mr. Lever, indeed, is careful to sound a note of warning, and points out that the use of this financial weapon should be limited to safeguarding investments in foreign countries which, owing to a rapidly fluctuating exchange, might depreciate, but at the same time his examples of seizing a profit by a rapid manipulation of foreign exchange rates in different foreign countries are perhaps more of a guide to speculation than to genuine investment financing.

Altogether these lectures form a practical description of present-day foreign exchange methods, and the student who reads with discrimination cannot fail to broaden his knowledge in that important sphere of finance.

J. C. C.

Reversions and Life Interests. By Frank Lakeman Collins, F.I.A. Published on behalf of the Institute of Actuaries. London: C. and E. Layton, 1925. Pp. 85.

This little volume consists of a course of four lectures delivered by Mr. Collins at the Institute of Actuaries, Staple Inn Hall, during the

session 1924-25. The first lecture is on "Legal Aspects," the second on "Actuarial Aspects," the third on "Practical Office Administration," and the fourth on "Funds in Reversion."

The type of lecture can be very well deduced from the following quotation: "Our literature teems with formulae, and as Mr. Lever pointed out in his paper read before the Students' Society a few years ago, the great thing is to grasp the underlying principles. Then it is at once seen whether the formula in question applies to the circumstances of the case." Three formulas only are mentioned: those of Nightingale, Jellicoe, and Baden; only the first of these is given, and its derivation is omitted.

It may be open to question as to whether it is desirable to entirely divorce an exposition of the principles from the mathematical formulas that apply, but where so many contingencies exist, and so many approximations must be resorted to, it does seem that good judgment and care not to omit any of the circumstances involved are more important than mathematical refinements.

Mr. Collins emphasizes the point that the interest rate to be used in the valuation of reversions is of much more importance than the mortality rate, and concludes that "the rate of interest ought to be at least one per cent higher than would be obtained by buying an investment or lending on a mortgage."

Following the plan of the *Actuaries Text Book* (Todhunter), Mr. Collins limits his treatment to the four most important interests in property created by deed or will, (1) Absolute Reversions, (2) Contingent Reversions, (3) Life Interests in Possession, and (4) Reversionary Life Interests. His discussion of each of these types of benefit is to the point, and reflects the practical actuary to a marked degree. The final lecture contains much valuable information and many suggestions for Trustees of Funds in Reversion.

J. F. R.

Handbook of Mathematical Statistics. By H. L. Rietz, Editor-in chief, and Eight Other Members of the National Research Council Committee on the Mathematical Analysis of Statistics. Boston: Houghton Mifflin Company, 1924. Pp. viii+221.

Now that the entire *Handbook* has been included in this Institute's syllabus of examinations, with references carefully distributed between associateship and fellowship sections, the Fellow of the future will necessarily have ideas of the volume decidedly more definite and pronounced than any a brief review can possibly convey. Eminent professors (nine of them), of statistics, of mathematics, of education, of insurance, of eco-

nomics, from Harvard, from Michigan and Illinois and Iowa, from the Pacific coast, have produced a dozen chapters on the important phases of the mathematical analysis of statistics, in which the gist of the three hundred or so items listed in the helpful bibliography is presented in conveniently condensed form for the student's reference. Numerical examples of the application of these methods to diversified data are generously interspersed.

Naturally, such a volume has less continuity than a good detective story, and is rather harder reading. Some chapters are barely more than a collection of formulas, others present a lucid exposition of some important branch of mathematical statistics. This diversity arises partly perhaps from the diversity of authorship, but chiefly I think from the very nature of the separate chapters, which range from the opening 19-page list of mathematical memoranda at the one extreme to the exceptionally successful treatment of simple correlation by Professors Rietz and Crathorne and the closing discussion of index numbers at the other. The actuary, by the way, tired and baffled after a hopeless quest of the ideal measuring-rod for mortality statistics, will derive a certain morose comfort in reading Professor Young's account of the difficulties involved in the choice of index numbers, or in relentlessly pursuing periodicities through chapters x and xi with Professor Persons and Professor Crum.

No doubt the same reader will be most at home in chapter iii, where Professor Glover in 28 pages manages to summarize admirably the fundamentals of interpolation, summation, and graduation which it took some of us so many years to learn—and alas forget. He will probably be convinced on finishing the book that if every chapter represents for its field as successful a summary as this and the others which he feels competent to judge, the *Handbook* is a treasure indeed; and that if some one chapter (I would modestly nominate the ninth, on "Partial and Multiple Correlation") means next to nothing to him after painful perusal, the fault lies in the reader's abysmal initial ignorance of that phase of the subject.

Because of the inclusion of *Handbook* in the examination syllabus it is proper to reprint in *The Record* the errata contained in Professor B. H. Camp's careful review of the work in *Bulletin of the American Mathematical Society*, July, 1925:

Errata (partly supplied by the Editor). Page 16, line 12: *For* at least *read* some. Page 16, line 14: *For* at least one 2-spot with two dice *read* a 1-spot or a 2-spot with one die. Page 16, line 7 from bottom: *For* $1/\sqrt{2\pi npq}$ *read* $1 \; 2npq$. Page 54, line 5: *For* coefficients *read* coefficient. Page 56, line 5 from bottom: The minus sign $(-)$ in $\frac{1}{2m}(u_0 - u_\omega)$ *should be changed to* $+$. Page 70, line 7 from bottom: *For* m^2 *read* m_1^2. Page 73,

line 15: *For* 688 *read* 668. Page 74, line 6 of Ex. 2: *After* number *insert* of women's numbers; *to avoid ambiguity from the use of number in two senses*. Page 78, line 4 of second paragraph: *For* fit *read* fitted. Page 92: The integrals are definite and should have upper limits. Page 97, line 15: *For* $_{N}{}^{y}$ *read* $_{N}{}^{\sigma}y$. Page 103, line 7: *For* 80 *read* 81. Page 105, line 3 from bottom: Omit x and the two commas following. Page 107, line 4: *For* V *read* VI *and for* VI *read* VII. Page 108, line 1: *For* VII *read* VIII. Page 121, line 4 from bottom: *For* $\frac{y_i'}{\sigma_y}$ *read* $\frac{y_i}{\sigma_y}$; *i.e., delete the prime.* Page 126, formula (9): *For* σ_x' *read* σ_x^2. Page 142: The last line should begin with $\beta_{12.34}$. Page 149, line 2: *For* (13) *read* (19). Page 187, formula (8): Insert Σ before parenthesis in the denominator.

H. H. J.

Economic Statistics. By William Leonard Crum and Alson Currie Patton. Chicago and New York: A. W. Shaw Company, 1925. Pp. xii+493.

This book is designed primarily for students of economics, who as a rule are not specialists in mathematics, to aid them in acquiring a working knowledge of the uses and limitations of statistical processes as a tool in economic analysis. The illustrative material is accordingly drawn entirely from the field of economic statistics. The discussion of the subject is under the heads: Part I, "Statistical Data"; Part II, "General Analytical Methods"; and Part III, "The Analysis of Time Series." In part I fundamental definitions are enunciated and the sources, compilation and collection of data, preparation and construction of tables, and the charting of categorical, time, and frequency series are discussed. Advanced readers of the work who might regard the treatment as unnecessarily diffuse will need to bear in mind that the work is intended for immature students. Some inconvenience will be experienced due to the multiplicity of tables and charts, making it impossible to keep them in proximity to the corresponding text. Part II deals in a non-technical way with the standard fundamental concepts incident to statistical analysis. Part III is the *raison d'être* of the work, dealing as exhaustively as possible in a brief work with the analysis of time series. Commendable conservatism is displayed at all times in results claimed for the method under consideration. The following wise caution appears on page 370: "The forecaster (should) avoid curves as a guide and look behind the statistical results at every given moment to assure himself that there are no factors present in the situation which need upset the normal sequence of movements." Chapters xxiii and xxiv treat the subjects of "Lagging Correlation and

Forecasting Sequence" and "Special Methods" made necessary by changes incident to the war in a non-technical way and should be of special interest to business men and others who are followers of the *Harvard Index Chart* and other guides.

Although apparently designed mainly as an aid to workers in the field of economic analysis, the work under review is one that might very profitably be perused by both professional actuaries and business men interested in commercial statistics. It very happily points out both the strength and weakness of the statistical method in the field under consideration.

H. C. M.

Mathematics of Life Insurance. By L. Wayland Dowling, Ph.D. New York: Hill Book Company, 1925. Pp. x+121.

The Preface informs us that this volume has grown out of lectures given at the University of Wisconsin and is intended as a first course for young men and women who wish to become trained actuaries; or as a final course for other students desiring an elementary knowledge of the fundamental principles underlying a vast and growing business.

The author has succeeded in condensing into this small volume a very satisfactory introduction to actuarial studies, and the examples and illustrations are well selected.

In the chapter on "Valuation of Policies" the term "net reserves" is used where "net level premium reserves" would be much more acceptable, and the section relating to preliminary term valuation is distinctly disappointing. Throughout the chapter on "Policy Options" the author fails to distinguish between surrender values and reserve values. In the chapter on "Contingent Functions," the symbol used for the net annual premium for term insurance is the symbol generally used for the net annual premium for endowment insurance.

In spite of the objections mentioned the book seems well suited for use in the classroom, and should prove a useful addition to the library of any actuarial student.

W. O. M.

OTHER RECENT PUBLICATIONS

Unemployment Insurance Act, 1925. London: H. M. Stationery Office, 1925. Pp. 8.

Report on Workmen's Compensation—May 1925. International Labour Conference—Seventh Session. Geneva: International Labour Office. Pp. 199.

Current Economic Problems. By W. H. Hamilton. (3d ed.) University of Chicago Press.

Inter-racial Variation in Infant Mortality. By J. DePorte, Johns Hopkins University, Department of Biometry and Vital Statistics. *Paper No. 114, Life Table Monogram. Paper No. 115*. Reprinted from *Am. Journal of Hygiene*, JC. 1925. Baltimore: 1925. Pp. 454, 496.

Manual of Problems and Tables in Statistics with Notes on Statistical Procedure. By C. F. Mills and D. H. Davenport. New York: Holt, 1925. Pp. 203.

Statistical Methods for Research Workers. By R. A. Fisher. Edinburgh: Oliver, 1925. Pp. 239.

MINUTES OF THE SEMIANNUAL MEETING

of the

AMERICAN INSTITUTE OF ACTUARIES

HELD AT THE EDGEWATER BEACH HOTEL, CHICAGO, ILLINOIS, NOVEMBER 5 AND 6, 1925

The Institute was called to order by the President at 10:45 A.M., November 5, and the following Fellows, Associates, and Nominees of Contributing Members were present:

FELLOWS

S. E. Allison
A. E. Babbitt
W. N. Bagley
C. H. Beckett
J. P. Bowerman
H. W. Buttolph
H. R. Corbett
H. W. Curjel
F. A. Draper
E. O. Dunlap
J. S. Elston
P. H. Evans
James Fairlie
E. G. Fassel
J. W. Glover
George Graham
Marcus Gunn
A. J. C. Huenergard
H. H. Jackson
Dorothy M. Jamison
W. M. Johnson
R. E. Kennon
J. M. Laird
M. A. Linton
E. L. Marshall
H. C. Marvin
R. C. McCankie
James McIntosh
F. B. Mead
P. V. Montgomery
W. O. Morris
W. E. Napier
J. H. Nitchie
J. G. Parker
G. B. Pattison
W. F. Poorman
E. A. Porter
H. L. Rietz
J. C. Rietz
J. C. Seitz
C. F. Stein
M. W. Torrey
R. M. Webb
W. A. P. Wood

ASSOCIATES

Henricka B. Beach
J. H. Birkenshaw
G. W. Bishop
L. T. Boyd
H. G. Brunnquell
Helen L. Clark
W. P. Coler
C. F. Cross
A. C. Eastlack
C. H. Foltz
J. S. Hale
W. J. Hiller
Valentine Howell
F. R. Jordan
Eloise B. Koch
A. T. Lehman
J. H. Lithgow
A. J. McAndless
W. H. McBride
Gertrude L. Pardieck
J. M. Powell
H. G. Sellman
B. E. Shepherd
L. O. Shudde
C. C. Sinclair
G. L. Stayman
F. D. Strudell
R. B. Sturtevant
V. L. Tickner
Margaret Walker
H. G. Walton
J. R. Young

NOMINEES OF CONTRIBUTING MEMBERS

P. P. Andrews.....Royal Union Mutual Life Insurance Company
G. D. Babbitt.....Bankers Reserve Life Company
S. J. Blashill.....Ohio National Life Insurance Company
Richard Boissard.....National Guardian Life Insurance Company
P. F. Bourscheidt.....Peoria Life Insurance Company
F. R. Brunner.....Security Mutual Life Insurance Company
T. F. Brunton.....John Hancock Mutual Life Insurance Company
L. D. Cavanaugh.....Federal Life Insurance Company
L. C. Cortright.....Old Line Life Insurance Company of America
L. C. Cox.....National Life & Accident Insurance Company
J. H. Evans.....Ohio National Life Insurance Company
T. H. Galbraith.....Central Life Insurance Company of Illinois
D. H. Garver.....Maryland Life Insurance Company
L. K. File.....Canada Life Assurance Company
M. N. Hanson.....Wisconsin Life Insurance Company
H. J. Hornberger.....Great Northern Life Insurance Company
L. H. Howe.....John Hancock Mutual Life Insurance Company
P. C. Irwin.....Equitable Life Insurance Company of Iowa
W. W. Lane.....LaFayette Life Insurance Company
J. R. Larus, Jr.....Phoenix Mutual Life Insurance Company
R. M. Malpas.....Reinsurance Life Company of America
E. W. Marshall.....Provident Mutual Life Insurance Company
H. E. Mizener.....Register Life Insurance Company
Helen Moon.....Central Life Assurance Society of the U. S.
G. C. Moore.....Imperial Life Assurance Company
T. M. Mott.....Liberty Life Insurance Company of Kansas
C. E. Nelson.....Missouri State Life Insurance Company
E. H. Nicholson.....Lincoln National Life Insurance Company
F. J. Owen.....Berkshire Life Insurance Company
J. G. Phippe.....Peoples Life Insurance Company
T. C. Rafferty.....International Life Insurance Company
E. B. Raub.....Indianapolis Life Insurance Company
J. D. Reeder.....Lincoln National Life Insurance Company
F. L. Rowland.....Lincoln National Life Insurance Company
J. B. Russell.....Peoria Life Insurance Company
L. L. Stevens.....Penn Mutual Life Insurance Company
A. C. Washburne.....Berkshire Life Insurance Company
Ethel Welsh.....National Life & Accident Insurance Company
H. R. Wilson.....American Central Life Insurance Company

Others in attendance were: Arthur B. Wood, vice-president and actuary, Sun Life Assurance Company of Canada, Montreal; W. D. Haller, actuary, Iowa Insurance Department, Des Moines, Iowa; W. H. Huehl, actuary, Insurance Department, State of Indiana, Indianapolis, Indiana.

The Minutes of the meeting held June 11 and 12, 1925, as printed in No. 29 of *The Record*, were approved.

The Secretary reported that the following had been admitted to Fellowship without examination: William Young, Victor Roy Smith; the following were admitted to Associateship without examination: Valentine Howell, John Walter Thomson; and that the following companies were admitted to Contributing Membership: The Monarch Life Assurance Company of Winnipeg, Manitoba, Canada; The Liberty Life Insurance Company of Kansas, Topeka, Kansas.

A new publication of the Institute, entitled the *Yearbook*, has been printed and issued to all members and students.

The Treasurer presented his report, which, upon proper motion, was accepted and filed.

Formal papers were then presented, followed by a discussion of the formal papers presented at the annual meeting in June.

The morning session adjourned at 12:15 P.M.

The meeting reconvened at 2:15 P.M. and continued the discussion of formal papers.

The afternoon session adjourned at 3:40 P.M.

Friday, November 6, the Institute reconvened at 10:15 A.M. and proceeded with the discussion of informal topics on the program.

The meeting adjourned at 12:30 P.M.

CONSTITUTION *and* BY-LAWS
OF THE
AMERICAN INSTITUTE *of* ACTUARIES

CONSTITUTION

NAME

ARTICLE I. This organization shall be called "American Institute of Actuaries."

OBJECT

ART. II. The object of the American Institute of Actuaries is to advance the Science of Insurance Mathematics and the knowledge of the theory and practice of Life Insurance and related interests by associating together persons of like interests.

MEMBERSHIP

ART. III. The membership of the Institute shall be composed of three classes, Fellows, Associates, and Contributing Members.

OFFICERS

ART. IV. The Officers shall consist of a President, a Vice-President, a Secretary, a Treasurer, a Librarian, and an Editor of *The Record*.

GOVERNORS

ART. V. The Board of Governors shall consist of the Officers, the ex-Presidents, if Fellows, and six other Fellows.

DUTIES OF OFFICERS AND GOVERNORS

ART. VI. The duties of the officers shall be such as usually appertain to their respective offices or as may be prescribed from time to time by the Board of Governors. The duties of the Board of Governors shall be in general to manage the affairs of the Institute. It shall have the power, if the Institute be not then in session in annual meeting, to fill all vacancies occasioned by death or resignation of Officers or Governors, such appointees to serve until the next annual meeting of the Institute and until their successors be elected. Appropriate subcommittees may be appointed by the Board of Governors for such matters as from time to time may be found desirable.

ELECTION OF OFFICERS AND GOVERNORS

ART. VII. The President, Vice-President, Secretary, and Treasurer shall be elected at each annual meeting to serve until their successors be elected. The President and Vice-President shall not be eligible for election to the same office at three successive annual meetings. The Librarian and the Editor of *The Record* shall be elected by the Board of Governors, subject to confirmation by the Institute at each annual meeting, and shall serve until their successors be elected. At each annual meeting two Governors shall be elected to serve until the third succeeding annual meeting and until their successors be elected. They shall not be eligible for election to succeed themselves at the same meeting at which their terms expire. Any vacancies existing on the Board shall be filled by election at each annual meeting, the Fellow so elected to serve during the unexpired term of his predecessor. All elections shall be by ballot.

MEETINGS

ART. VIII. The annual meeting of the Institute shall be held between the fifteenth day of May and the fifteenth day of June in each year on a date and at a place to be designated by the Board of Governors. Forty days' notice of the time and place of meeting shall be given the membership by the Secretary. Other meetings may be called by the Board of Governors or shall be called by the President upon the request in writing of not less than ten Fellows and not less than twenty days' notice shall be given of such meetings.

ASSOCIATES

ART. IX. Anyone at least twenty-one years of age pursuing actuarial studies and looking forward to a future Fellowship may be enrolled as an Associate under the following conditions: that he shall be nominated by at least two Fellows of the Institute; that a confidential notice of such nomination shall be given by the Secretary to each Fellow of the Institute; that this nomination shall be approved by the Board of Governors not less than twenty days after such notification of nomination has been given; that he shall pay such examination fee and pass such preliminary examination as shall be prescribed by the Board of Governors. If the Board of Governors finds that a candidate has passed an equivalent examination required by another recognized society of actuaries, or has had such

practical experience as in its opinion renders him qualified for Associateship, it may waive the examination herein required. An Associate shall be entitled to all the privileges of the Institute except those of voting and holding office.

ADMISSION TO FELLOWSHIP

ART. X. Any Associate whose application shall be approved by the Board of Governors may become a Fellow if at least twenty-five years of age under the following conditions: Every member of the Board of Governors shall have confidential notice of the application at least twenty days prior to the meeting at which it shall be acted upon, and the applicant shall pay such fee and pass such final examination as shall be prescribed by the Board of Governors. Otherwise no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Board of Governors followed by a ballot of the Institute either in person or by mail. Confidential notice of such recommendation shall be mailed by the Secretary to every Fellow, and after twenty days, if not more than seven negative votes and affirmative votes of not less than half the total number of Fellows have been cast, the applicant shall become a Fellow.

CONTRIBUTING MEMBERS

ART. XI. Contributing Members shall be legal reserve Life Insurance Companies applying for such membership and elected thereto by a majority vote of the Board of Governors. Contributing Members shall have the use jointly with other Members of the Library under the rules of the Institute governing the same. Any person nominated by a Contributing Member shall have the privileges of Associates at any of the annual or special meetings of the Institute except during executive sessions.

QUORUM

ART. XII. Four members of the Board of Governors shall constitute a quorum; at any duly called meeting of the Institute fifteen Fellows shall constitute a quorum.

EXPULSION OR SUSPENSION OF MEMBERS

ART. XIII. The Board of Governors shall receive and hear all complaints preferred by any Fellow of this Institute against any other Fellow or Associate for misconduct in his relations to the

Institute, or in his profession, or the practice thereof, or affecting the interests of the actuarial profession, provided such complaints shall be in writing plainly and specifically stating the matter complained of, and subscribed by the complainant.

The proceedings of the Board of Governors upon such complaint shall be deemed confidential and kept secret except so far as written or printed reports of the same shall be necessarily and officially made in the Institute.

Each party may appear personally and by counsel. Witnesses shall vouch for the truth of their statements on their word of honor. The Board of Governors shall make its decision, and if it finds the complaint to be true and of sufficient importance, it will so report to the Institute with its recommendations.

The Institute will take such action on the report as it shall see fit; but no member shall be expelled or suspended unless by a vote of at least two-thirds of the members present and voting.

ART. XIV. No resolution expressive of opinion shall be entertained at any meeting.

AMENDMENTS

ART. XV. Upon the written request of five Fellows of the Institute, the Secretary shall forthwith transmit by mail to the Fellows of the Institute any proposed amendment to this Constitution, and such amendment shall become effective provided it shall receive the favorable vote of two-thirds of the Fellows present and voting at the duly called meeting of the Institute next succeeding the mailing of such notice by the Secretary, and held not less than thirty days after the mailing of such notice.

BY-LAWS

ARTICLE I. *Order of business.*—At a meeting of the Institute the following order of business shall be observed, unless the Institute votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.
4. Summary of the business transacted by the Board since the last meeting of the Institute, presented for its information and approval.

5. Candidates for Fellowship considered and voted upon.
6. Reports of Committees and Officers.
7. At annual meeting, election of Officers and Governors.
8. Unfinished business; new business.
9. Reading of papers prepared for presentation to the Institute.
10. Discussions of papers presented.

The Board of Governors' meetings shall proceed in this order as far as it may apply to them.

ART. II. *Meetings of the Board of Governors* shall be called whenever the President or four members of the Board of Governors so request, but not without sending notice to each member of the Board ten or more days before the time appointed. Such notice shall state the subjects intended to be brought before the meeting; should other matters be passed upon, any member of the Board shall have the right to reopen the same at the next meeting.

During any interim between meetings of the Board of Governors the President may, and upon written request by three members of the Board, the President or Secretary shall submit to the Board by personal communication in writing, mailed or delivered to each member of the Board, any matter of business requiring consideration or action by the Board. The votes of the Board, communicated in like form and manner to the President or Secretary, shall have the same effect as though voted at a regular or called meeting, provided no member of the Board votes against such action or in favor of postponement to a subsequent meeting of the Board.

ART. III. *Duties of officers.*—The President, or in his absence the Vice-President, shall preside at meetings of the Institute and of the Board. At the Institute meetings the presiding Officer shall vote only in case of a tie, but at the Board meetings he may vote in all cases.

The Secretary shall keep a full and accurate record of the proceedings at the meetings of the Institute and of the Board, send out calls for the said meetings, and, with the approval of the President, carry on the correspondence of the Institute. Subject to the direction of the Board, he shall have immediate charge of the office and archives and other chattels of the Institute.

The Editor of *The Record*, under the general supervision of the Board, shall have charge of all matters connected with editing and printing the Institute publications. There shall be a publication

known as *The Record*, which shall contain only the proceedings of the meetings, original papers presented by Fellows and written by Fellows or Associates, and such discussions or other matters as may be expressly authorized by the Board of Governors.

The Treasurer shall send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Board of the Institute; keep a detailed account of all receipts and expenditures, and present an abstract of same at the annual meetings after it has been audited by a Committee of the Board.

ART. IV. *Dues.*—Each Fellow shall pay fifteen dollars and each Contributing Member twenty-five dollars upon entrance and a like amount on the date of each annual meeting thereafter, except in the case of Fellows not residing on this continent who shall pay seven dollars and fifty cents annually at the time stated. Each Associate shall pay five dollars upon entrance and a like amount on the date of each annual meeting thereafter until five such payments in all shall have been made. Beginning with the sixth annual meeting after the admission of an Associate as such the dues of any Associate heretofore or hereafter admitted shall be the same as those of a Fellow and shall be payable at the times when dues of Fellows are payable.

It shall be the duty of the Treasurer to notify by mail any Fellow, Associate, or Contributing Member whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow, Associate, or Contributing Member shall fail to pay such dues within three months from the time of mailing such notice, the Treasurer shall report the delinquent to the Board, by whom the name of such member shall be stricken from the rolls, and such member shall cease to be a Fellow, Associate, or Contributing Member of the Institute. Such Member may, however, be reinstated by vote of the Board and upon payment of arrears of dues.

And also except that any person who shall have attained the age of seventy years while a Fellow of the Institute, having been a Fellow on the date of the first annual meeting or otherwise for a period of at least twenty years, shall enjoy all the privileges of membership without further payment of dues.

ART. V. *Designation by initials.*—Fellows of the Institute are authorized to append to their names the initials F.A.I.A., and

Associates are authorized to append to their names the initials A.A.I.A.

Art. VI. *Amendments.*—Upon the written request of five Fellows of the Institute, the Secretary shall forthwith transmit by mail to the Fellows of the Institute any proposed amendment to the By-Laws, and such amendment shall become effective provided it shall receive the favorable vote of two-thirds of the Fellows present and voting at the duly called meeting of the Institute next succeeding the mailing of such notice by the Secretary, and held not less than thirty days after the mailing of such notice.

REQUIREMENTS *for* ADMISSION
TO THE
AMERICAN INSTITUTE *of* ACTUARIES

ASSOCIATESHIP

Any person pursuing actuarial studies and looking forward to a future Fellowship may be nominated for Associateship by two Fellows of the Institute on a blank form that will be furnished by the Secretary on request. The nomination paper must state which parts of the examinations the candidate will write at the next examination session and be mailed to the Secretary with the examination fee **not later than January 31** preceding the examination date. If the nomination is approved by the Board of Governors the candidate will be notified of the place and time of examination. If not approved, the fee will be refunded.

On passing one or more parts the candidate will be listed as a Student and his name continued on the list as long as he has at one of the last three examination sessions passed one or more parts.

Upon passing the Associateship examinations the candidate subject to final approval by the Board will be enrolled as an Associate of the Institute upon payment of dues for the current year provided he has reached the age of twenty-one years and has had such practical experience as in the judgment of the Board qualifies him for such enrolment. If at the time of passing the final examinations the candidate is less than twenty-one years of age he cannot be enrolled until attaining that age and until the Board of Governors has again approved his application. If on the date of passing the last of the Associateship examinations the candidate has not had such practical experience as in the judgment of the Board of Governors qualifies him for enrolment he cannot be enrolled as an Associate unless nor until he shall have completed such experience within three years and his application has again been approved by the Board of Governors. The experience usually required is at least one year of actuarial work of a kind to be approved by the Board.

FELLOWSHIP

Any Associate may apply for Fellowship on a blank form that will be furnished by the Secretary. This application must state which parts of the Fellowship examinations the candidate will write at the next examination session and be mailed to the Secretary with the examination fee **not later than January 31** preceding the examination date. Upon passing all of the Fellowship examinations the candidate may be admitted as a Fellow by final action of the Board and payment of the dues for the current year except that if less than twenty-five years of age such final action will not be taken until the candidate has attained that age.

EXAMINATIONS

Examinations are held unless otherwise ordered on the second Tuesday and the following Wednesday in April of each year at points within as convenient reach of the candidates as is possible. The schedule of examinations is as follows:

Tuesday, A.M.	Parts 1 and 5 of the Associateship and Part 9 of the Fellowship.
Tuesday, P.M.	Parts 2 and 6 of the Associateship and Part 10 of the Fellowship.
Wednesday, A.M.	Parts 3 and 7 of the Associateship and Part 11 of the Fellowship.
Wednesday, P.M.	Parts 4 and 8 of the Associateship and Part 12 of the Fellowship.

Candidates nominated for Associateship must designate in the nomination paper the parts they intend to write at the next examination. Similarly Associates applying for Fellowship must designate in the application the parts they intend to write at the next examination. At other times candidates desiring to write examination papers are required to file a written request with the Secretary **not later than January 31** preceding the examination date, specifying by number the parts the candidate desires then to write, accompanied by the examination fee.

The several parts of the examination must be taken in sequence. No candidate may present himself for any part unless he has previously passed or is then presenting himself for all previous parts. For example, a candidate who passed Parts 1, 3, and 4 but

failed in Part 2 may sit at the next examination for Parts 2 and 5 but not for 6 because of the conflict in time, and not for 7 because out of sequence. No candidate may take any part of the Fellowship examinations before he has passed all of the Associateship examinations. The Board has the right to deny the privilege of examination in its discretion to any one at any time.

EXAMINATION FEES AND MEMBERSHIP DUES

The examination fee required to accompany the nomination for Associateship, the application for Fellowship or an interim request for examination shall be $2 for each part but not less than a minimum fee of $5; thus, for one or two parts $5; for three parts $6 and for four parts $8. Examination fees will not be refunded as to any parts requested to which the candidate is admitted and duly notified of the place and date of examination unless a written request for refund is received by the Secretary on or before February 28 preceding the examination date.

The membership dues are, Fellows $15; Associates $5; payable on admission as such and a like amount at each annual meeting thereafter, except that beginning with the sixth annual meeting after the admission of an Associate as such the dues of any Associate then and thereafter shall be the same as those of a Fellow; provided, however, that for Fellows not residing on this Continent the dues shall be one-half of the amount otherwise payable.

SYLLABUS *of* EXAMINATIONS

OF THE

AMERICAN INSTITUTE *of* ACTUARIES

SUBJECTS

ASSOCIATESHIP

Part 1. Elementary Algebra
Plane Geometry
Part 2. Plane Trigonometry
Plane Analytical Geometry
Part 3. Advanced Algebra
Probability
Part 4. Differential and Integral Calculus
Finite Differences
Statistics
Part 5. Compound Interest
Accounting
Part 6. Life Contingencies Involving Single Life Functions Only
Part 7. Life Contingencies Involving Joint Life Functions
Part 8. Construction of Actuarial Tables
Multiple Decrement Tables
Disability and Accident Contingencies
History of Life Insurance

FELLOWSHIP

Part 9. Mortality and Disability Investigation
Selection of Risks and Treatment of Substandard Risks
Disability and Accident Contingencies. (Required only of candidates that passed Part 8 prior to 1926)
Part 10. Gross Premiums
Valuation of Liabilities
Non-Forfeiture Values and Change or Reinstatement of Life Insurance Contracts
Part 11. Analysis and Distribution of Surplus. (Candidates that passed Part 10 prior to 1926 are exempt from this subject)
Life Insurance Accounts

Investments and Valuation of Assets
Banking and Finance
Part 12. Life Insurance Law. (Candidates that passed Part 11 prior to 1926 are exempt from this subject)
Preparation of Policy Forms
Pension Funds
General Topics of Actuarial Interest

SCOPE OF EXAMINATIONS AND RECOMMENDED COURSE OF STUDY

ASSOCIATESHIP

Part 1

Elementary Algebra

—Hall and Knight: *Algebra for Colleges and Schools*, chaps. 1–35, and 37.

Plane Geometry

—Phillips and Fisher: *Elements of Geometry*, Books 1–3.

Part 2

Plane Trigonometry

—Hall and Knight: *Elementary Trigonometry*, chaps. 1–16.

Plane Analytical Geometry

—Smith and Gale: *Introduction to Analytic Geometry*, chaps. 1–5.

Part 3

Advanced Algebra

—Hall and Knight: *Higher Algebra*, chaps. 1–17, 21, and 29.

Probability

—Hall and Knight: *Higher Algebra*, chap. 32; Henry: *Calculus and Probability*, chap. 21.

Part 4

Differential and Integral Calculus

—Granville: *Elements of the Differential and Integral Calculus*, chaps. 1–6 (to end of Sec. 65), 7, 8, 11, 13 (to end of Sec. 107), 15, 17, 18 (to end of Sec. 147), 22, 24–27 (to end of Sec. 199), and 28 (to end of Sec. 210); Henry: *Calculus and Probability*, chaps. 1, 10–14, and 16–19.

Finite Differences

—Henry: *Calculus and Probability*, chaps. 2–9, 15, and 20; Rietz and others: *Handbook of Mathematical Statistics*, chap. 3 (omitting "Graduation"); *J.I.A.*, L, 15–33 (D. C. Fraser: "Notes on Finite Differences").

Statistics

Does not include graduation or curve fitting.

—Yule: *An Introduction to the Theory of Statistics*, chaps. 6–17; Rietz and others: *Handbook of Mathematical Statistics*, chaps. 1, 2, 5 (omitting "Test of Goodness of Fit"), 6 and 8–12; *R.A.I.A.*, XIII, 214–250 (H. L. Rietz: "Certain Applications of Statistics").

PART 5

Compound Interest

—Rietz, Crathorne, and Rietz: *Mathematics of Finance*, chaps. 1–6; Todhunter: *Institute of Actuaries' Text-Book, Part I*, chaps. 1–7 and 10.

Accounting

—Bogle: *Comprehensive Bookkeeping;* Dickinson: *Accounting Practice and Procedure*, chaps. 1–8.

PART 6

Life Contingencies Involving Single Life Functions Only

—Spurgeon: *Life Contingencies*, chaps. 1–4, 5 (omitting Arts. 24, 25), 6–9, 10 (from Art. 16), 11 (to Art. 13) and 12; Robertson and Ross: *Actuarial Theory*,* in Part II such sections of chaps. 1–16 and 18 as do not involve joint life functions, omitting Art. 29 and examples 24–26 of chap. 7; *J.I.A.*, LIV, 123–149 (P. H. McCormack: "Damaged Lives and Options").

PART 7

Life Contingencies Involving Joint Life Functions

—Spurgeon: *Life Contingencies*, chaps. 13–19; Robertson and Ross: *Actuarial Theory*,* in Part II such sections of chaps. 1–16 and 18 as involve joint life functions.

* Where *Actuarial Theory* in Part II refers to the "Text-Book," the *Institute of Actuaries' Text-Book, Part II*, is meant; inasmuch as the latter has been replaced by Spurgeon: *Life Contingencies*, the references are to be ignored.

PART 8

Construction of Actuarial Tables

—Todhunter: *Institute of Actuaries' Text-Book, Part I*, chap. 8; Spurgeon: *Life Contingencies*, chap. 20.

Multiple Decrement Tables

—Spurgeon: *Life Contingencies*, chap. 21.

Disability and Accident Contingencies

Only such disability and accident benefits as are included in life insurance policies.

—*T.A.S.A.*, XXIV, x–xii ("Disability Symbols"); Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, pp. 18–22 and 32–56; *T.A.S.A.*, XII, 66–71 ("Hunter's Disability Table"†); XVII, 211, 212, and

214–240 (M. Davis: "Disability Benefits"); XXIV, 92, and 99–125 (E. H. Hezlett: "Disability Benefits").

—For the theory of accident contingencies relating to benefits included in life insurance policies the published material is meager. Candidates should learn the practice of several companies in calculating double indemnity net premiums and reserves for individual policies on the common plans of insurance. Some help may be derived from *T.A.S.A.*, XIX, 82, 83, and 85 (W. A. Hutcheson: "Double Indemnity"); *R.A.I.A.*, VIII, 119 (H. W. Buttolph: "Death from Accident").

† It is important to change the headings of this table as indicated in Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, p. 36.

History of Life Insurance

—*T.A.S.A.*, XXI, 331–364 (W. A. Hutcheson: "Evolution of Life Insurance"); Maclean: *Life Insurance*, pp. 342–367 ("Historical Development of Life Insurance in the United States").

FELLOWSHIP

Part 9

Mortality and Disability Investigation

The principles to be observed in the investigation of mortality and disability experience and in the compilation and graduation of tables derived therefrom; also the sources, methods, and characteristics of the principal investigations that have been made and of the corresponding tables.

—Mortality. —Investigation of Experience and Compilation of Tables: Murphy and Papps: *Construction of Mortality Tables from the Records of Insured Lives; J.I.A.*, XLIII, 23–79 (C. H. Wickens: "Methods of Ascertaining Rates of Mortality amongst the General Population"); *T.A.S.A.*, XVIII, 260–286 (H. H. Wolfenden: "United States Census"); *J.I.A.*, XLVIII, 294–303 (G. King: "Abridged Mortality Table"); LIII, 104–166 (G. King: "Short Method of Constructing Select Mortality Tables").

—Graduation: Rietz and others: *Handbook of Mathematical Statistics*, pp. 58–70, 78–81, and 92–119; Hardy: *The Theory of the Construction of Tables of Mortality, etc ;* Henderson: *Graduation of Mortality and Other Tables; J.I.A.*, XXVI, 77–120 (T. B. Sprague: "Graphic Method of Adjusting Mortality Tables"); *T.A.S.A.*, XIV, 256–276 (J. B. Maclean: "Graduation by the Summation Method"); *J.I.A.*, L, 251–259 (W. P. Elderton: "Graduation by Makeham's Hypothesis"); *T.A.S.A.*, XXIV, 398–413 (F. B. Gerhard: "Makeham's Formula by Graphic Method").

—Principal Investigations and Tables: Moir: *Sources and Characteristics of the Principal Mortality Tables; Medico-Actuarial Mortality Investigation*, Vols. I–V; *American-Canadian Mortality Investigation, 1900–1915*, Vols. I and II; *T.A.S.A.*, XXI, 178–206 (V. Howell: "Two Graduations of American-Canadian Mortality Experience"); *United States Life Tables, 1890*,

1901, 1910, and 1901–1910, pp. 17–21 and 203–239; *T.A.S.A.*, XXIII, 269–273 ("Review of U.S. Life Tables"); XXI, 157–177 (A. Hunter: "Mortality of American Annuitants") and 535–543 (Discussion); *J.I.A.*, LIV, 43–90 and 269–284 (W. P. Elderton and H. J. P. Oakley: "Mortality of British Offices' Life Annuitants," 1900–1920); LV, 144–159 ("Mortality of British Government Life Annuitants," 1900–1920) and 160–177 (D. C. Fraser: "Notes on Recent Reports on the Mortality of Annuitants"); *T.A.S.A.*, XVIII, 311–338 (H. Moir: "Mortality Graphs"); XIX, 148–167 (Discussion); XXIII, 408–434 (H. Moir: "Recent Mortality"); *J.I.A.*, LVI, 199–202 ("Joint Mortality Investigations in America"); *R.A.I.A.*, XIII, 348–353 (Discussion); *Proceedings of the Seventh International Congress of Actuaries*, Vol. I, 527–552 (J. Burn and W. C. Sharman: "Changes in the Rates of Mortality amongst Assured Lives during the Past Century"); *T.A.S.A.*, XXII, 209–221 (J. B. Maclean: "Swedish Mortality Investigation"); XXI, 207–211 (S. A. Joffe: "Mortality Experience of Life Insurance Companies in Germany") and 378–391 (W. A. Hutcheson: "Mortality Experience of Mutual Life"); XXIV, 53–91 (M. Davis: "Mortality Experience of Equitable Life"); XXII, 466–482 (F. H. Johnston: "Convertible Term Business"); XXIV, 304–311 (P. C. H. Papps: "Mortality Experience under Term Policies and under Policies Issued at Old Ages"), 325–333 (E. B. Morris: "Comparison of Mortality Elements between Group and Regular Life Insurance") and 334–372 (E. E. Cammack: "Recent Developments in Group Insurance"; XIX, 53–59 (P. H. Evans: "Mortality by Order of Birth") and 319–326 (Discussion).

—Disability.—Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, pp. 1–32; *T.A.S.A.*, XII, 44–55 (A. Hunter: "Waiver of Premium on Permanent Disability"); XXII, 320–327 (W. A. Hutcheson: "Account of Friendly Societies"); XIX, 219–226 (A. Hunter: "Claims for Disability Benefits"); XX, 182–194 (Discussion); XXII, 200–208 (J. S. Thompson: "Causes of Total and Permanent Disability") and 442–465 (A. Hunter: "Claims for Permanent Total Disability"); XXIV, 92–99 (E. H. Hezlett: "Disability Benefits"); Watson: *An Account of an Investigation of the Sickness and Mortality Experience of the I.O.O.F. Manchester Unity, 1893–1897*, pp. 1–80.

Selection of Risks and Treatment of Substandard Risks

—Blank Application and Medical Examiner's Report of Several Companies; Maclean: *Life Insurance*, pp. 148–166 ("Selection"); *T.F.A.*, VIII, 103–162 (L. P. Orr: "Selection"); *T.A.S.A.*, XII, 1–17 (A. Hunter: "Selection"); 281 298 (Discussion); XX, 273–332 (A. Hunter and Dr. O. H. Rogers: "Numerical Rating"); XXIII, 322–346 (R. D. Murphy: "Selection").

—*T.A.S.A.*, XVII, 281–289 (A. Hunter and Dr. O. H. Rogers: "Need of Standards for Border-Line Risks"); XVIII, 164–169 (Discussion); Maclean: *Life Insurance*, pp. 167–182 ("Substandard Risks"); *T.A.S.A.*, XVI, 64-82 (A. Hunter: "Underaverage Lives") and 336–370 (Discussion); *R.A.I.A.*, XI, 158–186 (F. B. Mead: "Substandard Insurance"), 187–204 (A. Hunter: "Underaverage Business") and 204–262 (Discussion); X, 32–38 (A. Coburn: "Substandard Life Insurance") and 205–230 (Discussion); *T.A.S.A.*, XXIII, 114–160 (J. E. Hoskins: "Incidence of Substandard Mortality"); XXV, 11–28 (V

Howell: "Substandard Practice") and 281–292 (Discussion); *R.A.I.A.*, XIV, 42–67 (E. E. Cammack: "Substandard Business").

—*T.A.S.A.*, XXI, 365–377 (A. Hunter: "Is the Average the Same as the Normal?"); XV, 62–76 (A. Hunter and Dr. O. H. Rogers: "Interpretation of Results of M. A. Investigation") and 417–430 (Discussion); XXI, 16–63 (A. Hunter and Dr. O. H. Rogers: "Occupation"); XX, 353–363 (Discussion of "Overweight"); XXII, 512–531 ("Joint Committee Report on Height and Weight") and 342–384 (A. Hunter and Dr. O. H. Rogers: "Ratings for Principal Impairments"); XX, 83–100 (A. Hunter and Dr. O. H. Rogers: "Heart Murmurs") and 333–352 (Discussion); XXII, 532–537 ("Joint Committee Report on Heart Murmur and Albuminuria"); XXV, 210–218 (A. Hunter and Dr. O. H. Rogers: "Heart Impairments"); XXII, 538–540 ("Joint Committee Report on Family History"); XXIII, 347–367 (A. Hunter and Dr. O. H. Rogers: "Inflammatory Rheumatism, Tuberculosis and Blood Spitting"); XXI, 151–156 (O. W. Perrin and Dr. H. Toulmin: "Asthma, Renal Colic and Pleurisy"), 528–534 (Discussion) and 452–477 (J. D. Craig: "Colored Lives"); XXII, 283–285 (Discussion); *R.A.I.A.*, XIV, 86–89 (W. F. Poorman: "Aviation Hazard"); *T.A.S.A.*, XIX, 235–238 (P. H. Evans: "Habits"); XX, 197–206 (Discussion).

—Maclean, *Life Insurance*, pp. 255–257 ("Selection for Disability Benefits"); *T.A.S.A.*, XIX, 226 and 227 (A Hunter: "Selection for Disability Benefits"); XXIII, 394–396 (J. M. Laird: "Disability Underwriting"); XVII, 264–267 and 271–273 (W. J. Graham: "Selection for Group Insurance"); XXII, 120–139 (D. E. Kilgour: "Insurance without Medical Examination"), 140–144 (A. Hunter: "Savings in Expense Compared with Expected Extra Mortality under Insurance without Medical Examination") and 145–174 (Discussion); *R.A.I.A.*, XIV, 116–140 (Discussion of "Non-Medical Business").

Disability and Accident Contingencies

Required only of candidates that passed Part 8 prior to 1926.

—See Part 8.

Part 10

Gross Premiums

Does not include substandard premiums.

—Maclean: *Life Insurance*, pp. 80–86 ("Premiums"); Spurgeon: *Life Contingencies*, Arts. 24 and 25 of chap. 5; *T.F.A.*, II, 207–259 and 262–268 (H. Moir: "Office Premiums"); VII, 262–273 (W. Denham and C. K. Granger: "Office Premiums"); *T.A.S.A.*, XX, 379–409 (E. E. Cammack: "Premiums for Non-Participating Life Insurances"); XXI, 233–248 (Discussion); *J.I.A.*, XLVIII, 45–64 (F. J. Cameron: "Office Premiums for Whole Life Non-Profit Assurances"); *T.A.S.A.*, XVI, 272–278 (E. E. Rhodes: "Premium Loadings and Expense Limitations"); XX, 23–71 (A. Hunter: "Should the American Men Mortality Table Be the Basis for Premiums and Reserves?"); XXI, 175–177 (A. Hunter: "Premiums on Basis of New Annuity Rates") and 535–543 (Discussion); XXII, 49–52 (J. D. Buchanan: "Industrial Premiums"); *J.I.A.*, XXXVI, 51–81 (A. Hunter: "Effect of Using Lapse Element"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 97 ("Limitation of Expenses") and 85

("Actual Premium Less than Net Premium"); insurance laws of Wisconsin, Sec. 1950*m*, *n*, *o*, and *p* ("Limitation of Expense Provision in Premiums and of Expenses").

Valuation of Liabilities

—Method of at least one company in determining each item of liability in the "convention blank" annual statement; Maclean: *Life Insurance*, pp. 225–227 ("Liabilities"); Wolfe: *The Examination of Insurance Companies*, chaps. 10, 11, 14, and 15; Spurgeon: *Life Contingencies*, chaps. 10 (to Art. 15) and 11 (from Art. 14); *T.A.S.A.*, XXV, 219–245 (E. W. Marshall: "Office Methods of Obtaining Policy Reserve"); IX, 150–158 (H. Moir: "Group Valuation and Mortality Register") and 269 and 270 (Discussion); XI, 144–162 (R. Henderson: "Valuation by Attained Age") and 375–379 (Discussion); XXII, 43–47 (J. D. Buchanan: "Industrial Valuation").

—Maclean: *Life Insurance*, pp. 91–96 and 101–123 ("Reserves"); *J.I.A.*, XXXVII, 91–99 (W. A. Hutcheson: "American Valuation Tables"); *T.A.S.A.*, X, 179–200 (H. Moir: "Valuation"); XXIV, 312–324 (R. B. Robbins: "Reserve Standards for Life Insurance"); XXI, 125–133 (A. A. Welch: "Preliminary Term Valuation") and 503–514 (Discussion); *T.F.A.*, VII, 103–126 (A. T. Maclean: "Valuation with Allowance for Initial Expense"); *R.A.I.A.*, VIII, 22–25 (H. W. Buttolph: "Individual Reserves in Life Insurance").

—Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, pp. 57 and 58; *T.A.S.A.*, XXV, 80–94 (F. A. Shailer: "Approximate Methods of Valuation") and 327–333 (Discussion).

—Baldwin: *New York Insurance Law Annotated 1925*, § 84 ("Valuation").

Non-Forfeiture Values and Change or Reinstatement of Life Insurance Contracts

—Maclean: *Life Insurance*, pp. 126–136 ("Policy Provisions Relating to the Payment of Premiums"); *R.A.I.A.*, XI (Part I), 12–38 (P. H. Evans: "Surrender and Non-Forfeiture Values"); *J.I.A.*, XXXV, 199–241 (F. W. Fulford: "Surrender Values"); XLVIII, 71–81 (F. J. Cameron: "Surrender Values for Whole Life Non-Profit Assurances"); L, 313 and 314 (A. Fraser: "Surrender Values of Altered-Class Policies"); *T.A.S.A.*, XIII, 259–272 (J. F. Little: "Modern Surrender Values"); XIV, 128–151 (Discussion); *R.A.I.A.*, VII, 87–98 (Discussion of "Surrender Values in Policies on Substandard Lives"); XIII, 251–282 (F. S. Withington: "Using Standard Values for Substandard Policies"); XIV, 105–115 (Discussion); *T.A.S.A.*, X, 608 (E. E. Rhodes: "Conclusions from Mutual Benefit Experience under Extended Insurance"); XI, 73–98 (Discussion); XXIV, 386–397 (W. P. Barber: "Automatic Premium Loans"); XXV, 104–110 (Discussion); Baldwin: *New York Insurance Law Annotated 1925*, §§ 88 ("Surrender Value of Lapsed or Forfeited Policies") and 92 ("No Forfeiture without Notice").

—*T.F.A.*, II, 259–262 (H. Moir: "Changes in Policies"); *T.A.S.A.*, XI, 465–496 (W. Young: "Alteration of Life Insurance Contracts"); XII, 121–133 (Discussion); XXIII, 204–222 (J. S. Thompson: "Changes in Policy Con-

tracts"); XXV, 253–271 (J. E. Hoskins: "Office Practice on Changes in Policy Form").

—Maclean: *Life Insurance*, pp. 136 and 137 ("Reinstatement"); *T.A.S.A.*, XI, 296–303 (H. Moir: "Reinstatement of Policy on Impaired Risk") and 550–552 (Discussion); XVI, 284–296 (A. Hunter: "Reinstatement"); XVII, 134–139 (Discussion); XVIII, 246–259 (R. D. Murphy: "Reinstatement"); XIX, 127–133 (Discussion).

PART II

Analysis and Distribution of Surplus

Candidates that passed Part 10 prior to 1926 are exempt from this subject.

—Maclean: *Life Insurance*, pp. 183–199 ("Dividends"); *R.A.I.A.*, XI (Part I), 106–141 (J. C. Rietz: "Methods of Surplus Distribution"); XII, 35–65 (A. Coburn: "Distribution"); *T.A.S.A.*, XI, 49–54 (P. C. H. Papps: "Original Contribution Plan"); X, 200–223 (H. Moir: "Surplus Distribution"), 359–361 (Discussion) and 529–536 (Discussion); IX, 310–326 (R. W. Weeks: "Practical Rule for Calculating Annual Dividends"); X, 74–87 (Discussion); IX, 344–346 (M. M. Dawson: "A Distribution Formula"); X, 101–112 (Discussion); *J.I.A.*, XLIV, 415–452 (H. J. Rietschel: "Expenses"); LIII, 167–191 (C. H. Maltby: "Effect of Valuation Bases upon Surplus"); *T.A.S.A.*, XXIII, 83–113 (J. S. Elston: "Expense Investigations"); XXIV, 14–52 (A. Hunter: "Methods of Distributing Annual Dividends") and 291–298 (R. Henderson: "Equity in Surplus Distribution"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 83 ("Distribution of Surplus to Policyholders") and 87 ("Contingency Reserve").

—*T.A.S.A.*, XV, 40–48 (D. P. Fackler: "Annuities with Participation") and 391–408 (Discussion); XVI, 279–283 (D. P. Fackler: "Annuities with Participation"); XVII, 122–133 (Discussion); XV, 354–363 (C. C. Ferguson: "Participating Annuities"); XVI, 202–213 (Discussion).

—*R.A.I.A.*, XIV, 21–28 (E. G. Fassel: "Excess Interest under Annuity-Certain").

Life Insurance Accounts

—Method of at least one company in determining each item of the "convention blank" annual statement with schedules, except liabilities; Maclean: *Life Insurance*, pp. 218–225 and 227–235 ("Annual Statement, except Liabilities"); *T.A.S.A.*, XV, 328–353 (R. D. Murphy: "Life Insurance Bookkeeping and Annual Statements"); XVI, 197–201 (Discussion); Wolfe: *The Examination of Insurance Companies*, chaps. 6–8, 16–18, and 20; *Proceedings of the Insurance Institute of Toronto, 1922–1923*, pp. 84–128 (W. G. Reburn: "Life Assurance Bookkeeping"); *R.A.I.A.*, XIV, 68–85 (V. Howell: "Financial Statements"); IV (Part II), 79–88 (Discussion of "The Policy Exhibit"); VI, 104 and 106–111 (Discussion of "Excess Credits over Reserve"); XIV, 146–159 (Discussion of "New Gain and Loss Exhibit"); *T.A.S.A.*, XX, 101–111 (J. W. Tebbetts: "Gain and Loss on Annuity Business"), 112–133 (E. G. Fassel: "Formulae Used by

Various Companies in Computing Expected Mortality") and 471–487 (Discussion); XXI, 134–150 (J. S. Elston: "Treatment of Disability Claim Transactions in the Annual Statistics") and 515–527 (Discussion); Baldwin: *Insurance Law of New York Annotated 1925*, §§ 44, 45, and 103 ("Reports").

Investments and Valuation of Assets

—Jordan: *Investments*, chaps. 1–19 and 21–27; Maclean: *Life Insurance*, pp. 200–217 ("Assets"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 16–20*a* and 100 ("Investments"); Wolfe: *Examination of Insurance Companies*, chaps. 2–5 and 9; *T.F.A.*, IX, 1–38 (R. Murrie: "Investments of Insurance Offices"); *J.I.A.*, LVI, 121–152 (C. R. V. Coutts: "Life Assurance Investment Policy"); *R.A.I.A.*, XII, 96–150 (A. T. Maclean: "Course of Rate of Interest"); *T.F.A.*, VII, 171–213 (H. Moir: "Amortization of Bonds and Valuation of Assets"); *T.A.S.A.*, XXI, 95–117 (J. B. Gibb: "A Method of Valuing Bonds of a Life Insurance Company").

Banking and Finance

—Shearman: *Practical Economics;* Holdsworth: *Money and Banking* (Fourth Edition), chaps. 1–22 (omitting Appendix); Dewing: *Corporation Finance*, chaps. 1–5, 14–19, and 23–26.

PART 12

Life Insurance Law

Candidates that passed Part 11 prior to 1926 are exempt from this subject.

—Maclean: *Life Insurance*, pp. 323–341 ("Legal Aspects of Life Insurance"); *Corpus Juris*, §§ 1–756 of article on "Insurance," §§ 1–368 of article on "Life Insurance," §§ 1–54 of article on "Annuities," §§ 204–211 of article on "Bankruptcy," §§ 1–210 of article on "Accident Insurance" and § 195 of article on "Infants," omitting footnotes and matter that does not apply to the business of fixed premium life insurance companies; *Legal Notes* in last five volumes of *T.A.S.A.*, noting that in some cases the decisions do not accord with generally accepted doctrines or the law in other jurisdictions.

—Maclean: *Life Insurance*, pp. 307–322 ("Regulation and Taxation"); Baldwin: *New York Insurance Laws Annotated 1925*, §§ 1–15, 21–26*a*, 28–43, 46–82, 86, 89–91, 94–96*a*, 98, 99, 101*a*, 101*c*, 101*d*, 102, 104, 106 and Appendix (Tax Law, §§ 187–197), omitting all matter that does not apply to the business of fixed premium life insurance companies; *Regulations 65 Relating to the Income Tax under the Revenue Act of 1924*, §§ 242–247 and Arts. 661–693.

—At the option of the candidate, **either:** Matter relating to the business of fixed premium life insurance companies in the insurance laws of Illinois, Minnesota, and Ohio; **or:** *Proceedings of the Insurance Institute of Toronto, 1924–1925*, pp. 185–210 (H. J. Sims: "The Uniform Life Insurance Act, Canada"), *T.A.S.A.*, XXVI (Part I), "Address of the President" (A. B. Wood: "Uniform Life Insurance Act"), and matter relating to the business of fixed premium life insurance companies in the insurance laws of Ontario and the Dominion of Canada.

In the case of New York, a thorough knowledge of the laws is required, but for Illinois, Minnesota, and Ohio, or for Ontario and the Dominion of Canada, only a general knowledge is necessary.

Preparation of Policy Forms

—Insurance and annuity policy forms on the principal plans of at least one company that incorporates disability and double indemnity benefits in life policies, and the more important indorsements and riders used at issue or on subsequent change; Maclean: *Life Insurance*, pp. 124–147 ("Terms of the Policy"); *R.A.I.A.*, X, 118–126 (Discussion of "Corporation and Partnership Insurance"); *T.A.S.A.*, XII, 175–191 (H. Moir: "Liberality of Modern Policies"); XIII, 105–118 (Discussion); XVIII, 63–71 (O. W. Perrin: "On Modes of Settlement under Policy Provisions") and 383–393 (Discussion); XVII, 263, 264, 267, and 268 (W. J. Graham: "Group Insurance"); XXII, 52–56 (J. D. Buchanan: "Industrial Policy Contract"); Baldwin: *New York Insurance Laws Annotated 1925*, §§ 101, 101*b* and 101½ ("Standard Provisions").

Pension Funds

—*T.F.A.*, IV, 195–228 (J. J. M'Lauchlan: "Fundamental Principles of Pension Funds"); *J.I.A.*, XLV, 149–229 (H. W. Manly: "On Staff Pension Funds"); XXXIX, 129–206 (G. King: "On Staff Pension Funds"); *T.F.A.*, VII, 215–258 (J. J. M'Lauchlan: "On the Construction of Salary Scales and the Valuation of Prospective Pensions"); *J.I.A.*, LI, 106–117 and 233–244 (W. J. H. Whittall: "Pension Problems in America"); *T.F.A.*, II, 307–342 (V. Marr: "Widows' Funds"); *J.I.A.*, LII, 1–37 (C. E. Howell: "Widows' Funds"); *R.A.I.A.*, III, 33–52 (H. L. Rietz: "Status of Certain Current Pension Funds"); VIII, 1–12 (H. L. Rietz: "Pensions for Public Employees"); X, 1–14 (H. L. Rietz: "Pensions for Insurance Company Employees"); *T.A.S.A.*, XVI, 109–144 (D. A. Walker: "A Staff Pension Fund") and 388–395 (Discussion); XXI, 77–94 (J. B. Maclean: "Problems of Small Pension Funds"); XV, 306–314 (M. M. Dawson: "Mortality and Remarriage Tables"); XVI, 176–183 (Discussion).

General Topics of Actuarial Interest

—Riegel and Loman: *Insurance Principles and Practices*, chaps. 1–4 and 10–23; Maclean: *Life Insurance*, pp. 1–31 ("Life Insurance Systems and Companies") and 259–306 ("Group Insurance, Industrial Insurance and Internal Organization"); *T.A.S.A.*, XXII, 317–339 (W. A. Hutcheson: "Review of Disablement Insurance"); *J.I.A.*, LIV, 150–203 (P. N. Harvey: "National Health Insurance"); *T.A.S.A.*, XIX, 79–87 (W. A. Hutcheson: "Double Indemnity Clauses in Life Insurance Contracts") and 332–344 (Discussion); XVII, 263–273 (W. J. Graham: "Group Insurance"); XVIII, 132–160 (Discussion); XXIV, 168–219 (J. D. Craig: "Unemployment Insurance"); *R.A.I.A.*, XII, 1–34 (M. Gunn: "Unemployment Insurance"); *T.A.S.A.*, XXIII, 27–66 (W. N. Bagley and J. M. Laird: "Life Reinsurance"), 67–75 (M. W. Torrey: "Reinsurance") and 76–82 (Discussion); XXII, 75–119 (A. A. Rydgren: "Value of Business Reinsured in Bulk"); XXV, 5–10 (A. Hunter: "Reinsurance and Transfer

of Business in Foreign Countries") and 275–281 (Discussion); XXII, 36–74 (J. D. Buchanan: "Industrial Life Insurance"), 385–417 (E. B. Morris: "Some New Problems Affecting Life Insurance") and 13–35 (W. R. Halliday: "Problems of Smaller Companies"); *R.A.I.A.*, XIII, 283–347 (M. A. Linton: "Returns under Agency Contracts"); current issues of two or three well-known insurance journals; latest volumes of *R.A.I.A.*, *T.A.S.A.*, *J.I.A.*, *T.F.A.*, and such items of actuarial interest as may be found in the latest volumes reporting the proceedings of the Association of Life Insurance Presidents, National Convention of Insurance Commissioners, and American Life Convention.

PUBLICATIONS REFERRED TO IN THE RECOMMENDED COURSE OF STUDY

ACTUARIAL JOURNALS

R.A.I.A.—Record of the American Institute of Actuaries.
T.A.S.A.—Transactions of the Actuarial Society of America.
J.I.A.—Journal of the Institute of Actuaries.
T.F.A.—Transactions of the Faculty of Actuaries.

AUTHOR, TITLE, AND PUBLISHER OF TEXT-BOOKS

Baldwin, W. E., *New York Insurance Law Annotated 1925*, Banks Law Publishing Co., New York.

Bogle, A. M., *Comprehensive Bookkeeping*, The Macmillan Co., New York.

Dewing, A. S., *Corporation Finance*, Ronald Press Co., New York.

Dickinson, A. L., *Accounting Practice and Procedure*, Ronald Press Co., New York.

Granville, W. A., *Elements of the Differential and Integral Calculus*, Ginn and Co., Boston.

Hall, H. S., and Knight, S. R., *Algebra for Colleges and Schools*, The Macmillan Co., New York; *Elementary Trigonometry*, Macmillan and Co., London; *Higher Algebra*, Macmillan and Co., London.

Hardy, G. F., *The Theory of the Construction of Tables of Mortality, etc.*, C. & E. Layton, London.

Henderson, R., *Graduation of Mortality and Other Tables* ("Actuarial Studies No. 4"), Actuarial Society of America, New York.

Henry, A., *Calculus and Probability*, C. & E. Layton, London.

Holdsworth, J. T., *Money and Banking* (Fourth Edition), D. Appleton and Co., New York.

Hunter, A., *Total Permanent Disability Benefits in Relation to Life Insurance* ("Actuarial Studies No. 5"), Actuarial Society of America, New York.

Jordan, D. F., *Investments*, Prentice-Hall, Inc., New York.

Maclean, J. B., *Life Insurance*, McGraw-Hill Book Co., New York.

Moir, H., *Sources and Characteristics of the Principal Mortality Tables* ("Actuarial Studies No. 1"), Actuarial Society of America, New York.

Murphy, R. D., and Papps, P. C. H., *Construction of Mortality Tables from the Records of Insured Lives* ("Actuarial Studies No. 2"), Actuarial Society of America, New York.

Phillips, A. W., and Fisher, I., *Elements of Geometry*, American Book Co., New York.

Riegel, R., and Loman, H. J., *Insurance Principles and Practices*, Prentice-Hall, Inc., New York.

Rietz, H. L., and others, *Handbook of Mathematical Statistics*, Houghton Mifflin Co., Boston.

Rietz, H. L., Crathorne, A. R., and Rietz, J. C., *Mathematics of Finance*, H. Holt and Co., New York.

Robertson, W. A., and Ross, F. A., *Actuarial Theory*, The Spectator Co., New York.

Shearman, H. P., *Practical Economics*, McGraw-Hill Book Co., New York.

Smith, P. F., and Gale, A. S., *Introduction to Analytic Geometry*, Ginn and Co., Boston.

Spurgeon, E. F., *Life Contingencies*, C. & E. Layton, London.

Todhunter, R., *Institute of Actuaries' Text-Book, Part I*, C. & E. Layton, London.

Watson, A. W., *An Account of an Investigation of the Sickness and Mortality Experience of the I.O.O.F. Manchester Unity, 1893–1897*, I.O.O.F. M.U., 97 Grosvenor St., Manchester, England.

Wolfe, S. H., *The Examination of Insurance Companies*, Insurance Press, New York.

Yule, G. U., *An Introduction to the Theory of Statistics*, C. Griffin and Co., London.

OTHER PUBLICATIONS

American-Canadian Mortality Investigation, 1900–1915 (in two volumes), Actuarial Society of America, American Institute of Actuaries and National Convention of Insurance Commissioners.

Corpus Juris. (This is a voluminous reference work that may be consulted in any modern law library.)

Medico-Actuarial Mortality Investigation (in five volumes), Association of Life Insurance Medical Directors and Actuarial Society of America.

Proceedings of the Insurance Institute of Toronto, Apply to C. E. Elvins, Imperial Life Assurance Co., Toronto, Canada.

Proceedings of the Seventh International Congress of Actuaries (in two volumes), Roe Loffzen—Huebner & Van Santen, Amsterdam.

Regulations 65 Relating to the Income Tax Under the Revenue Act of 1924, Government Printing Office, Washington, D.C.

United States Life Tables, 1890, 1901, 1910 and 1901–1910, Government Printing Office, Washington. D.C.

The insurance laws of Illinois, Minnesota, Ohio, and Wisconsin, or of Ontario and the Dominion of Canada, may generally be consulted in company libraries and may be obtained from the respective insurance departments.

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Charles Harrison Beckett	1918–20	
George Graham	1920–22	
Lawrence M. Cathles	1922–24	

SCHEDULE OF MEMBERSHIP—DECEMBER 31, 1925

Particulars	Fellows	Associates	Company Members	Total
Membership—December 15, 1924	112	72	121	305
Increase:				
By examination	5	13	.....	18*
By election	4	6	5	15
	121	91	126	338
Decrease:				
By death	.....	1	.....	1
By other withdrawals	.....	1	4	5
Transfers from Associate to Fellow by examination	.....	5	.....	5
Membership—August 1, 1925	121	84	122	327

*Includes transfers from Associate to Fellow.

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(Key: † Charter Member at Organization in 1909; * Member by Examination)

FELLOWS

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1915 Allison, Sinclair Edward, A.A.S., Actuary, Pan-American Life Ins. Co., New Orleans, La.

1914 Allstrom, Henry Willard, A.A.S., Secretary and Actuary, Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1915 Ames, Ernest Capron, A.B., LL.B., Vice-President and Actuary, Bankers Life Ins. Co. of Nebraska, Lincoln, Neb.

† Anderson, Lewis Albert, B.L., Des Moines, Iowa.

1921 Angell, Charles Hart, A.B., A.A.S., Actuary, Massachusetts Mutual Life Ins. Co., Springfield, Mass.

† Appleby, Troy Wilson, A.M., President, The Ohio National Life Ins. Co., Cincinnati, Ohio.

† Arnold, Oswald James, B.S., President, Northwestern National Life Ins. Co., Minneapolis, Minn.

1924 *Babbitt, Albert E., A.M., Actuary, Lamar Life Ins. Co., Jackson, Miss.

1920 Bagley, W. Nelson, B.S., F.A.S., Assistant Actuary, The Travelers Ins. Co., Hartford, Conn.

† Barnard, William Francis, 1402 Midland Ave., Syracuse, N.Y.

† Barnett, Samuel, A.B., C.E., Insurance Lawyer and Consulting Actuary, 502 Forsyth Building, Atlanta, Ga.

† Beckett, Charles Harrison, A.B., Mem. A.M.S., Actuary, State Life Ins. Co., Indianapolis, Ind.

1912 Bowerman, Judah Philip, Actuary, George Washington Life Ins. Co., Charleston, W.Va.

1917 Buck, George Burton, LL.B., F.C.A.S., Consulting Actuary, 25 Spruce St., New York, N.Y.

ADMITTED

1913 Burton, Russell Conklin, A.B., Secretary and Actuary, New World Life Ins. Co., Spokane, Wash.

† Buttolph, Henry Wright, A.B., A.M., A.A.S., Actuary, American Central Life Ins. Co., Indianapolis, Ind.

1914 Cameron, John Campbell, M.A., F.F.A., A.A.S., Vice-President and Actuary, Great Southern Life Ins. Co., Houston, Tex.

1925 Cammack, Edmund Ernest, A.I.A., F.A.S., Vice-President and Actuary, Aetna Life Ins. Co., Hartford, Conn.

† Campbell, Dr. Donald Francis, A.M., Ph.D., Mem. A.M.S., Consulting Actuary, Suite 1832, 160 N. La Salle St., Chicago, Ill.

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1921 *Coates, Barrett Nelson, B.S., A.A.S., Consulting Actuary, 354 Pine St., San Francisco, Cal.

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1918 Craig, James Douglas, F.A.S., F.C.A.S., Actuary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

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1914 Epsteen, Saul Robinson, B.S., Ph.D., Special Deputy State Bank Commissioner, Antonito, Colo.

1919 Evans, Percy H., Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1914 Fairlie, James, M.A., A.A.S., Vice-President and Actuary, Mutual Life of Illinois, Springfield, Ill.

1922 *Fassel, Elgin George, A.I.A., F.A.S., Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

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1913 Graham, George, F.F.A., A.A.S., A.I.A., Vice-President, Central States Life Ins. Co., St. Louis, Mo.

1915 Greene, Winfield Wardwell, Ph.B., Actuary and Comptroller, General Reinsurance Corporation, 80 Maiden Lane, New York, N.Y.

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† Hann, Alfred Gilbert, Actuary, Pacific Mutual Life Ins. Co., Los Angeles, Cal.

1913 Harvey, Julian Corneille, LL.B., General Manager and Actuary, St. Louis Mutual Life Ins. Co., St. Louis, Mo.

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1920 HEMSING, FRANK C., A.A.S., Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1914 *HERFURTH, CARL E., Secretary and Actuary, Montana Life Ins. Co., Helena, Mont.

† HIGDON, JOHN E., A.B., American Bankers Ins. Co., Jacksonville, Ill.

1924 *HOHAUS, REINHARD ARTHUR, B.A., F.A.S., Assistant Actuary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

† HOMANS, I. SMITH, A.A.S., Vice-President and Actuary, Commonwealth Life Ins. Co., Louisville, Ky.

1918 HOPE, FRANCIS MOFFAT, F.F.A., A.I.A., A.A.S., Actuary, Occidental Life Ins. Co., Los Angeles, Cal.

1920 HUENERGARD, ARTHUR JACOB CONRAD, B.A., F.A.S., Assistant Actuary, Dominion Life Assurance Co., Waterloo, Ont., Canada.

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† HYDE, EDWARD WYLLYS, B.C.E., C.E. (Retired), Milford, Ohio.

1925 *JACKSON, HENRY HOLLISTER, M.A., F.A.S., Associate Actuary, National Life Ins. Co., Montpelier, Vt.

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1925 *JOHNSON, WILBUR MOORE, A.M., Vice-President and Actuary, Royal Union Life Ins. Co., Des Moines, Iowa.

1925 *KENNON, RALPH EUGENE, B.A., M.S., Mem. A.M.S., Secretary and Actuary, Medical Life Ins. Co. of America, Waterloo, Iowa.

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1916 King, Walter Irving, A.B., F.A.S., Secretary Group Ins. Dept., Connecticut General Life Ins. Co., Hartford, Conn.

1918 Laird, John Melvin, B.A., A.I.A., F.A.S., Secretary, Connecticut General Life Ins. Co., Hartford, Conn.

1923 *Larsen, Arthur William, M.A., A.A.S., Actuary, Volunteer State Life Ins. Co., Chattanooga, Tenn.

1924 Linton, Morris Albert, B.S., M.A., F.I.A., F.A.S., Vice-President, Provident Mutual Life Ins. Co., Philadelphia, Pa.

1924 Little, James Fulton, F.I.A., F.A.S., F.C.A.S., Associate Actuary, Prudential Ins. Co. of America, Newark, N.J.

1919 Macfarlane, William, F.F.A., F.A.S., Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

1921 Maclean, Alexander T., F.F.A., F.A.S., Associate Actuary, Massachusetts Mutual Life Ins. Co., Springfield, Mass.

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1924 *Marvin, Harry Cooper, A.M., Consulting Actuary, 2105 N. Meridian St., Indianapolis, Ind.

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1916 McIntosh, James, F.F.A., A.A.S., Associate Actuary, The Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1924 McKechnie, James Baldwin, M.A., F.I.A., F.A.S., General Manager, Manufacturers Life Ins. Co., Toronto, Canada.

1914 Mead, Franklin Bush, A.B., F.A.S., Secretary and Actuary, Lincoln National Life Ins. Co., Fort Wayne, Ind.

1919 Moir, Henry, F.F.A., F.I.A., F.A.S., President, United States Life Ins. Co., 105 Fifth Ave., New York, N.Y.

1913 Montgomery, Paul Vaughan, B.A., Vice-President and Actuary, Southland Life Ins. Co., Dallas, Tex.

1920 Morris, Edward Bontecou, Ph.B., F.A.S., Actuary, Life Dept., The Travelers Ins. Co., Hartford, Conn.

1913 Morris, William Oscar, Actuary, North American Life Ins. Co., North American Bldg., Chicago, Ill.

1913 Napier, William Ewart, Second Vice-President and Secretary, Scranton Life Ins. Co., Scranton, Pa.

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† Orr, Robert Kelly, Lansing, Mich.

1918 Papps, Percy Charles Herbert, A.I.A., F.A.S., Mathematician, The Mutual Benefit Life Ins. Co., Newark, N.J.

1920 Parker, John Gowans, B.A., A.I.A., F.A.S., Actuary, Imperial Life Assurance Co., Toronto, Canada.

1914 Pattison, George Benjamin, A.A.S., Secretary and Actuary, Peoria Life Ins. Co., Peoria, Ill.

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1918 Pipe, Sidney Herbert, F.A.S., A.I.A., Actuary, Temple Bldg., Toronto, Canada.

1925 *Poorman, William Frank, M.A., A.A.S., Actuary, Central Life Assurance Society, Des Moines, Iowa.

1924 *Porter, Elder Alexander, A.B., F.A.S., Actuary, The Indianapolis Life Ins. Co., Indianapolis, Ind.

† Reynolds, Joseph B., President, Kansas City Life Ins. Co., Kansas City, Mo.

1920 Rhodes, Edward Everett, F.A.S., Vice-President, Mutual Benefit Life Ins. Co., Newark, N.J.

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1912 Roslington, George, F.C.A., Vice-President, Occidental Life Ins. Co., Albuquerque, N.M.

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1925 Smith, Victor Roy, M.A., A.I.A., A.A.S., Actuary, Confederation Life Association, Toronto (2), Canada.

1915 Speers, Alexander Albert, A.B., F.A.S., Actuary and Treasurer, The Toledo Travelers Life Ins. Co., Toledo, Ohio.

1915 Stein, Calvert Frederick, Secretary-Actuary, Life Dept., Provident Life and Accident Ins. Co., Chattanooga, Tenn.

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1921 Strong, Wendell Melville, B.A., M.A., Ph.D., LL.B., F.A.S., Associate Actuary, Mutual Life Ins. Co., 34 Nassau St., New York, N.Y.

1913 Thomson, Gordon William, F.F.A., A.I.A., A.A.S., Vice-President, West Coast Life Ins. Co., 605 Market St., San Francisco, Cal.

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1916 Washburn, James Herman, A.A.S., Consulting Actuary, 165 Broadway, New York, N.Y.

1916 Watt, William Arthur, M.A., A.I.A., F.A.S., Secretary and Actuary, Pilot Life Ins. Co., Greensboro, N.C.

† Webb, Robert Montague, Kansas City Life Ins. Co., Kansas City, Mo.

1921 Welch, Archibald Ashley, A.B., A.M., F.A.S., President, Phoenix Mutual Life Ins. Co., Hartford, Conn.

1916 Williams, Frederick Alfred, F.A.S., A.I.A., F.S.S., Manager and Actuary, "La Nacional" Compañia de Seguros sobre la Vida, S.A., Apartado 1420, Mexico City, Mexico.

1923 *Williams, (Miss) Helen Jackson, A.B., M.S., Bureau of Statistics and Research, 510 Electric Bldg., Cleveland, O.

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† Withington, Frederic Scherer, F.A.I.A., Actuary, Midland Life Ins. Co., Kansas City, Mo.

1925 Wood, Arthur Barton, B.A., F.I.A., F.A.S., Vice-President and Actuary, Sun Life Assurance Co. of Canada, Montreal, Canada.

1924 Wood, William Archibald Porter, B.A., A.I.A., F.A.S., Actuary, Canada Life Assurance Co., Toronto, Canada.

1915 Woodward, Joseph Hooker, Ph.B., F.A.S., Consulting Actuary, Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y.

1925 Young, William, F.F.A., F.A.S., Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

ASSOCIATES

(Numerals indicate what parts of the Fellowship examinations have been passed.)

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1915 *Beach, (Miss) Henricka Bryant, M.A., Assistant Secretary and Actuary, Provident Life Ins. Co., Bismarck, N.D.

1913 Beecher, Benjamin Sanford, M.A., Vice-President, National Guardian Life Ins. Co., Madison, Wis.

1925 *Bell, David Wright, B.A., Imperial Life Assurance Co., Toronto, Canada.

1923 Birkenshaw, John Henry, B.A., F.A.S., Assistant Actuary, Confederation Life Association, Toronto, Canada.

† Bishop, George Walter, The Bankers Reserve Life Co., Omaha, Neb.

1923 *Bjorn, Walter, B.S., A.A.S., Connecticut General Life Ins. Co., Hartford, Conn. (*10*, *11*)

1915 *Bowerman, Walter Gregory, A.M., F.A.S., New York Life Ins. Co., 346 Broadway, New York, N.Y.

1922 *Boyd, Lorne Thomas, B.A., Secretary and Actuary, Commonwealth Life Ins. Co., Hamilton, Ont., Canada. (*9*, *10*)

† Brimstin, William Edgar, Secretary, Federal Life Ins. Co., Chicago, Ill.

1919 Brooks, Charles Edward, Ph.D., 613 Beavers Insurance Bldg., Madison, Wis.

1921 Brown, Everett Garrabrant, F.A.S., Actuary, Southwestern Life Ins. Co., Dallas, Tex.

ADMITTED

1917 Brown, (Mrs.) Helen Elizabeth, M.A., 702 S. Sixth St., Ironton, Ohio.

1919 Brunnquell, Helmuth G., A.C.A.S., Actuary, Wisconsin Insurance Dept., Madison, Wis.

1911 *Buckman, Harold Hibbs, Actuary, California State Life Ins. Co., Sacramento, Cal.

1924 *Budinger, Joseph Aloysius, A.A.S., Assistant Actuary, Franklin Life Ins. Co., Springfield, Ill. (*9*)

1914 Burkart, Charles J., Kansas City Life Ins. Co., Kansas City, Mo.

1919 *Clark, (Miss) Helen Lucy, A.B., Assistant Actuary, American Central Life Ins. Co., Indianapolis, Ind.

1914 *Coler, Wendell Phillips, A.B., Actuary, The Maccabees, 5065 Woodward Ave., Detroit, Mich. (*9, 10*)

† Copeland, John Alexander, A.B., Consulting Actuary, 407 Southeastern Trust Bldg., Atlanta, Ga.

1924 *Cross, Cecil Frank, A.B., Actuary, American Life Ins. Co., Detroit, Mich.

1925 *Dandy, John Percy, B.A., A.A.S., Confederation Life Association, Toronto, Canada.

1925 *Davis, Malvin Edward, B.A., A.A.S., A.C.A.S., Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

1924 *Dubuar, Charles Carroll, A.B., Assistant Actuary, New York Insurance Dept., Albany, N.Y. (*9*)

1925 *Eastlack, Allen Carlson, A.M., Actuary, Gem City Life Ins. Co., Dayton, Ohio.

1912 *Fetsch, Harry Christian, A.B., A.A.S., Actuary, Ohio State Life Ins. Co., Columbus, Ohio.

1924 *Folz, Clifford H., A.B., Actuary, Public Savings Ins. Co., Indianapolis, Ind.

1922 *Good, Alexander Caldwell, 1416 Chemical Bldg., St. Louis, Mo. (*9, 10*)

1921 *Goodrich, Clarence Robertson, Central States Life Ins. Co., St. Louis, Mo. (*9–11*)

† Grimes, Irvin Lorenzo, A.B., Actuary and Assistant Secretary, Mutual Trust Life Ins. Co., 77 W. Washington St., Chicago, Ill.

1916 Griswold, Robert N., B.A., A.A.S., Inter-Mountain Life Ins. Co., Salt Lake City, Utah.

ADMITTED

1923 HALE, JOHN STEWART, B.A., Actuary, Northwestern National Life Ins. Co., Minneapolis, Minn.

1919 *HANNA, JOHN VESTAL, Actuary and Assistant Secretary, United Life and Accident Ins. Co., Concord, N.H.

1915 *HILLER, WILLIAM JOHN, A.B., Assistant Actuary, Bankers Life of Nebraska, Lincoln, Neb.

1914 *HIPP, GRADY HAYNE, A.B., Actuary, New York Insurance Dept., Albany, N.Y.

1925 HOWELL, VALENTINE, B.S. in Econ., F.A.S., Associate Actuary, Guardian Life Ins. Co., 50 Union Sq., New York, N.Y.

† HUSKINSON, GEORGE WILLIAM H., Assistant Superintendent, Illinois Insurance Dept., Springfield, Ill.

1924 *JENKINS, WILMER ATKINSON, A.A.S., A.M., Instructor, Harvard University, 1 Craigie St., Cambridge, Mass.

1918 JORDAN, F. RUSSELL, B.A., A.A.S., Actuary, Franklin Life Ins. Co., Springfield, Ill.

1925 *KALMBACH, LELAND JOHN, A.B., Lincoln National Life Ins. Co., Fort Wayne, Ind.

1924 KATTELL, SHERMAN C., B.S., F.A.S., Assistant Actuary, Lincoln National Life Ins. Co., Fort Wayne, Ind.

1925 *KENIGSON, GEORGE, Actuary, Service Life Ins. Co., Lincoln, Neb.

1923 *KOCH, (MISS) ELOISE BENDER, A.B., Central States Life Ins. Co., St. Louis, Mo. (*9–11*)

1923 *LEHMAN, A. THOMAS, A.M., Actuary, Standard Life Ins. Co. of America, Pittsburgh, Pa. (*9, 10*)

1924 LITHGOW, JAMES HECTOR, F.A.S., A.I.A., Actuary, Manufacturers Life Ins. Co., Toronto, Canada

1925 *MACKINNON, WILL DONALD, B.S., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1925 MARTIN, LESLIE R., F.A.S., Assistant Actuary, Connecticut Mutual Life Ins. Co., Hartford, Conn.

1916 MAYNARD, JOSEPH DURYEA, Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1923 *MCANDLESS, A. J., B.A., Assistant Secretary, Lincoln National Life Ins. Co., Fort Wayne, Ind. (*9, 10*)

1923 MCBRIDE, WILLIAM HUGH, Actuary, National Life and Accident Ins. Co., Nashville, Tenn.

1925 *MCCLURG, D. RALPH, PH.B., Secretary and Treasurer, National Equity Life Ins. Co., Little Rock, Ark.

ADMITTED

1922 McConaghy, Charles Alexander, B.A., A.I.A., F.A.S., Actuary, Continental Life Ins. Co., St. Louis, Mo.

1925 *Moyer, Ross Emerson, A.B., A.A.S., Assistant Actuary, Columbian National Life Ins. Co., Boston, Mass.

1921 *Myers, George Elmer, A.A.S., Actuary, Cleveland Life Ins. Co., Cleveland, Ohio.

1923 *Oates, Merlin, A.B., Secretary and Actuary, Alamo Life Ins. Co., San Antonio, Tex. (*9, 10*)

1922 *Pardieck, (Miss) Gertrude L., A.B., Assistant Actuary, State Life Ins. Co., Indianapolis, Ind.

1925 Pedoe, Arthur, B.Sc. (London), F.I.A., A.A.S., Actuary, Continental Life Ins. Co., Toronto, Canada.

1925 Phillips, James Thompson, B.A., F.A.S., New York Life Ins. Co., 346 Broadway, New York, N. Y.

1918 *Powell, John Mark, A.B., F.A.S., Actuary, Columbian National Life Ins. Co., Boston, Mass. (*9, 10*)

1924 *Prentice, George Taylor, B.A., A.A.S., Imperial Life Assurance Co., Toronto, Canada.

1920 Reichgott, Henry, Ph.B., A.A.S., Second Vice-President, Missouri State Life Ins. Co., St. Louis, Mo.

1920 *Reilly, Dr. John Franklin, A.M., Ph.D., 212 Physics Bldg., Iowa City, Iowa.

1922 *Robbins, Dr. Rainard Benton, A.M., Ph.D., Assistant Actuary, New York Insurance Dept., 165 Broadway, New York, N.Y.

1916 *Rothschild, Stanford Z., A.B., A.A.S., Third Vice-President and Actuary, Sun Life Ins. Co. of America, Baltimore, Md.

1914 *Scott, Martin R., Reserve Loan Life Ins. Co., Indianapolis, Ind.

1916 Sellman, Henry G., A.B., Consulting Actuary, 6717 Virginia Ave., St. Louis, Mo. (*10–12*)

1924 *Shepherd, Bruce Edwin, Ph.B., Missouri State Life Ins. Co., St. Louis, Mo.

1916 *Shepherd, Clinton O., F.A.S., Actuary, Missouri State Life Ins. Co., St. Louis, Mo.

1925 *Shudde, Louis Otto, B.A., M.A., Actuary and Assistant Secretary, San Jacinto Life Ins. Co., Beaumont, Tex.

1921 Sinclair, Coll Claude, B.A., F.A.S., Assistant Actuary, Great-West Life Assurance Co., Winnipeg, Man., Canada.

ADMITTED

† STAYMAN, GUY L., P.O. Box 86, Indianapolis, Ind.

1925 STOWE, HUDSON J., B.A., F.A.S., Manufacturers Life Ins. Co., Toronto, Canada.

1913 STRUDELL, FRED DAVID, Consulting Actuary, 722 Chestnut St., St. Louis, Mo.

1915 *STURTEVANT, ROBERT BROWN, A.B., Assistant Secretary, American Central Life Ins. Co., Indianapolis, Ind.

1919 SUZUKI, TOSHIKAZU, M.S., Actuary, First Mutual Life Ins. Co., Kyobashi, Tokio, Japan.

1925 THOMSON, JOHN WALTER, F.F.A., A.I.A., F.S.S., Actuary, North American Reassurance Co., 250 Park Ave., New York, N.Y.

1915 TICKNER, VERNE L., Vice-President and Actuary, Roosevelt Life Ins. Co., St. Paul, Minn.

† VAIL, ROGER SHERMAN, A.B., 110 South Dearborn St., Chicago, Ill.

1925 *WALKER, (MISS) MARGARET, M.A., Lincoln National Life Ins. Co., Fort Wayne, Ind.

1925 *WALTON, HAROLD GRAHAM, B.S., American Central Life Ins. Co., Indianapolis, Ind.

1923 WARREN, PROFESSOR LLOYD ARTHUR HEBER, M.A., PH.D., F.R.A.S., F.C.A.S., A.A.S., University of Manitoba, Winnipeg, Man., Canada.

1917 WERKENTHIN, BERNARD, Texas Insurance Dept. Austin, Tex.

† WIGGINS, HORACE SCOVILLE, 514–15 Bankers Life Bldg., Lincoln, Neb.

1920 YORT, J. P., 2528 Thirteenth St., Washington, D.C.

1925 *YOUNG, JAY ROLLIN, Franklin Life Ins. Co., Springfield, Ill.

CONTRIBUTING MEMBERS

1921 AETNA LIFE INS. CO., Hartford, Conn.

1916 AMERICAN CENTRAL LIFE INS. CO., Indianapolis, Ind.

1920 AMERICAN LIFE INS. CO., Denver, Colo.

1923 AMERICAN LIFE INS. CO., Detroit, Mich.

1923 AMERICAN LIFE REINSURANCE CO., Dallas, Tex.

1917 AMERICAN NATIONAL INS. CO., Galveston, Tex.

1921 AMICABLE LIFE INS. CO., Waco, Tex.

1919 ATLANTIC LIFE INS. CO., Richmond, Va.

ADMITTED

1921 Bank Savings Life Ins. Co., Topeka, Kan.
1920 Bankers Life Co., Des Moines, Iowa.
† Bankers Reserve Life Co., Omaha, Neb.
1921 Berkshire Life Ins. Co., Pittsfield, Mass.
1922 Canada Life Assurance Co., Toronto, Canada.
1912 Capitol Life Ins. Co., Denver, Colo.
1919 Cedar Rapids Life Ins. Co., Cedar Rapids, Iowa.
1916 Central Life Assurance Society of the U.S., Des Moines, Iowa
† Central Life Ins. Co. of Illinois, Chicago, Ill.
1915 Central States Life Ins. Co., St. Louis, Mo.
† Cleveland Life Ins. Co., Cleveland, Ohio.
1920 Columbus Mutual Life Ins. Co., Columbus, Ohio.
1922 Confederation Life Association, Toronto, Canada.
1921 Connecticut General Life Ins. Co., Hartford, Conn.
1923 Continental Assurance Co., Chicago, Ill.
1922 Continental Life Ins. Co., St. Louis, Mo.
1920 Continental Life Ins. Co., Wilmington, Del.
1919 Dakota Life Ins. Co., Watertown, S.D.
1922 Detroit Life Ins. Co., Detroit, Mich.
1925 Dominion Life Assurance Co., Waterloo, Ont., Canada.
1918 Elkhorn Life & Accident Ins. Co., Norfolk, Neb.
1920 Equitable Life Ins. Co., Washington, D.C.
1920 Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.
1922 Excelsior Life Ins. Co., Toronto, Canada.
1920 Farmers & Bankers Life Ins. Co., Wichita, Kan.
1923 Farmers National Life Ins. Co., Chicago, Ill.
† Federal Life Ins. Co., Chicago, Ill.
1921 Fidelity Mutual Life Ins. Co., Philadelphia, Pa.
1916 Franklin Life Ins. Co., Springfield, Ill.
1914 George Washington Life Ins. Co., Charleston, W.Va.
1923 Great Northern Life Ins. Co., Chicago, Ill.
1916 Great Southern Life Ins. Co., Houston, Tex.
1921 Great-West Life Assurance Co., Winnipeg, Man., Canada.
1920 Guaranty Life Ins. Co., Davenport, Iowa.
1921 Guardian Life Ins. Co. of America, New York, N.Y.
1920 Idaho State Life Ins. Co., Boise, Idaho.
† Illinois Life Ins. Co., Chicago, Ill.

ADMITTED

1912 Indianapolis Life Ins. Co., Indianapolis, Ind.
† International Life Ins. Co., St. Louis, Mo.
1923 International Life & Trust Co., Moline, Ill.
1922 Imperial Life Assurance Co., Toronto, Canada.
1915 Jefferson Standard Life Ins. Co., Greensboro, N.C.
1921 John Hancock Mutual Life Ins. Co., Boston, Mass.
† Kansas City Life Ins. Co., Kansas City, Mo.
1913 LaFayette Life Ins. Co., LaFayette, Ind.
1925 Liberty Life Ins. Co., Topeka, Kan.
† Lincoln National Life Ins. Co., Fort Wayne, Ind.
1922 London Life Ins. Co., London, Ont., Canada.
1923 Manhattan Life Ins. Co., New York, N.Y.
1922 Manufacturers Life Ins. Co., Toronto, Canada.
1921 Maryland Life Ins. Co., Baltimore, Md.
1921 Massachusetts Mutual Life Ins. Co., Springfield, Mass.
1920 Merchants Life Ins. Co., Des Moines, Iowa.
1921 Metropolitan Life Ins. Co., New York, N.Y.
1915 Michigan Mutual Life Ins. Co., Detroit, Mich.
1920 Midland Life Ins. Co., Kansas City, Mo.
1915 Midland Mutual Life Ins. Co., Columbus, Ohio.
1919 Minnesota Mutual Life Ins. Co., St. Paul, Minn.
1912 Missouri State Life Ins. Co., St. Louis, Mo.
1925 Monarch Life Assurance Co., Winnipeg, Man., Canada.
1920 Montana Life Ins. Co., Helena, Mont.
1919 Mutual Benefit Life Ins. Co., Newark, N.J.
1921 Mutual Life of Illinois, Springfield, Ill.
1919 Mutual Trust Life Ins. Co., Chicago, Ill.
1919 National Life & Accident Ins. Co., Nashville, Tenn.
1921 National Life Ins. Co., Montpelier, Vt.
† National Life Ins. Co. of the U.S.A., Chicago, Ill.
1923 National Guardian Life Ins. Co., Madison, Wis.
1921 New England Mutual Life Ins. Co., Boston, Mass.
1925 North American Life Assurance Co., Toronto, Canada.
1915 North American Life Ins. Co., Chicago, Ill.
1920 Northern Life Ins. Co., Seattle, Wash.
1924 Northern States Life Ins. Co., Hammond, Ind.
1915 Northwestern National Life Ins. Co., Minneapolis, Minn.
1921 Occidental Life Ins. Co., Los Angeles, Cal.

ADMITTED

1915 Ohio National Life Ins. Co., Cincinnati, Ohio.
1912 Ohio State Life Ins. Co., Columbus, Ohio.
† Old Colony Life Ins. Co., Chicago, Ill.
1915 Old Line Life Ins. Co. of America, Milwaukee, Wis.
1917 Pacific Mutual Life Ins. Co., Los Angeles, Cal.
1916 Pan-American Life Ins. Co., New Orleans, La.
1923 Penn Mutual Life Ins. Co., Philadelphia, Pa.
1920 Peoples Life Ins. Co., Frankfort, Ind.
1915 Peoples Life Ins. Co. of Illinois, Chicago, Ill.
1915 Peoria Life Ins. Co., Peoria, Ill.
1921 Philadelphia Life Ins. Co., Philadelphia, Pa.
1921 Phoenix Mutual Life Ins. Co., Hartford, Conn.
1921 Provident Mutual Life Ins. Co., Philadelphia, Pa.
1921 Prudential Ins. Co. of America, Newark, N.J.
1913 Public Savings Ins. Co. of America, Indianapolis, Ind.
1916 Register Life Ins. Co., Davenport, Iowa.
1920 Reinsurance Life Co. of America, Des Moines, Iowa
1915 Reliance Life Ins. Co., Pittsburgh, Pa.
† Reserve Loan Life Ins. Co., Indianapolis, Ind.
† Royal Union Mutual Life Ins. Co., Des Moines, Iowa.
1923 San Jacinto Life Ins. Co., Beaumont, Tex.
1910 Scranton Life Ins. Co., Scranton, Pa.
† Security Life Ins. Co. of America, Chicago, Ill.
1922 Security Mutual Life Ins. Co., Binghamton, N.Y.
1923 Shenandoah Life Ins. Co., Roanoke, Va.
1922 Southern States Life Ins. Co., Atlanta, Ga.
1924 Southern Union Life Ins. Co., Fort Worth, Tex.
1912 Southland Life Ins. Co., Dallas, Tex.
1916 Southwestern Life Ins. Co., Dallas, Tex.
† State Life Ins. Co., Indianapolis, Ind.
1925 Sun Life Assurance Co. of Canada, Montreal, Canada.
1921 Travelers Ins. Co., Hartford, Conn.
1921 Two Republics Life Ins. Co., El Paso, Tex.
1921 Union Central Life Ins. Co., Cincinnati, Ohio.
1920 West Coast Life Ins. Co., San Francisco, Cal.
1912 Western Reserve Life Ins. Co., Muncie, Ind.
1911 Western States Life Ins. Co., San Francisco, Cal.
1917 Western Union Life Ins. Co., Spokane, Wash.
1914 Wisconsin Life Ins. Co., Madison, Wis.

DECEASED FELLOWS

ADMITTED		DIED
1924	CLARENCE EDMUND CALE	1924
†	CARROLL BOOTH CARR	1919
†	HORTON W. COCHNOWER	1913
1914	ISAAC DAVENPORT	1916
†	JOHN MAYNARD EMERY	1919
†	VIRGIL MORRISON KIME	1918
1918	LOUIS LINZMEYER	1918
†	LUCIUS MCADAM	1913
†	MALCOLM MCNEILL	1923
1914	ALBERT GARFIELD PORTCH	1918
†	HENRY SHERMAN VAIL	1919
†	GEERT ANDRIES VAN DER SLUIS	1917
1913	HARRIS ELIAS VINEBERG	1920
†	BENJAMIN A. WELSTEAD	1910
†	PAUL LIVINGSTON WOOLSTON	1924
†	WILBUR S. WYNN	1913

DECEASED ASSOCIATES

ADMITTED		DIED
†	NELS NELSON	1912
1919	ARTHUR ENOCH CLEMENT	1924

STUDENTS

THOSE WHO HAVE PASSED ANY PART OF THE EXAMINATIONS DURING THE LAST THREE YEARS AND HAVE NOT BEEN ADMITTED AS ASSOCIATES

(Italic numerals indicate what parts of the examinations have been passed; bold numerals indicate parts waived.)‡

AMBLER, ARTHUR B., St. Louis Mutual Life Ins. Co., St. Louis, Mo. (*1*)

AMOSS, G. BRUCE, Great-West Life Assurance Co., Winnipeg, Man., Canada. (*4*)

ANDERSON, T. LOYAL, care of D. F. Campbell, 160 N. LaSalle St., Chicago, Ill. (*1–6, 8*)

ANGERER, JOHN W., Michigan Insurance Dept., Lansing, Mich. (*1, 2, 3, 6*)

ARMANTROUT, EVERETT D., Provident Mutual Life Ins. Co., Philadelphia, Pa. (*1, 3*)

ATRUBIN, HARRY M., Dominion Life Assurance Co., Waterloo, Ont., Canada. (*1–4*)

ATWATER, ROBERT BURDICK, Bankers Life Ins. Co., Des Moines, Iowa. (*1, 2*)

BAILEY, NEIL, 99 Inkerman St., London, Ontario, Canada. (*2*)

BARNES, ORLIN M., Franklin Life Ins. Co., Springfield, Ill. (*1, 2*)

BATHO, ELGIN R., Great West Life Assurance Co., Winnipeg, Man., Canada. (*1–4*)

BELL, WRAY M., Burwash Hall, Victoria College, Toronto, Canada. (*1–3*)

‡ The parts are shown according to the revised syllabus in which former Part 2 was changed to Part 3 and former Part 3 to Part 2.

Benner, (Miss) Amy L., Peoria Life Ins. Co., Peoria, Ill. (*1–5*)
Beveridge, James R., Manufacturers Life Ins. Co., Toronto, Canada. (*1–4*)
Bittel, William H., 807 S. State St., Ann Arbor, Mich. (*1*)
Boissard, Richard, National Guardian Life Ins. Co., Madison, Wis. (*1–4*)
Bourscheidt, Paul F., Peoria Life Ins. Co., Peoria, Ill. (*1, 2*)
Bowes, George H., Great-West Life Insurance Co., Winnipeg, Man., Canada. (*1*)
Brahana, Henry R., 707 S. McCullough Ave., Urbana, Ill. (*1–7*)
Brockway, Parker W., Bankers Life Co., Des Moines, Iowa. (*1–3*)
Bronson, Dorrance C., 615 Monroe St., Ann Arbor, Mich. (*1–4*)
Bucknell, Earl F., 604 E. Madison St., Ann Arbor, Mich. (*2, 3*)
Buckner, David E., Jefferson Standard Life Ins. Co., Greensboro, N.C. (*1–4*)
Burgar, William J., 54 Murray St., Toronto, Canada. (*1–4*)
Carson, (Miss) Christiana E. ,235 Eugene St., Norwood, Man., Canada. (*1, 2*)
Chen, Ssu-tu, 402 Benjamin St., Ann Arbor, Mich. (*3*)
Clemo, Ambrose J., Montana Life Ins. Co., Helena, Mont. (*1–4*)
Coates, Clarence S., 2019 Los Angeles Ave., Berkeley, Cal. (*1–6*)
Coleman, John H., Lafayette Life Ins. Co., Lumberton, N.C. (*1, 2*)
Conrod, Stuart F., Great West Life Ass'ce. Co., Winnipeg, Man., Canada. (*1–4*)
Cox, Lewis C., National Life & Accident Ins. Co., Nashville, Tenn. (*1–4*)
Craig, Cecil C., 701 S. Division St., Ann Arbor, Mich. (*1–4*)
Crippen, Lloyd K., 1429 Euclid St., N.W., Washington, D.C. (*1–6, 8*)
Danzinger, Leo J., 24 Hart Ave., Winnipeg, Man., Canada. (*1, 2*)
Davidson, Waid J., Iowa Insurance Dept., Des Moines, Iowa. (*1–4*)
Dunlop, Burton E., Rm. 426, 195 Broadway, New York, N. Y. (*2–4*)
Drebin, Harry S., 228 S. Warren Ave., Big Rapids, Mich. (*3*)
Easley, Knight H., Amicable Life Ins. Co., Waco, Tex. (*1, 2*)
Ede, Cecil F., West Coast Life Ins. Co., San Francisco, Cal. (*1–3, 5*)
Ellsworth, Leon W., Southwestern Life Ins. Co., Dallas, Tex. (*1, 3*)
Estes, E. Forrest, Bankers Reserve Life Co., Omaha, Neb. (*1, 3, 4, 6*)
Evans, John H., Ohio National Ins. Co., Cincinnati, Ohio. (*1–6*)
Ferguson, Arthur A., Western Union Life Ins. Co., Spokane, Wash. (*1–3*)
Flesner, (Miss) Irene E., Board of Pensions and Relief, 820 Garland Bldg., Chicago, Ill. (*1–4*)
Frederick, Russell A., Franklin Life Ins. Co., Springfield, Ill. (*1*)
Glover, Lynn A., 29 S. LaSalle St., Chicago, Ill. (*1–5, 8*)
Graeser, A. Otis, 523 E. Madison St., Ann Arbor, Mich. (*1, 2*)
Green, Walter C., 2584 Park St., Salt Lake City, Utah. (*1–4*)
Hacking, Frank, 320 Brooklyn St., Winnipeg, Man., Canada. (*2*)
Hall, William D., National Bureau of Casualty and Surety Underwriters, 120 West 42nd St., New York, N.Y. (*1*)
Hammer, H. H., Dept. of Insurance and Banking, Austin, Tex. (*1–7*)
Hartwell, Maurice, care of The Maccabees, Detroit, Mich. (*3*)
Higginson, George W., Philadelphia Life Ins. Co., Philadelphia, Pa. (*1*)
Hogan, Frank J., Lincoln National Life Ins. Co., Fort Wayne, Ind. (*2, 4*)
Holtzman, Edward, Pan-American Life Ins. Co., New Orleans, La. (*1–4*)
Hondorp, Peter, 425 S. Division Ave., Ann Arbor, Mich. (*2–4*)
Howard, Wilkie A. M., Confederation Life Association, Toronto, Canada. (*1, 3*)
Huehl, Walter H., Indiana Insurance Dept., Indianapolis, Ind. (*1–6, 8*)
Huth, George, North American Life Ins. Co., Chicago, Ill. (*1*)
Irwin, Philip C., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa. (*1–6*)
Jekel, Alfred H., Cragmor Sanatorium, Colorado Springs, Colo. (*1–4*)
Johanssen, Tryggve,Jennestad, Norway. (*1*)
Johnson, George F., Whitewood Banking Co., Whitewood, S. D. (*1*)
Johnson, Herbert E., 5118 Huron St., Chicago, Ill. (*3*)
Jones, W. Ralph, 502 County Bldg., Chicago, Ill. (*1–4*)
Kirkpatrick, Clair C., Iowa Insurance Dept., Des Moines, Iowa. (*2, 4*)

Larsen, Murray N., 140 Ethelbert St., Winnipeg, Man., Canada. (*2*)
Laurimore, George F., State Life Ins. Co., Indianapolis, Ind. (*2*)
Lee, Frank H., The Maccabees, Detroit, Mich. (*1–5*)
Li, Shou-Kun, 1127 Church St., Ann Arbor, Mich. (*2, 3*)
Linder, Joseph, care of Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y. (**1–4**, *5, 6*)
Little, John E., The Maccabees, 5065 Woodward Ave., Detroit, Mich. (*1–6*)
Longeneker, Joshua Van S., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa. (*1–4*)
Louis, Pak Him, 811 Catherine St., Ann Arbor, Mich. (*3*)
Lyle, (Miss) Annie M., Cotton States Life Ins. Co., Nashville, Tenn. (*1–4*)
Menge, Walter O., 444 S. State St., Ann Arbor, Mich. (*1–4*)
Merkle, John W., care of L. A. Glover & Co., 29 S. LaSalle St., Chicago, Ill. (*1–3*)
Messinger, Leroi W., Great-West Life Assurance Co., Winnipeg, Man., Canada. (*1–3*)
Moon, (Miss) Helen, Central Life Ass'ce. Society, Des Moines, Iowa. (*1–4*)
Morse, John J., Central States Life Ins. Co., St. Louis, Mo. (*1, 2*)
Mott, A. B. C., Midland Life Ins. Co., Kansas City, Mo. (*1–4, 6*)
Mott, Thomas M., Liberty Life Ins. Co., Topeka, Kan. (*1–4*)
Murdoch, John M., 293 St. John's Ave., Winnipeg, Man., Canada. (*1, 3*)
Myers, Owen J., Occidental Life Ins. Co., Los Angeles, Calif. (*1, 2*)
Nelson, Carroll E., Missouri State Life Ins. Co., St. Louis, Mo. (*1–4*)
Nicholson, Earl H., Lincoln National Life Ins. Co., Fort Wayne, Ind. (*1, 2*)
Pall, Gordon, 108 Hallett St., Winnipeg, Man., Canada. (*1–4*)
Parks, George A., Bankers Life Co., Des Moines, Iowa. (*1–4*)
Peterson, Ray M., Equitable Life Assurance Society, 393 Seventh Ave., New York, N.Y. (*1–4*)
Pollard, Harry S., 1821 University Ave., Madison, Wis. (*1–4*)
Prenner, Myron R., North Dakota Insurance Dept., Bismarck, N. D. (*1–4*)
Preston, Robert F., Northwestern Nat. Life Ins. Co., Minneapolis, Minn. (*1–5, 8*)
Raiford, Theodore E., 620 Oxford Road, Ann Arbor, Mich. (*1–4*)
Richter, Otto C., Room 426, 195 Broadway, New York, N.Y. (*1–7*)
Ruben, Isadore, 1404 Bushwick Ave., Brooklyn, N.Y. (*1–3*)
Ruby, Harlan H., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa. (*1*)
Rust, (Miss) Eleanor, Guardian Life Ins. Co., 50 Union Sq., New York, N.Y. (*1–3*)
Sarason, Harry M., Missouri State Life Ins. Co., St. Louis, Mo. (*1–3*)
Schillig, (Miss) Eva M., North American Reassurance Co., 250 Park Ave., New York, N. Y. (*1–4*)
Schroeder, Gordon D., Confederation Life Assoc., Toronto, Canada. (*1*)
Shaw, (Miss) Victoria E., Martha Cook Bldg., Ann Arbor, Mich. (*1, 3*)
Shepherd, Pearce, North American Reassurance Co., 250 Park Ave., New York, N. Y. (*1–4*)
Skelding, George A., Confederation Life Assoc., Toronto, Canada. (*1–3*)
Smith, Lewis G., Confederation Life Assoc., Toronto, Canada. (*1, 3–6*)
Tuck, Edward B., 479 Raglan Road, Winnipeg, Man., Canada. (*1, 2*)
Turnbull, David Mc I., 178 Anderson Ave., Winnipeg, Man., Canada (*2*)
Turner, Paul A., Philadelphia Life Ins. Co., Philadelphia, Pa. (*1*)
Van Fleet, George, 420 Thompson St., Ann Arbor, Mich. (*2, 3*)
Veatch, Virgil V., Merchants Life Ins. Co., Des Moines, Iowa. (*3*)
Vetter, Lester, Minnesota Mutual Life Ins. Co., St. Paul, Minn. (*1*)
Wadleigh, William L., Metropolitan Life Ins. Co., New York, N.Y. (*1–3*)
Wallace, William, State Insurance Department, Columbia, S.C. (*1–4*)
Waters, L. L., National Accident Ins. Co., Lincoln, Neb. (*1–7*)
Wellman, Alexander C., Royal Union Life Ins. Co., Des Moines, Ia. (*2–4*)
Whipple, (Miss) Ruth, 220 N. Spring Ave., St. Louis, Mo. (*1–3*)

WHITBREAD, ROBERT B. T., 736 Valour St., Winnipeg, Man., Canada. (*1*)
WILKS, LEE, Acacia Mutual Life Assoc., Washington, D.C. (*1, 2*)
WILSON, (MISS) ELIZABETH W., 1431 Meridian St., N.W., Washington, D.C. (*1–6*)
WOOD, DOUGLAS, Missouri State Life Ins. Co., St. Louis, Mo. (*1, 2*)
WOOLERY, J. MYRON, Protective Life Ins. Co., Birmingham, Ala. (*1–4*)
YOUNG, FLOYD E., Western Union Life Ins. Co., Spokane, Wash. (*1–7*)
YUDKOFF, HARRY, 627 Kenilworth Ave., Detroit, Mich. (*2, 3*)

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Eloise B Koch

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ADDRESS BY THE PRESIDENT, PERCY H. EVANS

As this is my last opportunity to speak to you as President, it seems appropriate to offer some comments on the work of the Institute during the past two years and later on to refer to some of the problems that have engaged our attention during that period.

At the time of the annual meeting in June, 1924, the Institute had 114 Fellows, 72 Associates, 95 Students, and 121 Company Members. At the present time the membership numbers 132 Fellows, 89 Associates, 166 Students, and 124 Company Members. Students, so called, have no standing as members of the Institute. For a period of three years after passing one or more parts without becoming Associates such persons are presumed to be reading for further examination and are continued on the student list. The increase in the number of Students from 95 to 166 during the past two years is very encouraging as is also the marked improvement in the quality of the papers.

The value and volume of the work of the Examination Committee is indicated by the number of papers written during the past six years, as follows:

1921	153	1924	357
1922	179	1925	455
1923	298	1926	629

In view of the rapidly increasing number of young men and women who are reading for actuarial examinations it may not be out of place to offer some remarks on the subject. Nearly thirty years ago Mr. Emory McClintock, in his address as President of the

Actuarial Society, discussed the opportunities open to actuarial students and mentioned that the number of life insurance companies employing actuaries had then for a long period shown no increase. The Pocket Index for 1897 listed only 57 United States and 2 Canadian companies. There was immediately after the close of the Civil War considerable activity in the promotion of new life-insurance companies. Many of these promotions lacked technical management and after a few years reinsured or otherwise terminated their business. At the end of the nineteenth century the number of successful companies had shown no increase for a long period as Mr. McClintock pointed out, and furthermore, it was the general impression that the number probably never would be materially increased. No one then foresaw the development of life insurance history in the early part of the new century when a large number of new companies were organized under more favorable conditions than existed at the close of the Civil War. The significant differences between the two periods are to be found in the supply of, and demand for, technically competent officials, the improvement in government supervision, and also, of course, in the relative stability of financial conditions. As a result of this promotion activity under more favorable conditions the Pocket Index for 1926 lists 268 United States and 28 Canadian companies. Not only has there been this great increase in the number of successful life-offices, and a ten-fold increase in the amount of life insurance in force, but there have been other changes which have greatly increased the need for actuarial training, and extended such requirements into other departments of the life insurance business. Seventy-five years ago life insurance was a new kind of business in this country. The companies were organized by business men who generally regarded its technical requirements as more or less of a nuisance and in many cases disregarded them entirely with disastrous consequences. Even now there are business managements inclined to consider actuarial points of view as more theoretical than practical. Nevertheless it appears to be true that there is in this country, as in Great Britain, a growing tendency to fill official positions other than actuarial with men having substantial actuarial training. At all events there is no reason why young men specializing in actuarial science should not look for positions in the secretarial, agency, and other departments where their special knowledge will be far from superfluous and indeed exceedingly useful.

Returning now to the activities of the Institute and the growth indicated by the increase in membership, and in the number of students reading for examination, it may also be mentioned that during the last four sessions the Institute accepted for presentation eighteen formal papers. There can be no doubt that each one of these papers contained information or suggestions that were helpful to actuaries and to students. Each of these papers was an effort to repay the traditional debt that every actuary owes to his profession. This leads me to urge upon our new members the obligation to prepare at least one paper, and to those who are exceptionally gifted a series of papers as subjects appear to which such men can make a contribution.

In speaking of the affairs of the Institute during the past two years it would be a serious omission if I failed to refer to the very successful joint meeting held at the suggestion of the Actuarial Society in November, 1924. A considerable percentage of the membership of the two organizations is common and as time passes this percentage will increase. It seems not impossible that eventually the common membership will become so large that the two bodies will deem it advisable to combine their labors in some way with considerable advantage to both in economy of time and effort. So long as the two societies in this large country continue to function separately there will be, and there ought to be, differences in method and point of view but there is one point wherein it seems to me there should be a practical identity of practice and that is in the scope and order of the examinations. The new syllabus of the Institute printed in the *Year Book of 1925*, by opening the way to more frequent revision, seems to have been a step in that direction.

In addition to such matters as uniform mortality cards and codes, disability and blood pressure investigations, and the new Gain and Loss Exhibit there have also appeared a number of questions and tendencies of interest and importance to actuaries.

The agitation for a change in the basis of valuation to the American Men Table is perhaps the most important of these questions. There is wide difference of opinion on the subject and while the discussion may not result in any radical changes in practice it must have brought to the foreground the fact that the American system of writing into the statutes rules for conducting the minutest details of the business has the disadvantage of placing serious difficulties in the way of otherwise desirable changes in practice.

Restrictions placed upon freedom of action may stand in the way of unwise and perhaps inequitable practices but such restrictions will inevitably interfere also with freedom of action when advantageous changes are contemplated.

The very general adoption of substandard practice; the recent introduction of non-medical acceptances and its application to the so-called salary deduction plan may be mentioned without comment except perhaps to point out that the interrelations of the actuarial and agency departments are much closer than was formerly supposed.

Judging by the volume of the discussions in the insurance press one of the interesting new questions is that relating to the basis of premium calculation. Shall the companies continue to calculate their premiums on the basis of yearly intervals or adopt rates on a continuous or monthly basis? If there exists a sound actuarial reason for such a change in the basic assumptions and long-established practice we shall probably find here another example of the disadvantages of statutory regulation of life insurance in minute details.

The manner in which disability annuity benefits have been liberalized without a corresponding increase in the rates charged was, as has been pointed out, a departure from an old principle that in the case of benefits of unknown or uncertain value we should begin with a redundant premium and reduce it as experience permits. The situation has had and will have its repercussions. Among other things, for example, it led the commissioner of one state to propose to his legislature that in the case of mutual companies the commissioner be empowered to direct the discontinuance of the benefit (or any form or class of policies) if, in the opinion of the insurance department, it was being granted at a loss. It was a new idea that a law was necessary to prevent mutual companies from writing policies at a loss. But who shall say that the proposal was, under the circumstances, entirely devoid of common sense?

A somewhat similar situation appears to have arisen in the sale of group insurance where the tendency to make competitive rates resulted in an amendment fixing minimum group premiums at the American Men (Ultimate) $3\frac{1}{2}$ per cent net rates plus a leading to be determined by the state commissioner of insurance. The laws of

Wisconsin fix the maximum loading, but the official determination of a minimum loading is another new idea.

Insurance department rulings forbidding the announcement of dividend scales more than thirty days prior to the time of payment and prohibiting extra dividends at regular (stipulated) intervals other than annual, are examples of the tendency toward regulation of details. It may be that the intention at the bottom of these rulings was justified by existing practices, or incidents liable to become practices. In any case the limits of discretion were thereby narrowed in a new way.

Two other questions related to the actuarial task of equitable surplus distribution were also raised by one of the insurance departments. These questions related to the practice of making the payment of a dividend contingent upon the payment of the premium for the ensuing year and the theory that acquisition and other expenditures should be charged against the individual policy and no dividend allotted until the net asset share showed a sufficient surplus. It is evident that the latter question involves the very technical and difficult problem of expense allocation and is in conflict with the weight of practice under which the expense of acquiring a new group of select members is considered a proper charge against the ultimate group until the excess cost is liquidated by the realization of select margins.

The recent events and questions referred to in this short review, as of interest to actuaries, are not mentioned in any partisan or critical spirit but merely as reflecting the existence of a tendency in the direction of closer supervision of actuarial details. I do mean, however, to raise the question at this time whether the legislative habit of minute regulation has not already gone farther than is favorable to the broadest and best development of life insurance practice in its actuarial aspects.

PREMIUM RATES AND SURRENDER VALUES: THEIR RELATION TO COMPANY POLICY

By C. O. SHEPHERD

According to popular conception in life insurance circles, gross premiums are determined by adding a constant expense loading to net valuation premiums, and surrender values are determined by deducting a surrender charge from legal reserves, the implication being that the loading and surrender charge are determined by a separate preliminary operation. In the earlier days of the life insurance business, when the legal standard of mortality and interest represented the best opinion regarding the mortality and interest rates that could be relied on, and the expenses of the life insurance business had not yet taken definite and determinable forms, the process referred to was followed with entire justification. In the light of the changed conditions and additional knowledge of today, the problem takes quite a different form and legal standards instead of serving as a basis for our calculations only add to our difficulties.

Another and more subtle fallacy is the assumption that a company's premiums are based on its past experience and are determined by a purely mathematical process. On the contrary, premiums are based on future experience, and intelligent rate making must be founded on a well-defined policy of company management.

In the early days of company development premiums are determined almost entirely by competition. There comes a time, however, when a company is faced with the task of correlating its general policy with its premium rates. In some degree the problem exists in all companies, and intelligent, successful management can scarcely prevail unless it is recognized.

The desire to grow, to swell the insurance account, affects the amount which the company is willing to pay for new business and development of agency organization, and also affects its overhead or general expense. The selection policy adopted is reflected in its mortality; some companies are noted for a rigid selection of risks, others for a broader classification. A company needs to know the

full effect of the various courses of action open to it in order to chart its course and develop its operating program to the best advantage.

In an effort to obtain information of this character I have investigated the effect of variations in mortality, interest rate, and the different classes of expenses, on both premium rates and surrender values. Each of the factors mentioned is to a greater or less extent under control, and to such extent this study should be a valuable guide in fixing company policy. I have also shown the peculiar part played by present day surrender values fixed as they are by competition and legal requirements.

GROSS PREMIUMS COMPUTED DIRECTLY

Mr. Cammack, in his excellent paper on "Premiums for Non-Participating Life Insurance" (*T.A.S.A.*, XX, 379), showed that gross or office premiums and surrender values are determined directly by a simultaneous evaluation of mortality costs and expenses, without any predetermination of net premiums, loadings, or surrender charges. I have adopted Mr. Cammack's principles and methods with only slight modifications. I have not attempted a justification of methods or assumptions which I believe he has already justified.

Until the American Men Mortality Table was published, Mr. Cammack's paper could hardly have been written. Until that time we had had no accepted standard for measuring recent mortality on insured lives in this country. Certainly we had no satisfactory means of measuring the effects of selection, and this latter fact alone, in my estimation, was an insurmountable barrier to an intelligent study of either rates or values. The American Experience table reflects mortality experienced during the middle of the nineteenth century. The changes in mortality since that time have made it almost worthless. Its other fatal shortcoming is that it makes no pretense of showing mortality on recently selected lives; it is an ultimate table only.

EFFECT OF MORTALITY, EXPENSES AND INTEREST RATES ON PREMIUMS. "STANDARD ASSUMPTIONS"

Every one of us knows that lower mortality rates, lower expense rates, and higher interest rates decrease the cost of insurance, but we know this only in a very general way. How much difference does

a margin of 10 or 20 points in mortality make? How much effect does a higher first commission have on our premiums? How much effect does a higher overhead cost have? Innumerable questions of this character arise from time to time in company management.

Using Mr. Cammack's formulas, I have computed premium rates on different fractional multiples of the A.M. (American Men) Select Mortality Table and on various expense rates, at different rates of interest. I have first computed rates on the basis of the following factors, which for convenience I have called standard factors. Later, I have considered the effect of variations in these factors so that it may be possible from the tables presented to closely estimate rates on any set of factors likely to be required in practice. No undue importance should be attached to the term "standard"; it is not intended to have any special significance but has been adopted solely for convenience of reference throughout this paper. On the whole, I have had in mind conditions as they exist in the younger companies operating under the general agency system.

Mortality.—90 per cent A.M. Select.

Interest.—First 20 years, $4\frac{1}{2}$ per cent; after 20 years, $3\frac{1}{2}$ per cent.

Medical and inspection fees.—$3.00 per thousand of insurance.

General expense—first year and renewal.—(*a*) $2.00 per thousand of insurance. (*b*) 3 per cent of gross premiums.

First-year commissions.—Ordinary and twenty-payment life, 70 per cent; twenty-year endowment, 50 per cent.

Renewal commissions.—Years 2–10, $7\frac{1}{2}$ per cent; years 11–15, 5 per cent; years 16 and subsequent, 2 per cent.

Before proceeding I offer a few words of explanation regarding mortality and interest assumptions.

MORTALITY ASSUMPTIONS; RECENT MORTALITY

The A.M. table of mortality was based on deaths which occurred during the period 1900–1915 on insurance written during this period and on insurance written prior to 1900. When it first appeared in 1918 it was the lowest standard of mortality on insured lives that had ever been published and we were frequently cautioned against its use for computation of premium rates. It was pointed out that it was derived from an experience that did not include any of the heavy losses due to war and the influenza pandemic. It was pointed out, too, that the mortality in certain large sections of the

country was considerably higher than for the country as a whole and that the mortality of some companies was necessarily higher than this combined experience which was an average of nearly all.

Every mortality experience of any considerable size is out of date before it is published on account of the time required to accumulate the experience and the additional time required to compile and publish it. This was true of the A.M. Table; the mortality during the latter part of the period covered was lighter than that during the early part; likewise the mortality on risks selected in later years was better than on risks selected many years previously.

Since 1918 we have acquired much additional information on mortality from the several experiences of individual companies which have been published and from the sidelights on other experiences cited in the discussions of this Institute and the Actuarial Society. Briefly, we know that there has been a continuous improvement in mortality over the past 25 years extending down almost, if not quite, to the present time. One of the conclusions drawn from a recent study[1] of current mortality rates, including such sources as I have mentioned, was "The ultimate mortality to be expected at the present time does not exceed 75 per cent of the A.M. (5) Table." I cite this for the convenience of those who may not have kept in touch with the later developments in this subject. Undoubtedly, the author's conclusion implies careful selection and favorable, or at least not unfavorable, habitat and as it was the result of a study of mortality trend it applies not to even the recent past but to the present. This explanation is intended as a warning to anyone who might otherwise be led into unwarranted optimism.

I have been particularly anxious not to detract from the value of my investigation by over-conservatism. After all, the test of boldness or cautiousness does not lie in the standard of mortality or the rate of interest assumed but in the premium rates actually adopted. Conservatism in the assumption of a high mortality is apt to be offset by a corresponding lack of conservatism elsewhere.

Undoubtedly, there is a certain element of presumption in the use of a 90 per cent or an 80 per cent A.M. Table. Granted that we may have an aggregate experience as favorable as 80 per cent A.M., have we a right to assume that such a per cent is applicable to all

[1] C. D. Rutherford, "A Note on Mortality Variations," *T.A.S.A.*, XXVI, 58.

ages alike? May not the mortality at the younger ages be less than 80 per cent and the mortality at the higher ages more than 80 per cent? It is common knowledge that, as compared with the American Experience Table, the A.M. Table showed a marked improvement in mortality at the younger ages, but practically none at the older ages. The more recent improvement appears to have been more evenly spread and it is safe to assume that the older ages have benefited materially. It would be difficult to deal with more favorable mortality on any incidence assumption other than the one used and if the possible exaggeration of the improvement at older ages is kept in mind the use of such tables for the purpose at hand seems defensible.

Since the foregoing comment was written new evidence of improvement in mortality comes to hand in the report of the Joint Committee of the Life Insurance Medical Association and the Actuarial Society on their "Blood Pressure Investigation." The business contributed by twenty-six companies for purposes of study was presumably average standard business and according to the report the great majority of the policies in the data under investigation were issued in the years 1918–20 inclusive. The business was observed to the 1924 anniversary. The loss ratio by amounts was 72 per cent of the A.M. Select Table including influenza deaths and 67 per cent with influenza deaths excluded. The loss ratio for ages at issue, 50 and over, was 9 points higher than for all ages combined. At ages below 50 the experience paralleled the A.M. Select rather closely.

INTEREST AND EXPENSE ASSUMPTIONS

Mr. Cammack computed premiums on the basis of $4\frac{1}{4}$ per cent interest throughout. Accumulation of premiums at this rate resulted in a marked deficiency as compared with legal reserves and he, therefore, also showed accumlations of these same $4\frac{1}{4}$ per cent premiums at $4\frac{3}{4}$ per cent. Even at this higher rate, many years were required before the company had the legal reserve in hand. Though it may be safe to assume high interest rates today, we shall not be able to take full advantage of them, so long as we have our present legal standard of reserve, unless our companies have such an abundance of surplus as will enable them to finance their new business for many years until the book loss is overcome. It will be even more necessary to modify the many published cash values that are equal

to legal reserves. This question will be taken up more fully later. My assumption of $3\frac{1}{2}$ per cent after 20 years may be ascribed to the foregoing facts and also to the feeling that the assumption of $4\frac{1}{4}$ per cent beyond such a length of time is a trifle bold. Due partly to this change I have not considered it necessary to make any provision for general expenses beyond the premium paying period. This has necessitated a slight change in Mr. Cammack's formulas for limited payment forms.

The other factors assumed do not seem to require much comment. A general expense rate of $2.00 per thousand of insurance is higher than the rate Mr. Cammack used ($1.50), but not at all too high for the younger companies.

One assumption in Mr. Cammack's formula which may be criticized is the assumption of the same constant general expense per thousand of insurance, first year and renewal. A good argument can be made for assessing a larger proportion as first year expense and, in fact, there has been considerable controversy around this point in times past. The question is important and worthy of further investigation and study, though it is unlikely that there will ever be unanimity of opinion in regard to it. The result of a change such as I have indicated would be only a slight variation in premium rates but a decided decrease in values in early policy years. So long as we retain high cash-surrender values in early policy years, practical considerations would interfere with our theory. We are adopting Mr. Cammack's assumptions, but we shall have occasion to revert to this question in some of our further discussions.

Mr. Cammack's formula provides for an assessment of medical and inspection fees and general expenses as a constant at all ages per $1,000 of insurance. An assessment of these expenses as a constant per policy would be more logical, but the use of the latter basis would result in a lower premium rate per $1,000 to the man who buys a $10,000 policy than to the man who buys a $1,000 policy—a practice contrary to the state laws under which we operate.

However, we can, and I believe we should, give effect in our formula to the variation which occurs in average size of policy with variation of age and plan. Mr. Elston discussed this question and presented a table of average amounts by age and plan in his paper on "Expense Investigations," *T.A.S.A.*, XXIII, 83. Variation by age,

it seems to me, is more fundamental than variation by plan. No doubt the term plan deserves special consideration but if any extended differentiation by plan is attempted, great care must be used to avoid incongruities between rates for different plans at the same age. In the case of endowments the effect of too great variation with age may produce rates that decrease with age. As my chief purpose is to show the effect of variation in premium factors, I have not considered it advisable to complicate my paper by giving effect to variations in the average size of policies.

PREMIUMS ACCORDING TO "STANDARD ASSUMPTIONS"

Table I shows premium rates derived according to what I have called "Standard Assumptions." For comparison I have added a column (*b*) showing the average rates used for a similar purpose by Mr. Cammack and described by him as an average of the rates of five leading companies writing non-participating insurance in the United States in 1919. In column (*c*) I have shown a similar average of rates in use today by several leading companies. The reduction which has taken place in the past 6 years is apparent. The figures in column (*d*) will be fully explained when we take up the subject of reserves and surrender values (see Table X).

It is hardly necessary to call attention to the fact that I have not attempted anything resembling a comprehensive treatment of the subject of premium rates. I have merely tried to throw light on certain important phases of the problem. I have omitted the term plan from consideration, in spite of its decided and increasing importance, for the reason that it involves certain features which are peculiar to it and lacks features which are common to life and endowment forms. I may say again that I have not recommended any premiums for adoption nor tried to produce premiums suitable for adoption. The premiums in column (*a*) are compared with those in (*b*) and (*c*) only to show that they are close enough to premiums in use so that changes resulting from variations in our factors will produce rates that are fairly within practicable limits.

The formula I have used makes no provision for profit. In my judgment, profit for stockholders should come from the margin of safety to be taken account of in determining each factor. On profit-sharing business, cost is not determined by premiums alone and as there is not the occasion for close estimating that exists in the case

of non-participating premiums, more substantial margins may be left in all factors from which satisfactory dividends will naturally result.

I will now take up the effect of varying the different factors entering into the premium formula. In finding the effect of a varia-

TABLE I

Age	(*a*) Gross Premiums "Standard Assumptions"	(*b*) Average Rate Five Companies 1919	(*c*) Average Rate Leading Companies 1926	(*d*) Gross Premiums According to "Standard Assumptions" Adjusted
	Ordinary Life			
15	$11.83	.....	$12.03	$12.43
20	13.30	$13.75	13.33	13.77
30	17.40	17.55	17.09	17.68
40	24.62	24.33	23.47	24.62
50	37.47	37.35	35.63	38.49
60	60.54	62.40	58.76	62.59
65	78.41	.....	80.44	80.63
	Twenty-Payment Life			
15	$17.99	.....	$19.06	$20.16
20	19.88	$21.27	20.57	21.76
30	24.62	25.43	24.61	26.02
40	32.07	32.08	30 91	33.04
50	44.07	43.68	42.16	45.35
60	64.72	65.69	62.67	66.26
65	81.05	.....	83.60	82.43
	Twenty-Year Endowment			
15	$39.67	.....	$39.67	$41.80
20	39.87	$40.51	39.84	41.98
30	40.42	41.20	40.42	42.13
40	42.76	43.20	42.53	44.06
50	49.44	49.68	48.80	50.45
60	65.57	67.56	64.64	66.24
65	80.08	.....	84.35	80.43

tion in one factor, it will be undertsood that all other factors remain standard.

I am showing below the effect of, first, a reduction of ten points in mortality from 90 per cent American Men to 80 per cent American Men and, second, an increase of ten points from 90 per cent American Men to 100 per cent American Men. The reduction in rate is very nearly equal to the corresponding increase in rate. This

principle of practically constant first difference holds rather generally and I have, therefore, contented myself with showing the results of a single variation in most of the factors.

If anyone has been disposed to minimize the importance of favorable mortality these figures should correct the impression. The practical range within which mortality may be fixed by the standard of selection adopted is probably wider than ten points and it is evident that a company's rate policy and its underwriting policy must go hand in hand. The peculiar importance of favorable mortality will appear again in connection with surrender values. Table

TABLE II

VARIATION IN MORTALITY

Variation in Premium Rates Due To:

AGE	REDUCTION OF 10 PTS. FROM 90% TO 80% A.M. SELECT	INCREASE OF 10 PTS. FROM 90% TO 100% A.M. SELECT	REDUCTION OF 10 PTS. FROM 90% TO 80% A.M. SELECT	INCREASE OF 10 PTS. FROM 90% TO 100% A.M. SELECT	REDUCTION OF 10 PTS. FROM 90% TO 80% A.M. SELECT	INCREASE OF 10 PTS. FROM 90% TO 100% A.M. SELECT
	Ordinary Life		Twenty-Payment Life		Twenty-Year Endowment	
15....	$— .62	$ .59	$— .86	$ .82	$— .28	$ .27
20....	— .70	.67	— .93	.87	— .30	.29
30....	— .91	.88	—1.07	1.01	— .35	.36
40....	—1.38	1.33	—1.39	1.32	— .61	.62
50....	—2.35	2.26	—2.13	2.03	—1.37	1.37
60....	—4.27	4.15	—3.80	3.72	—3.20	3.20
65....	—5.85	5.71	—5.36	5.28	—4.83	4.82

II emphasizes, too, the fact that ten points mortality at the higher ages is a much more weighty matter from a dollars-and-cents standpoint than at younger ages. On the Ordinary Life plan the ratio of premium difference at age 65 to premium difference at age 15 is as 10 to 1.

The effect of changes in interest rates varies greatly as between different plans of insurance and as between different ages for the same plan. Generally, the effect of changes in interest rates is greater on the higher premium plans. At the younger ages changes after 20 years are more significant than changes within the first 20 years. At the older ages, where changes in the later years are heavily discounted by high mortality, the contrary is the case.

In column (*d*) I have departed from the plan of showing only

deviation from the standard values adopted and have shown the effect of a variation throughout.

A reduction of 10 per cent in first commissions at the younger ages has only a slight effect on the premium. At the older ages the effect is important and is particularly interesting on account of the practice of modifying commissions above age 60 as followed by some companies.

The column showing the effect of a reduction in renewal com-

TABLE III

VARIATION IN INTEREST RATES

Variation in Premium Rates Due To:

Age	Ordinary Life				Twenty-Payment Life				Twenty-Year Endowment	
	(a) Change from 4½% to 5% 1st 10 Years	(b) Change from 4½% to 5% 1st 20 Years	(c) Change to 4½% Throughout	(d) Change from 4½% Throughout to 4% Throughout	(a) Change from 4½% to 5% 1st 10 Years	(b) Change from 4½% to 5% 1st 20 Years	(c) Change to 4½% Throughout	(d) Change from 4½% Throughout to 4% Throughout	(a) Change from 4½% to 5% 1st 10 Years	(b) Change from 4½% to 5% 1st 20 Years
15..	$−.05	$− .18	$−1.21	$ .72	$−.17	$− .61	$−2.64	$1.77	$−.53	$−1.81
20..	−.06	− .22	−1.28	.80	−.20	− .70	−2.70	1.89	−.53	−1.80
30..	−.10	− .35	−1.33	.97	−.26	− .91	−2.58	2.09	−.53	−1.79
40..	−.17	− .56	−1.20	1.13	−.35	−1.14	−2.10	2.14	−.55	−1.77
50..	−.30	− .81	− .85	1.22	−.47	−1.33	−1.34	1.99	−.59	−1.68
60..	−.49	−1.04	− .39	1.25	−.60	−1.36	− .55	1.67	−.67	−1.52
65..	−.61	−1.12	− .20	1.25	−.68	−1.32	− .27	1.50	−.74	−1.42

missions is included for the value it may have in the working out of modified forms of renewal compensation.

Within practicable limits the item of Medical and Inspection Fees is unimportant.

As would be expected, a reduction of a certain amount per thousand of insurance in General Expenses affects premiums by about the same amount at all ages. A reduction in the share of General Expenses expressed as a percentage of gross premiums is of considerable consequence. We can readily see, for instance, the effect of a 2 per cent premium tax.

Various tests made show that the resultant effect of combination

of changes in factors will be closely approximated by combining the effects of the changes taken singly. For example, in the case of a 10 per cent reduction in mortality combined with a reduction of $\frac{1}{2}$

TABLE IV

VARIATION IN EXPENSE RATES

Variation in Premium Rates Due To:

Age	(*a*) Reduction of 10% in 1st Com.	(*b*) Reduction of 2½% in Ren'l. Com. 2d–10th Yr.	(*c*) Reduction of $1 per M in Med. & Insp. Fees	(*d*) Reduction of $.50 per M in General Expense	(*e*) Reduction of 1% Gross Premium in General Expense
	Ordinary Life				
15.....	$— .07	$— .11	$— .06	$— .56	$— .13
20.....	— .07	— .13	— .06	— .56	— .15
30.....	— .10	— .18	— .06	— .56	— .19
40.....	— .16	— .29	— .07	— .56	— .27
50.....	— .31	— .53	— .09	— .58	— .43
60.....	— .64	—1.03	— .11	— .59	— .71
65.....	— .97	—1.47	— .12	— .60	— .92
	Twenty-Payment Life				
15.....	— .16	— .28	— .09	— .58	— .21
20.....	— .18	— .31	— .09	— .58	— .23
30.....	— .21	— .38	— .09	— .58	— .28
40.....	— .28	— .50	— .09	— .58	— .37
50.....	— .43	— .73	— .10	— .59	— .51
60.....	— .73	—1.17	— .11	— .59	— .76
65.....	—1.03	—1.57	— .13	— .60	— .96
	Twenty-Year Endowment				
15.....	— .34	— .61	— .09	— .57	— .45
20.....	— .34	— .61	— .09	— .57	— .45
30.....	— .34	— .61	— .08	— .57	— .45
40.....	— .38	— .67	— .09	— .57	— .49
50.....	— .47	— .81	— .10	— .57	— .56
60.....	— .73	—1.17	— .11	— .58	— .75
65.....	—1.00	—1.51	— .13	— .59	— .93

per cent interest for twenty years the error in premium did not exceed $.04 per thousand of insurance at any age. As another test the premiums computed by Mr. Cammack, involving simultaneous changes in four factors, were reproduced with an error varying from nothing to $.10. I should mention, perhaps, that a special calculation was necessary to allow for the slight difference in commission

after sixteen years and on the Twenty-Payment Life form a special allowance was required on account of the difference in formula previously mentioned.

The full effect of the various changes indicated is not to be found in Tables I–IV. Certain changes, having a comparatively slight influence on premiums directly, have a much more pronounced effect on the company's ability to accumulate reserve funds and thereby affect premiums indirectly.

Neither have we, as yet, considered all the elements that enter into the determination of premium rates. The question of deficiency reserves must be considered. Surrender values fixed under stress of competition or by virtue of legal restrictions may require an increase in premiums.

NET PREMIUM RESERVES AND "NATURAL RESERVES"

Having determined our premiums, and for the moment ignoring statutory requirements, what reserves should we carry? What sums shall we need to have in hand to carry out our contracts, and what sums shall we be able to accumulate out of our premiums? We find our answer by using the same assumption of interest, mortality, and expense as we used to determine premiums. The term "natural reserves" has been applied to reserves so computed; it seems more suggestive and more definite than the word "accumulation" used by Mr. Cammack.

I have first computed natural reserves according to "standard assumptions" and have then shown the effect of varying the more important factors in much the same manner as I have already done for premiums.

EFFECT OF VARIATION IN FACTORS

It is not considered necessary to show the effect of variation in factors on reserves to the same extent as we did for premiums. Certain variations can be disposed of on the strength of general considerations. An increase or decrease in the general expense factors will have little effect. Slight variations in renewal commissions will not make changes of any moment, though a high second-year commission, say 15 per cent, such as some companies pay, would result in a reduction of reserves for the second and several following years. Such variations as are likely to occur in practice in Medical and In-

TABLE V

NATURAL RESERVES PER $1,000 INSURANCE ACCORDING TO "STANDARD ASSUMPTIONS"

Ordinary Life

YEAR	AGE						
	15	20	30	40	50	60	65
	GROSS PREMIUM						
	$11.83	$13.30	$17.40	$24.62	$37.47	$60.54	$78.41
1.....	$−4.17	$−3.99	$−3.09	$−1.70	$− .90	$−1.90	$−3.90
2.....	1.64	2.88	7.37	14.46	23.21	33.23	38.24
3.....	7.57	9.90	18.17	30.96	47.20	66.94	77.79
4.....	13.66	17.11	29.41	47.89	71.27	99.79	115.68
5.....	19.89	24.51	40.99	65.04	95.01	130.99	150.85
6.....	26.25	32.12	52.85	82.19	118.02	159.86	182.43
7.....	32.82	40.07	65.16	99.76	141.20	188.42	213.32
8.....	39.63	48.38	77.93	117.71	164.53	216.61	243.47
9.....	46.71	57.07	91.16	136.06	187.98	244.36	272.79
10.....	54.08	66.17	104.86	154.77	211.53	271.62	301.21
11.....	62.07	76.05	119.48	174.48	236.10	300.00	330.84
12.....	70.42	86.39	134.58	194.57	260.76	327.93	359.68
13.....	79.14	97.22	150.14	215.02	285.48	355.38	387.68
14.....	88.28	108.52	166.19	235.81	310.23	382.34	414.84
15.....	97.85	120.30	182.71	256.93	335.00	408.76	441.05
16.....	108.23	132.99	200.24	279.17	360.99	436.71	469.07
17.....	119.12	146.20	218.28	301.75	387.08	464.41	496.67
18.....	130.51	159.91	236.80	324.69	413.28	491.94	523.98
19.....	142.40	174.16	255.82	348.00	439.63	519.45	551.22
20.....	154.80	188.94	275.32	371.65	466.17	547.08	578.65

Twenty-Payment Life

YEAR	AGE						
	15	20	30	40	50	60	65
	GROSS PREMIUM						
	$17.99	$19.88	$24.62	$32.07	$44.07	$64.72	$81.05
1.....	$−2.43	$−2.13	$−1.05	$ .41	$ .98	$− .70	$−3.14
2.....	9.25	11.00	16.28	23.67	31.41	38.50	41.60
3.....	21.33	24.58	34.30	47.65	62.09	76.59	83.99
4.....	33.87	38.68	53.10	72.43	93.27	114.17	125.01
5.....	46.86	53.33	72.62	97.85	124.56	150.54	163.68
6.....	60.31	68.53	92.83	123.74	155.66	185.13	199.23
7.....	74.33	84.44	113.91	150.53	187.50	220.00	234.63
8.....	88.95	101.10	135.89	178.24	220.12	255.19	269.93
9.....	104.24	118.56	158.81	206.91	253.58	290.77	305.21
10.....	120.22	136.86	182.69	236.56	287.93	326.85	340.55

TABLE V—*Continued*

TWENTY-PAYMENT LIFE—*Continued*

YEAR	AGE						
	15	20	30	40	50	60	65
	GROSS PREMIUM						
	$17.99	$19.88	$24.62	$32.07	$44.07	$64.72	$81.05
11	$137.40	$156.57	$208.22	$268.10	$324.40	$365.35	$378.44
12	155.39	177.22	234.80	300.76	362.03	404.86	417.10
13	174.24	198.87	262.48	334.61	400.93	445.68	456.94
14	194.00	221.53	291.30	369.74	441.28	488.19	498.50
15	214.72	245.25	321.31	406.20	483.28	532.90	542.50
16	237.00	270.69	353.35	445.16	528.62	582.67	592.82
17	260.36	297.29	386.73	485.76	576.33	636.50	648.64
18	284.86	325.09	421.52	528.17	626.83	695.66	712.14
19	310.51	354.16	457.82	572.58	680.66	761.87	786.48
20	337.37	384.54	495.70	619.21	738.48	837.42	876.16

Twenty-Year Endowment

YEAR	AGE						
	15	20	30	40	50	60	65
	GROSS PREMIUM						
	$39.67	$39.87	$40.42	$42.76	$49.44	$65.57	$80.08
1	$ 12.01	$ 11.87	$ 11.89	$ 12.40	$ 12.90	$ 13.43	$ 13.66
2	44.71	44.45	44.69	46.30	49.05	54.37	58.75
3	78.83	78.42	78.93	81.46	85.79	94.38	101.63
4	114.49	113.91	114.75	118.00	123.40	134.08	143.28
5	151.73	150.99	152.15	155.83	161.57	172.84	182.77
6	190.64	189.76	191.14	194.83	200.06	210.13	219.38
7	231.36	230.39	231.94	235.47	239.82	248.04	256.06
8	274.01	273.00	274.65	277.81	280.96	286.63	292.91
9	318.70	317.69	319.36	321.98	323.60	326.09	330.06
10	365.54	364.56	366.17	368.07	367.88	366.57	367.69
11	415.68	414.77	416.24	417.35	415.30	410.15	408.34
12	468.26	467.44	468.67	468.90	464.87	455.53	450.40
13	523.40	522.68	523.60	522.89	516.86	503.19	494.42
14	581.25	580.62	581.18	579.55	571.62	553.72	541.21
15	641.93	641.38	641.56	639.10	629.58	607.92	591.77
16	706.84	706.38	706.22	703.20	692.84	669.05	650.37
17	774.94	774.55	774.14	770.90	760.61	736.57	716.80
18	846.39	846.09	845.55	842.61	833.67	812.39	794.03
19	921.34	921.17	920.74	918.79	913.05	899.06	886.38
20	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00

spection Fees can have only slight effect. The factors of special interest are Mortality, Interest, and First Commission. Tables VI, VII, and VIII will give a good idea of the effect of variations in these factors. All factors except the one under consideration have been held constant.

TABLE VI

INCREASE IN GROSS PREMIUMS AND NATURAL RESERVES DUE TO INCREASE IN MORTALITY FROM 90 PER CENT A.M. SELECT TO 100 PER CENT A.M. SELECT

PLAN	ORDINARY LIFE			TWENTY-PAYMENT LIFE			TWENTY-YEAR ENDOWMENT		
Age	20	40	60	20	40	60	20	40	60
Premium Increase	$ .67	$1.33	$4.15	$ .87	$1.32	$3.72	$ .29	$ .62	$3.20
Reserve Increase at end of year:									
1	— .10	0.00	— .36	— .04	0.00	— .48	—.13	— .07	.07
3	.41	1.45	2.93	.88	1.42	1.97	—.31	.04	1.66
5	.91	2.86	5.51	1.83	2.86	3.64	—.50	.05	2.39
10	2.27	6.08	9.07	4.62	6.33	4.67	—.94	— .50	.55
15	4.12	9.17	10.95	8.47	10.37	4.61	—.88	—1.09	—3.44
20	6.50	12.06	11.94	13.68	16.10	11.93	0.00	0.00	0.00

TABLE VII

INCREASE IN GROSS PREMIUMS AND NATURAL RESERVES DUE TO DECREASE IN INTEREST RATES FROM $4\frac{1}{2}$ PER CENT TO $3\frac{1}{2}$ PER CENT DURING FIRST TWENTY YEARS

PLAN	ORDINARY LIFE			TWENTY-PAYMENT LIFE			TWENTY-YEAR ENDOWMENT		
Age	20	40	60	20	40	60	20	40	60
Premium Increase	$ .44	$1.15	$2.17	$1.48	$2.45	$2.90	$3.87	$3.79	$3.23
Reserve Increase at end of year:									
1	.15	.33	.57	.42	.67	.76	1.77	1.72	1.40
3	.84	2.05	3.69	2.91	4.63	5.16	8.01	7.77	6.20
5	1.31	3.24	5.91	5.02	7.96	8.68	13.38	12.92	10.11
10	1.21	3.55	7.96	8.26	12.90	14.05	21.52	20.70	16.33
15	—1.39	— 1.00	4.11	7.35	11.44	13.48	19.04	18.53	16.01
20	—7.70	—12.63	—10.81	0.00	0.00	0.00	0.00	0.00	0.00

Table VI shows, as we should expect, that reserves according to a 90 per cent American Men table are generally smaller than reserves on a 100 per cent American Men table. The reserves on the Twenty-Year Endowment plan at the younger ages constitute an exception to the rule. It is not true that a lower mortality will always result in smaller reserves though it will be true if, as in this case, the lower mortality table is constructed by applying a con-

stant percentage to the q column of the higher table. It follows that we should be cautious in drawing deductions regarding reserves which are derived from an artificial table of the character in question.

Table VII shows the effect of assuming an interest rate as low as $3\frac{1}{2}$ per cent for the entire life of the policy instead of the higher rate of $4\frac{1}{2}$ per cent which we used for twenty years. The difference is quite considerable on the Twenty-Payment Life and the Twenty-Year Endowment forms reaching a maximum about the tenth year and receding to zero at the end of the twentieth year.

The effect of a decrease in first commission shown in Table VIII is a rather substantial increase in natural reserves, the increase

TABLE VIII

INCREASE IN GROSS PREMIUMS AND NATURAL RESERVES DUE TO DECREASE IN FIRST COMMISSION RATES

PLAN	ORDINARY LIFE			TWENTY-PAYMENT LIFE			TWENTY-YEAR ENDOWMENT		
Decrease in Commission	From 70 to 55%			From 70 to 50%			From 50 to 40%		
Age	20	40	60	20	40	60	20	40	60
Premium Increase	$-.11	$-.25	$-.96	$-.35	$-.57	$-1.44	$-.34	$-.38	$-.73
Reserve Increase at end of year:									
1	2.04	3.77	9.20	3.99	6.45	12.99	3.98	4.26	6.51
3	2.03	3.67	8.60	3.74	6.00	11.95	3.71	3.97	5.99
5	2.03	3.57	8.06	3.44	5.33	10.96	3.43	3.66	5.49
10	2.01	3.32	6.89	2.59	4.18	8.48	2.59	2.76	4.25
15	1.97	3.04	5.78	1.46	2.41	5.31	1.48	1.60	2.66
20	1.91	2.74	4.74	0.00	0.00	0.00	0.00	0.00	0.00

having a maximum value the first year and diminishing very gradually. The advantage of a reasonable first commission is obvious because it relieves the strain of deficient reserves at a time when it is greatest.

There is danger that the casual reader of the preceding tables may seriously underestimate the unfavorable effect of a higher mortality or a higher first commission on the company's surplus. These tables show the effects produced *when premiums are increased to cover the increased cost.* In company management where an increase in premium is not to be considered the problem is quite different and the effect of higher mortality or higher first commission in reducing accumulated funds during the early policy years is much greater.

GROSS PREMIUMS AND NATURAL RESERVES ACCORDING TO "STANDARD ASSUMPTION" COMPARED WITH NET PREMIUMS AND RESERVES ON AMERICAN EXPERIENCE $3\frac{1}{2}$ PER CENT ILLINOIS STANDARD BASIS

I now wish to make some comparison between natural reserves, computed according to "Standard Assumptions" on the one hand with surrender values and legal reserves on the other. For the latter I have used American Experience $3\frac{1}{2}$ per cent net reserves modified on the Illinois Standard Basis, hereafter referred to as Illinois Standard reserves.

TABLE IX

EXCESS OF GROSS PREMIUMS AND NATURAL RESERVES ACCORDING TO "STANDARD ASSUMPTIONS" OVER ILLINOIS STANDARD NET RENEWAL PREMIUMS AND RESERVES

ORDINARY LIFE

AGE	15	20	30	40	50	60	65
PREMIUM EXCESS	$—.60	$—.47	$—.28	$.26	$.88	$.62	$—.61
Reserve Excess at End of Year:							
1	— 4.17	— 3.99	— 3.09	— 1.70	— .90	— 1.90	— 3.90
3	— 3.12	— 3.23	— 1.90	— .21	— .51	— 1.23	— 1.32
5	— 2.28	— 2.69	— .53	.85	— 1.76	— 4.83	— 4.92
10	— .58	— .83	3.21	.29	—11.77	—27.05	—33.05
15	4.52	6.11	11.06	3.27	—15.94	—38.18	—58.34
20	15.61	19.18	23.75	13.44	— 7.08	—37.04	—73.61

TWENTY-PAYMENT LIFE

AGE	15	20	30	40	50	60	65
PREMIUM EXCESS	$—2.17	$—1.88	$—1.40	$—.40	$.85	$1.28	$.34
Reserve Excess at End of Year:							
1	— 2.43	— 2.13	— 1.05	.41	.98	— .70	— 3.14
3	— 5.83	— 5.57	— 3.59	— .88	.08	— .64	1.05
5	— 9.70	— 9.42	— 6.14	— 2.72	— 2.40	— 2.08	— .56
10	—20.90	—19.59	—12.98	—10.66	—14.83	—20.34	—20.59
15	—29.63	—25.40	—15.77	—13.91	—19.33	—23.99	—28.75
20	—33.18	—25.49	—12.79	— 7.71	— 8.50	—12.55	—19.62

TABLE IX—*Continued*

TWENTY-YEAR ENDOWMENT

AGE	15	20	30	40	50	60	65
PREMIUM EXCESS	$.01	$−.06	$−.40	$−.14	$.58	$.33	$−1.27
Reserve Excess at End of Year:							
1	− 8.33	− 7.09	− 3.56	1.50	6.98	11.52	12.97
3	−12.01	−11.11	− 7.36	− 1.29	5.09	12.28	16.42
5	−15.68	−15.16	−10.97	− 4.51	1.71	9.13	14.33
10	−23.51	−23.36	−19.00	−14.70	−12.03	− 9.20	− 5.40
15	−20.30	−19.97	−17.36	−15.91	−15.33	−11.75	−10.51
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00

On the ordinary Life Plan the younger ages offer a striking illustration of the fact that smaller reserves do not always go with smaller premiums. With a premium $.47 lower at age 20 we have a reserve at the end of the 20th year $19.18 higher. Our gross premiums are under the net premiums at ages 15, 20, 30, and rather strangely at 65. The difference at the younger ages is due to difference in the two mortality tables while the difference at age 65 is due to the increased effect of the Illinois Standard method of valuation at the higher ages. There is a deficiency in natural reserves at all ages at the end of the first year—in other words, the first premium is not sufficient to meet first year expenses and mortality. There is a deficiency at the end of the 10th year for every age except 30 and 40 and at the older ages where much of the Ordinary Life business is written the deficiency is large.

On the Twenty-Payment Life form gross premiums are under Illinois Standard net renewal premiums for ages 15 to 40, in amounts decreasing from $2.17 to $.40. First year reserves are deficient at all ages except 40 and 50 and the deficiency grows from year to year until at the 10th year it varies from a minimum at age 40 of $10.66 to a maximum at age 15 of $20.90. At the end of the 20th year we should note that we are comparing American Experience $3\frac{1}{2}$ per cent net single premiums with 90 per cent American Men $3\frac{1}{2}$ per cent single premiums and the deficiency varies from a minimum at age 40 of $7.71 to a maximum at age 15 of $33.18.

On the Twenty-Year Endowment plan the rate of mortality is a

less important factor in our premium rates and both sets of reserves increase from year to year to $1,000 at the end of the 20th year. The first year deficiency is found only at ages 15, 20, and 30 and the tide turns rapidly as we move to the older ages until we have excesses of $11.52 and $12.97 for ages 60 and 65. At the end of the 10th year we have a deficiency at all ages varying from $23.51 at age 15 to $5.40 at age 65. This decided sag shortly after the middle of the twenty-year period is due principally to the higher interest rate. The more favorable showing at higher ages, particularly in the first year, is due principally to the fact that the Illinois Standard method gives a very high allowance at advanced ages. For example, the Illinois Standard reserve at age 65 is only $.69—almost full preliminary term.

It goes without saying that if similar comparisons were made with reserves computed on the net level basis the deficiencies we have found would be very greatly increased.

SOME PRACTICAL EFFECTS OF FOREGOING CONDITIONS

What are some of the practical effects of these conditions?

Deficiency reserves.—No matter how confident we may be that our basic assumptions are safe and sound, we cannot adopt office premiums which are lower than our legal valuation net renewal premiums unless we are prepared to set up "deficiency" reserves. According to Mr. Corcoran in his paper read at the meeting of the National Convention of Insurance Commissioners, September, 1925, the latter would be required by six states. Such deficiency reserves are substantial and practically prohibitive if the deficiency in the premium is material

An Illinois Standard company that reduced its gross premiums to the point reached by the companies shown in column (*c*), Table I, could not use the Illinois Standard method of valuation for ages below 60 without setting up deficiency reserves required on the ground that gross premiums are less than Illinois Standard net renewal premiums. At ages where the gross premiums are equal to net level premiums the combined Illinois Standard and deficiency reserves would be exactly equal to the net level reserve. Net Level companies meet with the same embarrassment if their gross rates drop below net level premiums. The desire to avoid this embarrassment has resulted in relatively high rates at the younger ages

for the non-participating companies, giving the profit sharing companies an unfair advantage.

If the Illinois Standard method is sound, as it is generally conceded to be, we should not be prevented from using it as we are at present and as we will be to a much greater extent in the future.

The requirement of deficiency reserves is nothing less than regulation of premium rates and in my judgment is not implied in the requirement of minimum legal reserves. I believe the latter is a sufficient means of regulation, for if premiums are inadequate it will be impossible to accumulate adequate reserves and the weakness will soon come to light. If it is necessary to regulate premiums it should be done directly. The question of deficiency reserves will not arise unless there is a wide gap between the basis of legal reserves and actual conditions upon which premiums are based.

Insufficient natural reserves.—So much for the effect of gross premiums which are less than net valuation premiums. There are two other forces which drive us to higher premiums. The second force, not to be confused with the one we have just described, is natural reserves which are deficient as compared with legal reserves.

If we remember that natural reserves are the maximum amounts (the exact amounts) which can be accumulated from premiums if all assumptions are realized, we see the seriousness of deficient natural reserves. If the deficiency is small and is overcome within a few years as the policy continues, the strain on surplus may not be too great, but substantial deficiencies such as we have shown, persisting for 10 and even 20 years, must be overcome by increasing our rates.

The essential nature of this increase should be clearly understood. It is not an increase required to enable the company to carry out its contracts. It is required to cover up a book loss which would or might easily lead to legal insolvency. The additional premium will be an additional source of ultimate profit if the resulting increase in reserves is not made the basis of increased surrender values. Even so it will yield a profit in the case of policies terminated other than by surrender.

Why such considerable deficiencies should exist is a natural question and an old question, altogether too big for consideration here.

The effect of surrender values.—The third condition requiring an

increase in premiums is the minimum surrender value permitted by law and the stress of competition. The common state requirement is a surrender value differing from the legal reserve by not more than $25 per thousand. When a company has to set up a certain reserve under a policy there is quite a temptation to use the same sum as the basis for attractive surrender values. There is a striking difference, however, between the effect of putting up excessive reserves and the effect of paying excessive surrender values. Both have the same immediate effect on the surplus but the first is only a book loss, temporary in character and bound to be made good ultimately while the second is an absolute and final loss. It is of vital importance to keep this distinction in mind in fixing surrender values for the early policy years when the lapse rate is high as it is particularly apt to be with the younger company.

As the life insurance business developed, surrender values were introduced to protect the withdrawing policyholder from a serious forfeiture. The terms of the option to withdraw should be such that its exercise will not be unfavorable to the company apart from the loss of probable future profits arising from continuance. This principle, I believe, is generally accepted. If we assume that the withdrawing policyholder is an average risk, and this has been found to be a reasonably satisfactory assumption in practice, the *maximum* amount we can afford to pay him is his pro rata share of the natural reserve. This has given rise to the American conception of individual reserves. There are many situations in which the law now requires us to grant surrender values in excess of this amount. Surrender values should be incidental to the level premium insurance business but under present conditions they have become a direct additional benefit for which we are obliged to charge an extra premium. This is not a natural or a desirable state of affairs.

ADJUSTMENT IN PREMIUMS REQUIRED

We have shown that premiums computed according to "Standard Assumptions" as shown in column (*a*), Table I, in some cases require deficiency reserves or lead to natural reserves altogether inadequate to cover legal reserves or surrender values.

Few companies would feel that they could operate on a basis which would require them to wait more than 10 years for the legal reserves on their new business to be made up out of corresponding

TABLE X

ADJUSTED GROSS PREMIUMS (SEE COLUMN (*a*) TABLE I) AND ADJUSTED EXCESS OF NATURAL RESERVES OVER ILLINOIS STANDARD NET RESERVES (SEE TABLE IX)

Ordinary Life

	AGE						
	15	20	30	40	50	60	65
Gross premium (Col. (*a*) Table I)	$ 11.83	$ 13.30	$ 17.40	$ 24.62	$ 37.47	$ 60.54	$ 78.41
Increase (*a*)	.60	.47	.28	.....	.....	.....	.....
Increase (*b*)	.....	.....	.....	.....	1.02	2.05	2.22
Adjusted premium. (Col. (*d*) Table I)	12.43	13.77	17.68	24.62	38.49	62.59	80.63

Adjusted Excess (See Table IX)

YEAR							
1	$ −4.00	$ −3.86	$ −3.01	$ −1.70	$ − .61	$ −1.31	$ −3.26
3	−1.78	−2.18	−1.27	− .21	1.79	3.49	3.89
5	.35	− .63	.70	.85	2.81	4.75	5.87
10	5.86	4.22	6.23	.29	0.00	0.00	0.00
15	15.90	15.04	16.44	3.27	6.98	22.62	26.94
20	33.38	33.16	32.30	13.46	35.01	105.16	176.48

Twenty-Payment Life

	AGE						
	15	20	30	40	50	60	65
Gross premium (Col. (*a*) Table I)	$ 17.99	$ 19.88	$ 24.62	$ 32.07	$ 44.07	$ 64.72	$ 81.05
Increase (*a*)	2.17	1.88	1.40	.40	.....	.....	.....
Increase (*b*)	.....	.....	.....	.57	1.28	1.54	1.38
Adjusted premium. (Col. (*d*) Table I)	20.16	21.76	26.02	33.04	45.35	66.26	82.43

Adjusted Excess (See Table IX)

YEAR							
1	$ −1.82	$ −1.60	$ − .65	$ .68	$ 1.34	$ − .26	$ −2.74
3	− .99	−1.37	− .46	1.29	2.97	4.19	4.29
5	− .20	−1.19	0.00	1.55	3.33	5.11	6.15
10	2.39	.61	2.12	0.00	0.00	0.00	0.00
15	11.52	10.31	11.12	5.53	9.43	21.68	24.26
20	31.09	30.42	29.96	24.52	44.32	94.26	135.84

TABLE X—*Continued*

Twenty-Year Endowment

	Age						
	15	20	30	40	50	60	65
Gross premium (Col. (*a*) Table I)	$ 39.67	$ 39.87	$ 40.42	$ 42.76	$ 49.44	$ 65.57	$ 80.08
Increase (*a*)	.....	.06	.40	.14	.....	.....	.....
Increase (*b*)	2.13	2.05	1.31	1.16	1.01	.67	.35
Adjusted premium. (Col. (*d*) Table I)	41.80	41.98	42.13	44.06	50.45	66.24	80.43
Adjusted Excess (See Table IX)							
Year							
1	$−7.28	$−6.05	$−2.72	$ 2.14	$ 7.48	$ 11.85	$ 13.15
3	−6.77	−5.91	−3.14	1.92	7.61	13 99	17.33
5	−5.82	−5.38	3.04	1.55	6 50	12.45	16.13
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	20 96	20.98	16.19	10.71	7 89	8.63	3.31
20	64.20	63.85	53.13	43.98	42.49	47.54	40.45

premium income. During such a period, of course, the deficiency has to be made up out of surplus derived from other sources. We will assume then that gross premiums are not to be less than net renewal premiums on the legal valuation standard and that natural reserves must be at least equal to legal reserves at the end of the tenth year. In Table X increases in premiums due to the first cause are marked (*a*) and additional increases due to the second cause are marked (*b*).

For convenience of comparison the adjusted premiums are shown in column (*d*) of Table I.

Obviously the conditions which we adopted are not consistent with the low rate now in actual use as shown in column (*c*). Whether such rates can be adopted and, if so, on what basis, and whether the necessary conditions can be met, are questions that each company must answer for itself. I hope that the tables I have supplied may simplify the consideration of problems of this kind.

In considering the question of surrender values it is well to remember that our formulas, undoubtedly, err by understating first-year general expenses and slightly overstating general renewal expenses, so that our natural reserves overstate the sums which the company will be able to accumulate out of premiums. The higher the company's lapse rate the more important it is that surrender

TABLE XI

ADDITION TO NATURAL RESERVES RESULTING FROM AN INCREASE OF $1 PER $1,000 IN GROSS PREMIUMS

Ordinary Life and Twenty-Payment Life

YEAR	AGE						
	15	20	30	40	50	60	65
1	$ .283	$ .283	$ .283	$ .283	$ .284	$ .286	$ .288
2	1.234	1.235	1.235	1.237	1.243	1.259	1.274
3	2.232	2.233	2.234	2.239	2.257	2.304	2.349
4	3.278	3.281	3.283	3.294	3.333	3.435	3.534
5	4.376	4.380	4.383	4.404	4.478	4.671	4.859
6	5.528	5.534	5.540	5.578	5.705	6.037	6.365
7	6.736	6.745	6.754	6.816	7.017	7.544	8.071
8	8.004	8.015	8.031	8.125	8.425	9.218	10.022
9	9.335	9.348	9.372	9.511	9.940	11.088	12.274
10	10.731	10.747	10.783	10.980	11.578	13.195	14.900
11	12.223	12.241	12.295	12.566	13.382	15.614	18.023
12	13.788	13.809	13.886	14.254	15.346	18.382	21.743
13	15.431	15.454	15.564	16.054	17.496	21.575	26.224
14	17.155	17.182	17.334	17.977	19.859	25.291	31.681
15	18.964	18.997	19.204	20.037	22.470	29.657	38.414
16	20.894	20.934	21.212	22.282	25.403	34.870	46.860
17	22.919	22.971	23.339	24.700	28.679	41.114	57.545
18	25.045	25.113	25.593	27.313	32.361	48.673	71.250
19	27.278	27.367	27.986	30.145	36.525	57.918	89.083
20	29.623	29.740	30.533	33.227	41.265	69.364	112.654

Twenty-Year Endowment

YEAR	AGE						
	15	20	30	40	50	60	65
1	$ .492	$ .492	$ .493	$ .493	$ .494	$ .498	$ .501
2	1.454	1.454	1.455	1.457	1.465	1.484	1.503
3	2.462	2.464	2.465	2.471	2.491	2.546	2.598
4	3.520	3.522	3.525	3.537	3.580	3.695	3.805
5	4.629	4.633	4.637	4.660	4.740	4.951	5.156
6	5.793	5 800	5.806	5.847	5.983	6.341	6.693
7	7.014	7.023	7.034	7.099	7.313	7.874	8.436
8	8.296	8.307	8.324	8.424	8.740	9.578	10 430
9	9.641	9.655	9.680	9.826	10.276	11.483	12.733
10	11.052	11.069	11.107	11.312	11.937	13.630	15.420
11	12.560	12.579	12.634	12.917	13.766	16.096	18.616
12	14.141	14.163	14.243	14.625	15.759	18.917	22.423
13	15.802	15.826	15.940	16.446	17.939	22.173	27.010
14	17.544	17.572	17.729	18.392	20.337	25.964	32.600
15	19.372	19 406	19.619	20.478	22.987	30.419	39.496
16	21.322	21.364	21.649	22.750	25.965	35.737	48.148
17	23.368	23.422	23.799	25.197	29.290	42.110	59.095
18	25.516	25.587	26.078	27.842	33.029	49.825	73.138
19	27.772	27.864	28.498	30.710	37.257	59.263	91.413
20	30.142	30.263	31.072	33.830	42.071	70.948	115.569

values be kept within natural reserves. If there is a loss of $5 per thousand on business surrendered at the end of the second year on a lapse rate say as high as 20 per cent then there is a loss of $6.25 per thousand to be made up by each $1,000 of business persisting.

Table XI has been constructed to facilitate the computation required for Table X and others of a similar nature. It shows the increase in natural reserves corresponding to an increase of $1 per thousand in premiums on the three plans according to "Standard Assumptions." It will be noted that the same figures apply to both the Ordinary Life and the 20-Payment Life forms.

EFFECT OF LAPSE RATE

If negative natural reserves could be altogether avoided and if surrender values were neither greater nor less than natural reserves, the rate of lapse would be of no interest except as it affected the volume of insurance. These ideal conditions do not exist in practice and as a final test an accumulation of proposed premiums, on a basis giving effect to the lapse rate, may be of advantage. It seems worth while to give the following simple formula for such accumulation:

$$ {}_{n+1}V = \{({}_nV + \pi_{n+1})(1+i) - (q_{[x]+n}[1+i]^{\frac{1}{2}} + \lambda w_{n+1})\} \frac{1}{p_{[x]+n} - \lambda} $$

where

${}_nV$ = the accumulation per $1 of insurance at the end of the nth policy year.

π_n = the effective premium for the nth year; i.e., the gross premium less expenses.

i = the rate of interest assumed.

$$ \lambda = \frac{\text{number lapsing for non-payment of } (n+1) \text{ st premium}}{\text{number paying } n \text{ premiums}}. $$

w_n = surrender value per $1 of insurance at the end of the nth policy year.

q and p have the usual significance.

λ differs from the lapse rate customarily used in that the denominator of the fraction is the number entering at the beginning of the nth year rather than the number living to complete the nth year. Expression of the lapse rate in this form simplifies the formula.

FORMULAS

The formulas used for the computation of premiums and reserves appear below; the terms used are as follows:

i = the rate of interest during first twenty years.

i' = the rate of interest after twenty years.

m = medical and inspection fees per $1 of insurance.

c = general expenses, first year and renewal, per $1 of insurance.

t = general expenses, first year and renewal, per cent of premiums (Same as Mr. Cammack's)

k_1 = commission rate, first year.

k_2 = commission rate, 2d to 10th years inclusive.

k_3 = commission rate, 11th to 15th years inclusive.

k_4 = commission rate, 16th and subsequent years.

$$M_{[x]\overline{20}|} = M_{[x]} - M_{x+20}$$

$$N_{[x]\overline{20}|} = N_{[x]} - N_{x+20}$$

$$v'' = \frac{v}{v'}$$

$$r = 1 + i\ .$$

Natural Reserves:

Reserves were computed by a modified form of Fackler's formula:

(7) *a)* $${}_1V = [(1-t-k_1)P - (c+m)]U_{[x]} - r^{\frac{1}{2}}K_{[x]}$$

b) $${}_nV = [(1-t-k_2)P - c + {}_{n-1}V]U_{[x]+n-1} - r^{\frac{1}{2}}K_{[x]+n-1}$$

$n = 2$–10 inclusive.

c) $${}_nV = [(1-t-k_3)P - c + {}_{n-1}V]U_{x+n-1} - r^{\frac{1}{2}}K_{x+n-1}$$

$n = 11$–15 inclusive.

d) $${}_nV = [(1-t-k_4)P - c + {}_{n-1}V]U_{x+n-1} - r^{\frac{1}{2}}K_{x+n-1}$$

$n = 16$ and subsequent.

The following formulas were used to check premiums and final reserve values computed by formulas (7).

Gross Premiums:

(1) $$P_{[x]}=\frac{M_{[x]\overline{20}|}r^{\frac{1}{2}}+v''^{x+20}M'_{x+20}r'^{\frac{1}{2}}+mD_{[x]}+c(N_{[x]\overline{20}}+v''^{x+20}N'_{x+20})}{(1-t-k_1)N_{[x]}+(k_1-k_2)N_{[x]+1}+(k_2-k_3)N_{x+10}+(k_3-k_4)N_{x+15}+(1-t-k_4)(v''^{x+20}N'_{x+20}-N_{x+20})}$$

(2) $$_{20}P_{[x]}=\frac{M_{[x]\overline{20}|}r^{\frac{1}{2}}+v''^{x+20}M_{x+20}r'^{\frac{1}{2}}+mD_{[x]}+cN_{[x]\overline{20}|}}{(1-t-k_1)N_{[x]}+(k_1-k_2)N_{[x]+1}+(k_2-k_3)N_{x+10}+(k_3-k_4)N_{x+15}-(1-t-k_4)N_{x+20}}$$

(3) $$P_{[x]\overline{20}|}=\frac{M_{[x]\overline{20}|}r^{\frac{1}{2}}+D_{x+20}+mD_{[x]}+cN_{[x]\overline{20}|}}{(1-t-k_1)N_{[x]}+(k_1-k_2)N_{[x]+1}+(k_2-k_3)N_{x+10}+(k_3-k_4)N_{x+15}-(1-t-k_4)N_{x+20}}$$

When the interest rate remains constant throughout the duration of the policy, these formulas become:

(4) $$P_{[x]}=\frac{M_{[x]}r^{\frac{1}{2}}+mD_{[x]}+cN_{[x]}}{(1-t-k_1)N_{[x]}+(k_1-k_2)N_{[x]+1}+(k_2-k_3)N_{x+10}+(k_3-k_4)N_{x+15}}$$

(5) $$_{20}P_{[x]}=\frac{M_{[x]}r^{\frac{1}{2}}+mD_{[x]}+cN_{[x]\overline{20}|}}{(1-t-k_1)N_{[x]}+(k_1-k_2)N_{[x]+1}+(k_2-k_3)N_{x+10}+(k_3-k_4)N_{x+15}-(1-t-k_4)N_{x+20}}$$

(6) $P_{[x]\overline{20}|}=$ Same as (3).

RETROSPECTIVE

$$(8)\qquad {}_{20}V=\frac{\begin{array}{r}[(1-t-k_1)P-(c+m)]N_{[x]}+[(k_1-k_2)P+m]N_{[x]+1}\\ +(k_2-k_3)PN_{x+10}+(k_3-k_4)PN_{x+15}\\ -[(1-t-k_4)P-c]N_{x+20}-r^{\frac{1}{2}}M_{[x]\overline{20|}}\end{array}}{D_{x+20}}$$

PROSPECTIVE

$$(9)\qquad {}_{20}V_{[x]}=\frac{M'_{x+20}r'^{\frac{1}{2}}-[(1-t-k_4)P_{[x]}-c]N'_{x+20}}{D'_{x+20}}$$

$$(10)\qquad {}_{20:\overline{20}|}V_{[x]}=\frac{M'_{x+20}r'^{\frac{1}{2}}}{D'_{x+20}}$$

$$(11)\qquad {}_{20}V_{[x]\overline{20|}}=1$$

OSCULATORY INTERPOLATION WITH UNEQUAL INTERVALS

By J. F. REILLY

All the osculatory interpolation formulas in the literature with one exception[1], so far as I have been able to find them, are based on the assumption that values of the argument proceed by equal steps, or, as we shall say, are based on equal intervals; non-osculatory formulas with unequal intervals, however, are well known. The object of this paper is to show that the change from equal to unequal intervals does not materially complicate the problem of developing osculatory interpolation formulas and to point out that with unequal intervals the same degree of generality is attainable as with equal intervals. We will briefly indicate how to develop an osculatory interpolation formula analogous to Sprague's formula, but based on unequal intervals.

The polynomial of fourth degree with given values $f(x)$, $x=a_1$, a_2, a_3, a_4, a_5, may be written[2]

$$\begin{aligned} u_x \equiv f(a_1) &+ (x-a_1)\,f(a_1,\ a_2) + (x-a_1)(x-a_2)\,f(a_1,\ a_2,\ a_3) \\ &+ (x-a_1)(x-a_2)(x-a_3)\,f(a_1,\ a_2,\ a_3,\ a_4) \\ &+ (x-a_1)(x-a_2)(x-a_3)(x-a_4)\,f(a_1,\ a_2,\ a_3,\ a_4,\ a_5)\ , \qquad (a) \end{aligned}$$

where $f(a_1,\ a_2)$, $f(a_1,\ a_2,\ a_3)$, $f(a_1,\ a_2,\ a_3,\ a_4)$, and $f(a_1,\ a_2,\ a_3,\ a_4,\ a_5)$ are divided differences of orders one, two, three, and four respectively.

Similarly, the polynomial of fourth degree with given values $f(x)$, $x=a_2$, a_3, a_4, a_5, a_6, may be written

$$\begin{aligned} v_x \equiv u_x + (a_6-a_1)(x-a_2)(x-a_3)(x-a_4)(x-a_5) \\ f(a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)\ . \qquad (b) \end{aligned}$$

[1] *J. I. A.*, XLIX, 369 (Actuarial Notes). In the note "On Osculatory Interpolation where the given values of the function are at unequal intervals," the writer, T. G. A., develops a formula involving third order differences, in which the slopes, but not the curvatures, are equal at the junction points.

[2] Whittaker and Robinson, *The Calculus of Observations* (1924), p. 25; A. Henry, *Calculus and Probability* (1922), p. 52.

FIRST METHOD

As in the case of equal intervals the problem is to determine the coefficients in a polynomial of the fifth degree

$$y_x = c_0 + c_1(x-a_3) + c_2(x-a_3)^2 + c_3(x-a_3)^3 + c_4(x-a_3)^4 + c_5(x-a_3)^5,$$

so that the following six conditions will be satisfied: that this polynomial will have the same value as the polynomial (a) when $x=a_3$, and the same value as the polynomial (b) when $x=a_4$; that the first and second derivatives of this polynomial will have the same values as the corresponding derivatives of (a) when $x=a_3$, and the same values as the corresponding derivatives of (b) when $x=a_4$.

Imposing these conditions we are led to the following system of equations to determine the c's:

$$c_0 = u_{a_3}$$
$$c_1 = u'_{a_3}$$
$$2c_2 = u''_{a_3}$$

$$\begin{aligned} c_0 + c_1(a_4-a_3) + c_2(a_4-a_3)^2 + c_3(a_4-a_3)^3 + c_4(a_4-a_3)^4 + c_5(a_4-a_3)^5 &= v_{a_4} \\ c_1 + 2c_2(a_4-a_3) + 3c_3(a_4-a_3)^2 + 4c_4(a_4-a_3)^3 + 5c_5(a_4-a_3)^4 &= v'_{a_4} \\ 2c_2 + 6c_3(a_4-a_3) + 12c_4(a_4-a_3)^2 + 20c_5(a_4-a_3)^3 &= v''_{a_4}. \end{aligned}$$

The values of c_0, c_1, and c_2 are given directly by the first three equations, and then the values of c_3, c_4, and c_5 are determined from the equations:

$$\begin{aligned} c_3 + c_4(a_4-a_3) + c_5(a_4-a_3)^2 &= F_1(a_1, a_2, a_3, a_4, a_5) \\ 3c_3 + 4c_4(a_4-a_3) + 5c_5(a_4-a_3)^2 &= F_2(a_1, a_2, a_3, a_4, a_5, a_6) \\ 6c_3 + 12c_4(a_4-a_3) + 20c_5(a_4-a_3)^2 &= F_3(a_1, a_2, a_3, a_4, a_5, a_6) \end{aligned}$$

where F_1, F_2, F_3 are known functions of the a's.

On comparing these equations with the corresponding equations for equal intervals,[1] we see that the two systems are identical in type, if c_3, $c_4(a_4-a_3)$, $c_5(a_4-a_3)^2$ be here regarded as the unknown quantities. This system of equations can, therefore, be solved by the methods employed for equal intervals, and the c's being determined the fifth degree polynomial will serve as an osculatory interpolation formula.

[1] Reilly, *The Record*, XIII, Part I, No. 27 (1924), p. 11.

SECOND METHOD

If we follow Lidstone's method the problem would then be to determine a fifth degree polynomial, ψ_x, so that $u_x+\psi_x$ will satisfy the same six conditions as before.

From four of these conditions we infer that ψ_x contains $(x-a_3)^3(x-a_4)$ as a factor, and hence we write

$$\psi_x=(x-a_4)[c_3(x-a_3)^3+c_4(x-a_3)^4] .$$

Imposing the remaining two conditions we find that

$$c_3(a_4-a_3)^3+\ c_4(a_4-a_3)^4=v'_{a_4}-u'_{a_4}$$
$$6c_3(a_4-a_3)^2+8c_4(a_4-a_3)^3=v''_{a_4}-u''_{a_4} .$$

Hence c_3 and c_4 are determined from the equations

$$c_3+\ c_4(a_4-a_3)=\phi_1(a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$$
$$3c_3+4c_4(a_4-a_3)=\phi_2(a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6) ,$$

when φ_1 and φ_2 are known functions of the a's.

Comparing these with the corresponding equations[1] for equal intervals, we see that they are identical in type if c_3, $c_4(a_4-a_3)$ be here regarded as the unknown quantities.

Solving these equations we are led to the same osculatory interpolation formula as before.

The methods here employed are general, and will apply equally well when divided differences to order $2h+1$ are used, and when the osculatory interpolation curve has contact of order k with its partial interpolation curves. That is, the osculatory formula based on unequal intervals lends itself to the same generalizations as the formula based on equal intervals.

OSCULATORY INTERPOLATION FORMULAS FOR SPECIAL VALUES OF THE a'S

When the values of the function corresponding to -3, -2, 0, 1, 2, 4 of the argument are given, and we wish to interpolate between 0 and 1, either of the methods described will lead to the formula:

$$y_x=f(-3)+(x+3)f(-3,\ -2)+(x+3)(x+2)f(-3,\ -2,\ 0)$$
$$+(x+3)(x+2)xf(-3,\ -2,\ 0,\ 1)$$

[1] Reilly, *The Record*, XIV, Part 1, No. 29 (1925), p. 15.

$$+(x+3)(x+2)x(x-1)f(-3,\ -2,\ 0,\ 1,\ 2)$$
$$+x^3(x-1)(2x-5)f(-3,\ -2,\ 0,\ 1,\ 2,\ 4)\ .$$

This is the ordinary divided-differences formula except for the final term which has been modified for osculation purposes.

When the set of argumental values is $-3,\ -1,\ 0,\ 1,\ 3,\ 4$ the formula is

$$y_x=f(-3)+(x+3)f(-3,\ -1)+(x+3)(x+1)f(-3,\ -1,\ 0)$$
$$+(x+3)(x+1)xf(-3,\ -1,\ 0,\ 1)$$
$$+(x+3)(x+1)x(x-1)f(-3,\ -1,\ 0,\ 1,\ 3)$$
$$+28x^3(x-1)(2x-3)f(-3,\ -1,\ 0,\ 1,\ 3,\ 4)\ ,$$

where, again, only the final term of the divided-differences formula has been modified.

In proceeding from interval to interval it will be necessary to change the formula at each step, whereas Sprague's formula for equal intervals remains the same.

VALUATION CONSTANTS FOR AN ATTAINED-AGE VALUATION, ILLINOIS STANDARD

By WILBUR M. JOHNSON

The growth of life insurance in recent years with its multiplicity of policy forms has called for a corresponding revision and improvement in office systems. The desire for increased efficiency with decreased overhead expense and lower net costs has directed many to a study of systems and methods. The importance of study and planning increases with the expanding business.

For many years the attained-age method of valuation has been discussed, but it appears not to have been used much until the installation of perforated cards demonstrated its practical advantages. A recent paper by Edward W. Marshall (*T.A.S.A.*, XXV, 219) gives a description of the attained-age method of valuation and sets out its advantages for the modern office.

It is interesting to note that, for Mr. Marshall's Company, a valuation by the attained-age method requires about ten units of work as against one hundred units under the American-group method. A test of this method on a small section of our own company's business showed about twenty units of work as against one hundred units by the American-group method. The saving effected by an attained-age method obviously depends on the policy plans of a company's business. Policies with increasing coupons or other varying benefits are not so easily handled as those with level benefits. However, the saving effected by those companies using this method is evidence of the practical importance of the question to a life office. Moreover the method lends itself readily to the practice of doing most of the work of the valuation throughout the year, so that the valuation may be completed in a very short time at the end of the year. Altogether the topic appears worthy of careful consideration by actuaries.

The general formula is given as

$$(1) \qquad \sum_{n=1}^{n=y} {}_nV_{y-n} = (\Sigma S)A + (\Sigma\, {}^s\theta)B - (\Sigma\, {}^sP)C$$

where $A = A_y$, $B = 1/D_y$, and $C = a_y$.

θ is the valuation constant and is based on the plan of insurance and the age at issue. ${}^{s}\theta$ is θ multiplied by the amount of insurance (S). For obtaining mean reserves, y being the attained age on the current year's anniversary, the general formula is used except that for A we write $\frac{A_y+A_{y+1}}{2}$; for B, $\frac{1}{2}\left(\frac{1}{D_y}+\frac{1}{D_{y+1}}\right)$; and for C, $\frac{1+a_y+a_{y+1}}{2}$.

Heretofore, most of the papers dealing with this topic, as well as the constants published in the actuarial literature, have to do with the valuation of policies on the level premium basis. I do not find much available particularly for policies valued on the Preliminary Term or on the Illinois Standard modification.

That the *Record* may make them accessible to others interested in this method, I append valuation constants for renewal years for policy forms valued according to the Illinois Standard. As many companies value their term policies by the level premium method, the constants appended for term policies are on the level premium basis. It should be noted that the constants for Paid-up Term Insurance given on page 49 may be taken directly from the commutation columns. They are included here for convenient reference. Values of $1{,}000\left(\frac{A_y+A_{y+1}}{2}\right)$, $100{,}000\,(\frac{1}{2})\left(\frac{1}{D_y}+\frac{1}{D_{y+1}}\right)$, and $\frac{a_y+a_{y+1}+1}{2}$ are added also for convenient reference.

One very simple method of developing a reserve formula for the attained-age method is first to write down a general formula for a level premium policy retrospectively. Such a formula may be written as follows, where ${}_tV_x$ represents the reserve at the end of the tth year for the policy in question and π the net annual premium for such policy:

$$(2) \qquad {}_tV_x=\frac{\pi(N_x-N_{x+t})-(M_x-M_{x+t})}{D_{x+t}}$$

which we may write, remembering $M_x=P_x \cdot N_x$ in the form

$$(3) \qquad {}_tV_x=A_{x+t}-\pi \cdot a_{x+t}+(\pi_x-P_x)N_x \cdot \frac{1}{D_{x+t}}.$$

If the premium for the first year be π and for the renewal years $\pi+r$ we may write for the end of any renewal year

$$(4) \qquad {}_tV_x=\frac{\pi(N_x-N_{x+t})+r(N_{x+1}-N_{x+t})-(M_x-M_{x+t})}{D_{x+t}}$$

which may be written

$$(5) \qquad {}_tV_x = A_{x+t} - (\pi + r)a_{x+t} + [(\pi - P_x)N_x + rN_{x+1}] \cdot \frac{1}{D_{x+t}} .$$

Generally if the net annual premium be π for the first m years, $\pi + r$ from the $(m+1)$th to the nth year, and $\pi + s$ from the $(n+1)$th to the qth year the reserve at the end of any year t during the first m years will be

$$(6) \qquad {}_tV_x = \frac{\pi(N_x - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}$$

$$(7) \qquad = A_{x+t} - \pi \cdot a_{x+t} + (\pi - P_x)N_x \cdot \frac{1}{D_{x+t}}$$

The value at the end of any year t, $(m < t \leqq n)$, will be

$$(8) \qquad {}_tV_x = \frac{\pi(N_x - N_{x+t}) + r(N_{x+m} - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}$$

$$(9) \qquad = A_{x+t} - (\pi + r)a_{x+t} + [(\pi - P_x)N_x + rN_{x+m}] \frac{1}{D_{x+t}}$$

and the value at the end of any year t, $(n < t \leqq q)$ will be

$$(10) \quad {}_tV_x = \frac{\pi(N_x - N_{x+t}) + r(N_{x+m} - N_{x+t}) + s(N_{x+n} - N_{x+t}) - (M_x - M_{x+t})}{D_{x+t}}$$

$$(11) \qquad = A_{x+t} - (\pi + r + s)a_{x+t} + [(\pi - P_x)N_x + rN_{x+m} + sN_{x+n}] \frac{1}{D_{x+t}}$$

For an extended discussion of retrospective valuation the reader is referred to a paper by Frederick Bell, *J.I.A.*, XXXIX, 17.

Many simplifications in the foregoing formulas will occur in actual practice. For example the reserve for a 20 Pay Life Preliminary Term policy may be written

$$(12) \qquad {}_{t:19}V_{x+1} = \frac{{}_{19}P_{x+1}(N_{x+1} - N_{x+t}) - (M_{x+1} - M_{x+t})}{D_{x+t}}$$

$$(13) \qquad = A_{x+t} - {}_{19}P_{x+1} \cdot a_{x+t} + ({}_{19}P_{x+1} - P_{x+1})N_{x+1} \cdot \frac{1}{D_{x+t}}$$

$$(14) \qquad = A_{x+t} - {}_{19}P_{x+1} \cdot a_{x+t} + {}_{19}P_{x+1} \cdot N_{x+20} \cdot \frac{1}{D_{x+t}}$$

The constant ${}_{19}P_{x+1} \cdot N_{x+20}$ may be computed very readily by machine.

The value of θ is given per 1,000 of insurance and two correct digits to the right of the decimal have been retained. In making the original computation the net premium per thousand to the nearest one-hundredth of a cent was used and the constant computed to four decimal places to the right of the decimal found of which the first three were usually accurate. I do not believe it practicable to attempt to carry on the perforated card the constant to more than two places to the right of the decimal.

From the nature of this method it is apparent that we should not expect to secure an individual reserve accurate to the cent without carrying the net premium and constant to more decimals, but any such inaccuracies in an individual reserve obtained from the accompanying constants should not materially affect the accuracy of the aggregate reserves.

I desire to express my appreciation to Mr. Alexander C. Wellman for his assistance in computing and checking the constants given herein.

TABLE I

VALUE OF θ FOR RENEWAL YEARS—LIMITED PAYMENT LIFE—AMERICAN EXP. $3\frac{1}{2}$ PER CENT—ILLINOIS STANDARD

Age	10 Payment	15 Payment	20 Payment
15	256.78	146.12	92.10
16	247.75	140.68	88.43
17	239.02	135.42	84.88
18	230.56	130.34	81.44
19	222.41	125.41	78.11
20	214.48	120.64	74.88
21	206.81	116.01	71.75
22	199.37	111.51	68.70
23	192.17	107.17	65.76
24	185.17	102.93	62.88
25	178.38	98.82	60.10
26	171.78	94.82	57.39
27	165.37	90.94	54.76
28	159.14	87.17	52.20
29	153.08	83.49	49.70
30	147.17	79.91	47.28
31	141.43	76.42	44.91
32	135.83	73.02	42.62
33	130.36	69.70	40.38
34	125.03	66.47	38.20
35	119.82	63.32	36.08
36	114.74	60.24	34.02
37	109.77	57.23	32.01
38	104.91	54.30	30.07
39	100.15	51.44	28.17
40	95.50	48.65	26.34
41	90.95	45.93	24.56
42	86.50	43.28	22.84
43	82.14	40.69	21.18
44	77.88	38.18	19.58
45	73.72	35.74	18.03
46	69.65	33.37	16.55
47	65.67	31.07	15.12
48	61.78	28.84	13.76
49	57.98	26.68	12.46
50	54.28	24.59	11.22
51	50.66	22.58	10.05
52	47.15	20.64	8.95
53	43.73	18.79	7.91
54	40.42	17.01	6.95

TABLE I—*Continued*

Age	10 Payment	15 Payment	20 Payment
55	37.22	15.33	6.05
56	34.13	13.73	5.23
57	31.16	12.22	4.47
58	28.31	10.80	3.78
59	25.59	9.49	3.16
60	23.01	8.26	2.60
61	.....	.....	2.11
62	.....	.....	1.68
63	.....	.....	1.31
64	.....	.....	.99
65	.....	.....	.73

TABLE II

VALUE OF θ FOR RENEWAL YEARS—ENDOWMENT INSURANCE
AMERICAN EXP. $3\frac{1}{2}$ PER CENT—ILLINOIS STANDARD

Age	10 E	15 E	20 E	25 E		30 E	
				2d to 20th year	thereafter	2d to 20th year	thereafter
15	935.99	532.61	335.70	222.83	218.60	150.76	146.52
16	890.57	505.71	317.88	210.27	206.18	141.61	137.52
17	847.05	479.93	300.81	198.26	194.30	132.87	128.91
18	805.34	455.25	284.46	186.76	182.94	124.51	120.69
19	765.35	431.59	268.81	175.76	172.07	116.54	112.85
20	727.05	408.93	253.83	165.24	161.68	108.92	105.36
21	690.32	387.23	239.49	155.18	151.75	101.65	98.22
22	655.15	366.45	225.77	145.57	142.27	94.72	91.41
23	621.45	346.56	212.65	136.40	133.21	88.13	84.93
24	589.14	327.49	200.08	127.62	124.56	81.83	78.77
25	558.21	309.24	188.07	119.25	116.30	75.86	72.91
26	528.58	291.78	176.59	111.26	108.44	70.17	67.35
27	500.21	275.08	165.62	103.65	100.94	64.78	62.07
28	473.03	259.09	155.14	96.40	93.80	59.67	57.07
29	447.02	243.80	145.14	89.49	87.00	54.83	52.34
30	422.11	229.18	135.59	82.93	80.55	50.26	47.88
31	398.28	215.21	126.48	76.70	74.42	44.46	44.46
32	375.47	201.86	117.80	70.78	68.61	40.43	40.43
33	353.65	189.10	109.54	65.17	63.10	36.64	36.64
34	332.78	176.92	101.67	59.86	57.89	33.10	33.10
35	312.82	165.30	94.19	54.85	52.98	29.78	29.78
36	293.74	154.22	87.09	50.11	48.34	26.68	26.68
37	275.52	143.65	80.35	45.66	43.98	23.81	23.81
38	258.10	133.59	73.97	41.46	39.88	21.15	21.15
39	241.47	124.02	67.93	37.53	36.04	18.68	18.68
40	225.61	114.92	62.22	33.85	32.45	16.42	16.42
41	210.48	106.28	56.84	30.41	29.10	14.35	14.35
42	196.06	98.09	51.77	27.21	25.99	12.46	12.46
43	182.33	90.33	47.01	24.25	23.10	10.74	10.74
44	169.28	82.99	42.55	21.50	20.44	9.20	9.20
45	156.87	76.05	38.37	18.97	17.98	7.81	7.81
46	145.08	69.51	34.47	16.16	16.16	6.58	6.58
47	133.91	63.35	30.84	14.05	14.05	5.48	5.48
48	123.32	57.56	27.47	12.15	12.15	4.52	4.52
49	113.30	52.13	24.35	10.42	10.42	3.68	3.68
50	103.83	47.04	21.47	8.87	8.87	2.96	2.96
51	94.88	42.28	18.83	7.49	7.49	2.34	2.34
52	86.46	37.85	16.41	6.26	6.26	1.81	1.81
53	78.54	33.74	14.21	5.18	5.18	1.38	1.38
54	71.10	29.93	12.22	4.23	4.23	1.02	1.02
55	64.14	26.41	10.43	3.40	3.40	.73	.73
56	57.63	23.18	8.82				
57	51.57	20.23	7.39				
58	45.94	17.53	6.13				
59	40.73	15.10	5.02				
60	35.93	12.90	4.06				

TABLE III

VALUE OF θ FOR RENEWAL YEARS—ENDOWMENT INSURANCE AT AGE EIGHTY-FIVE
AMERICAN EXP. $3\frac{1}{2}$ PER CENT—ILLINOIS STANDARD

Age	10 E 85	15 E 85	20 E 85	Cont. Prem. End. 85
15	257.59	146.70	92.58	.43
16	248.55	141.27	88.91	.43
17	239.81	136.01	85.36	.43
18	231.36	130.92	81.92	.44
19	223.17	125.98	78.58	.44
20	215.26	121.21	75.35	.44
21	207.58	116.58	72.22	.45
22	200.14	112.08	69.17	.45
23	192.94	107.74	66.23	.45
24	185.92	103.49	63.34	.45
25	179.13	99.37	60.56	.46
26	172.53	95.38	57.85	.46
27	166.12	91.49	55.21	.47
28	159.88	87.71	52.64	.47
29	153.81	84.03	50.15	.47
30	147.90	80.44	47.72	.48
31	142.15	76.95	45.35	.48
32	136.54	73.55	43.05	.49
33	131.06	70.22	40.81	.49
34	125.73	66.98	38.62	.50
35	120.51	63.83	36.50	.50
36	115.42	60.74	34.44	.51
37	110.44	57.73	32.43	.52
38	105.57	54.79	30.48	.52
39	100.81	51.93	28.58	.53
40	96.15	49.13	26.74	.54
41	91.59	46.40	24.96	.55
42	87.13	43.75	23.23	.56
43	82.77	41.16	21.57	.57
44	78.50	38.64	19.96	.58
45	74.32	36.19	18.41	.59
46	70.24	33.81	16.92	.60
47	66.25	31.51	15.49	.61
48	62.35	29.27	14.13	.62
49	58.55	27.10	12.82	.64
50	54.83	25.01	11.58	.65
51	51.21	22.99	10.41	.67
52	47.68	21.05	9.30	.69
53	44.26	19.19	8.26	.70
54	40.94	17.41	7.29	.72

TABLE III—*Continued*

Age	10 E 85	15 E 85	20 E 85	Cont. Prem. End. 85
55	37.72	15.72	6.39	.74
56	34.62	14.11	5.56	.77
57	31.64	12.60	4.80	.79
58	28.78	11.17	4.11	.81
59	26.06	9.85	3.48	.84
60	23.46	8.62	2.92	.87
61	21.01	7.48	2.43	.90
62	18.69	6.44	1.99	.94
63	16.53	5.50	1.62	.97
64	14.51	4.64	1.31	1.02
65	12.65	3.88	1.05	1.05

TABLE IV

VALUE OF $-\theta$ FOR ALL YEARS—TERM INSURANCE
AMERICAN EXP. $3\frac{1}{2}$ PER CENT—NET LEVEL PREMIUMS

Age	5 Year	10 Year	15 Year	20 Year
15	59.51	58.38	57.07	55.54
16	59.15	57.98	56.64	55.08
17	58.77	57.56	56.20	54.59
18	58.35	57.14	55.75	54.10
19	57.93	56.71	55.29	53.58
20	57.50	56.27	54.81	53.05
21	57.07	55.82	54.33	52.50
22	56.66	55.37	53.83	51.94
23	56.21	54.92	53.33	51.35
24	55.79	54.44	52.81	50.74
25	55.34	53.96	52.26	50.12
26	54.87	53.46	51.70	49.46
27	54.41	52.96	51.13	48.77
28	53.94	52.43	50.52	48.04
29	53.45	51.89	49.90	47.28
30	52.94	51.33	49.25	46.47
31	52.43	50.74	48.57	45.61
32	51.90	50.13	47.86	44.70
33	51.34	49.50	47.10	43.73
34	50.76	48.85	46.31	42.70
35	50.17	48.16	45.47	41.62
36	49.54	47.45	44.57	40.48
37	48.88	46.70	43.62	39.29
38	48.21	45.90	42.61	38.03
39	47.52	45.07	41.55	36.72
40	46.78	44.18	40.42	35.36
41	46.02	43.23	39.24	33.95
42	45.21	42.21	38.00	32.48
43	44.35	41.14	36.70	30.97
44	43.43	40.00	35.35	29.43
45	42.45	38.80	33.95	27.85
46	41.39	37.55	32.50	26.24
47	40.27	36.23	31.01	24.60
48	39.08	34.87	29.48	22.95
49	37.83	33.46	27.92	21.30
50	36.53	32.00	26.33	19.65
51	35.17	30.51	24.73	18.01
52	33.78	28.99	23.11	16.40
53	32.35	27.45	21.49	14.82
54	30.88	25.88	19.87	13.30

As the values above are for $-\theta$ they should be applied negatively.

TABLE IV—*Continued*

Age	5 Year	10 Year	15 Year	20 Year
55	29.38	24.30	18.26	11.85
56	27.86	22.71	16.68	10.47
57	26.32	21.12	15.13	9.18
58	24.76	19.54	13.63	7.97
59	23.20	17.97	12.18	6.85
60	21.63	16.43	10.81	5.81
61	20.07	14.92	9.52	4.87
62	18.53	13.45	8.32	4.01
63	17.00	12.04	7.20	3.25
64	15.50	10.69	6.17	2.58
65	14.04	9.43	5.23	2.00

As the values above are for $-\theta$ they should be applied negatively.

TABLE V

VALUES OF θ FOR PAID-UP INSURANCE
AMERICAN EXPERIENCE $3\frac{1}{2}$ PER CENT

Age	Paid-Up Endowment Insurance*	Paid-Up Term Insurance†	Age	Paid-Up Endowment Insurance*	Paid-Up Term Insurance†
15	422.38	152.34	55	42.23	55.11
16	402.94	148.10	56	38.94	53.36
17	384.31	144.02	57	35.82	51.59
18	366.44	140.10	58	32.86	49.78
19	349.31	136.32	59	30.07	47.95
20	332.89	132.67	60	27.43	46.09
21	317.15	129.16	61	24.94	44.19
22	302.05	125.78	62	22.60	42.26
23	287.58	122.51	63	20.41	40.30
24	273.72	119.35	64	18.36	38.31
25	260.42	116.31	65	16.44	36.29
26	247.69	113.38	66	14.66	34.25
27	235.48	110.54	67	13.00	32.18
28	223.77	107.80	68	11.48	30.10
29	212.56	105.15	69	10.07	28.01
30	201.82	102.59	70	8.78	25.93
31	191.52	100.11	71	7.61	23.85
32	181.66	97.71	72	6.54	21.79
33	172.21	95.39	73	5.59	19.77
34	163.16	93.14	74	4.73	17.81
35	154.50	90.95	75	3.97	15.91
36	146.20	88.83	76	3.29	14.10
37	138.25	86.76	77	2.71	12.38
38	130.64	84.76	78	2.20	10.76
39	123.36	82.80	79	1.76	9.25
40	116.39	80.89	80	1.39	7.85
41	109.71	79.02	81	1.07	6.56
42	103.33	77.20	82	.81	5.39
43	97.23	75.41	83	.61	4.34
44	91.39	73.65	84	.44	3.43
45	85.81	71.93	85	.32	2.63
46	80.47	70.23	86	.21	1.97
47	75.38	68.54	87	.13	1.41
48	70.51	66.87	88	.08	.96
49	65.86	65.21	89	.05	.61
50	61.43	63.55	90	.02	.36
51	57.21	61.89	91	.01	.19
52	53.18	60.22	92	.00	.09
53	49.34	58.53	93	.00	.03
54	45.70	56.83	94	.00	.01

* Maturing at age shown in age column.

† Expiring at age shown in age column. As values in this column are for $-\theta$, they should be applied negatively.

TABLE VI

y	$1{,}000 \cdot \frac{1}{2}(A_y + A_{y+1})$	$100{,}000 \cdot \frac{1}{2}\left(\frac{1}{D_y} + \frac{1}{D_{y+1}}\right)$	$\frac{a_y + a_{y+1} + 1}{2}$
15	266.9186	1.777366	21.1782
16	270.6836	1.853750	21.0669
17	274.5902	1.933471	20.9514
18	278.6410	2.016687	20.8316
19	282.8383	2.103566	20.7075
20	287.1884	2.194274	20.5788
21	291.6941	2.288998	20.4456
22	296.3582	2.387933	20.3077
23	301.1876	2.491274	20.1648
24	306.1893	2.599224	20.0170
25	311.3710	2.711996	19.8638
26	316.7364	2.829832	19.7051
27	322.2899	2.952985	19.5408
28	328.0395	3.081706	19.3708
29	333.9900	3.216279	19.1948
30	340.1463	3.357003	19.0128
31	346.5174	3.504176	18.8244
32	353.1092	3.658135	18.6295
33	359.9235	3.819259	18.4280
34	366.9666	3.987930	18.2197
35	374.2486	4.164529	18.0044
36	381.7727	4.349516	17.7818
37	389.5424	4.543380	17.5521
38	397.5612	4.746652	17.3150
39	405.8332	4.959901	17.0704
40	414.3628	5.183742	16.8181
41	423.1546	5.418837	16.5582
42	432.2139	5.665908	16.2902
43	441.5426	5.925770	16.0144
44	451.1394	6.172809	15.7306
45	461.0033	6.487700	15.4389
46	471.1302	6.791974	15.1394
47	481.5125	7.113535	14.8324
48	492.1431	7.453900	14.5180
49	503.0044	7.814932	14.1968
50	514.0788	8.198743	13.8694
51	525.3523	8.607673	13.5360
52	536.8106	9.044384	13.1972
53	548.4396	9.511920	12.8532
54	560.2215	10.013853	12.5048

TABLE VI—*Continued*

y	$1{,}000 \cdot \frac{1}{2}(A_y + A_{y+1})$	$100{,}000 \cdot \frac{1}{2}\left(\frac{1}{D_y} + \frac{1}{D_{y+1}}\right)$	$\frac{a_y + a_{y+1} + 1}{2}$
55	572.1384	10.554304	12.1524
56	584.1716	11.138067	11.7966
57	596.3018	11.770735	11.4379
58	608.5115	12.458777	11.0768
59	620.7788	13.209949	10.7141
60	633.0798	14.033477	10.3504
61	645.3910	14.940311	9.9863
62	657.6883	15.943533	9.6226
63	669.9502	17.058747	9.2600
64	682.1530	18.304908	8.8992
65	694.2672	19.705466	8.5410
66	706.2644	21.289055	8.1862
67	718.1198	23.090785	7.8356
68	729.8042	25.154690	7.4901
69	741.2918	27.535584	7.1504
70	752.5592	30.302510	6.8172
71	763.5879	33.542600	6.4910
72	774.3784	37.364509	6.1720
73	784.9492	41.905037	5.8594
74	795.3290	47.339592	5.5524
75	805 5476	53.898185	5.2502
76	815.6306	61.888195	4.9520
77	825.5922	71.730898	4.6575
78	835.4308	84.016171	4.3666
79	845.1416	99.579201	4.0794
80	854.6834	119.662100	3.7972
81	864.0250	146.095444	3.5210
82	873.1946	181.595814	3.2498
83	882.2568	230.330078	2.9818
84	891.2770	299.035099	2.7151
85	900.2319	399.497711	2.4503
86	908.9986	553.651606	2.1910
87	917.4035	804.810207	1.9425
88	925.3444	1,242.866764	1.7076
89	932.9162	2,067.249832	1.4838
90	940.2838	3,782.174133	1.2659
91	947.3366	7,959.979508	1.0574
92	953.5203	20,999.68858	.8745
93	958.9078	75,931.39445	.7152
94	963.8498	498,120.3106	.5690

REINSTATEMENT OF POLICIES IN FORCE AS EXTENDED TERM INSURANCE

By E. G. FASSEL

If an application for reinstatement has been received and the policy is in force as extended term insurance, it is possible that the application can properly be approved even though the papers show the insured to have become an impaired risk. The fact that the company is obliged to carry the impaired risk during the remaining period of extension may compensate for failure to meet the principle generally applied to reinstatements, viz., "to reinstate policies which will, on the average, give as good a mortality as that of continuing policies issued at the same age and in the same year."[1]

If on calculation it should be found that the present value of what the company pays minus what it receives is not increased by making the reinstatement, then the request could be granted. This would meet the wishes of the insured without penalizing the general body of policyholders.

It is desirable to avoid such calculations for individual cases, not only because of the labor, but also on account of the assumption they would involve as to the mortality to be experienced under the particular risk in hand. Such individual calculations will not be necessary and the assumption as to mortality will be avoided if rules can be framed from advance calculations for typical cases in such a way as to be independent of the extent of impairment. One of the objects of the calculations reported herein was to find in how far this is possible.

Not only is the special mortality under the reinstated policy involved, but the mortality of the general body of policyholders into which the insured is to be reinstated must also be considered. After reinstatement the policy will be on the normal basis and the insurance will be on the regular terms. To see the effect most clearly we may consider a participating policy and note that upon receiving regular dividends the reinstated policyholder will be enjoying insurance at a mortality charge on a par with the company's normal risks.

[1] R. D. Murphy, "Reinstatement of Policies," *TASA*, XVIII, 247.

This means that for a given period of unexpired term extension with a given special mortality, then the better the mortality of the general body of policyholders the less freely can such a reinstatement be made. On the other hand, it is probable that the mortality to be anticipated under applicants for reinstatement tends toward the same improvement or deterioration as the company's general mortality, so that fixed practical rules for reinstatement of term extension cases may satisfy a wide range of actual general mortality experience. The present calculations have been made in a way that recognizes the improved mortality rates on which the companies have been operating in recent years.

References to this subject will be found in contributions by Mr. Henry Moir,[1] Mr. Arthur Hunter,[2] and Mr. R. D. Murphy.[3] Mr. Moir and Mr. Murphy treat the problem at some length and indicate methods for numerical solution on the assumption that the mortality of the group into which the impaired risk is being reinstated is according to the table on which reserves are based. Mr. Hunter suggests an empirical rule "to reinstate policies with more than five years of term extension, irrespective of the condition of health of the insured," and on this point Mr. Murphy finds that a reinstatement rule that considers only the unexpired period of term extension without regard to age, plan, duration, and indebtedness "is probably too rough a method to be just between different policyholders." The chief object of the present calculations has been to throw light on this point and if possible to derive rules suitable for use in practice.

ANALYSIS

For simplicity the analysis is developed for the ordinary life plan with full level premium reserves, the modifications for other plans and reserve systems being evident. In order that the expressions may be general, it is assumed that the policy is participating and that dividends are allowed both while in full force and while in force as extended insurance. Plain symbols are at the rates of mortality and interest used for the reserves, single accents (′)denote the rates assumed in determining dividends, i.e., the rates experienced on the general body of policyholders, and double accents (″) denote the rates actually experienced on the reinstated policy.

[1] *TASA*, XI, 296. [2] *Ibid.*, XVI, 284. [3] *Loc. cit.*

A policy was issued at age x, and on default in payment of premium at duration n the full reserve was applied to purchase extended insurance at the face amount for $(s+t)$ years of which period t years remain.

If the policy is reinstated:

The company from now on pays 1 on death at any time, regular dividends, and such expenses as are incurred on policies in full force. There is in hand the reserve for the unexpired term. The company now receives the payment required because of back premiums less back dividends and must reckon with back expenses, credit being taken for dividends and expenses incurred under the extended insurance.

If the policy is not reinstated:

The company from now on pays 1 on death within the term, the dividends under extended insurance, and such expenses as are incurred under extended insurance. There is in hand the reserve for the unexpired term. From now on nothing is received.

The present value of 1 to be paid on death at any time is A_{x+n+s} and on death within the term, $A^{1}_{x+n+s:\overline{t}|}$.

Let Π_x be the regular gross premium, ${}_r\Delta_x$ and ${}_rE_x$ the regular dividend and expense of the rth year, and $\Delta^{1}_{x+r+1:\overline{n+s+t-r-1}|}$ and $E^{1}_{x+r+1:\overline{n+s+t-r-1}|}$ the dividend and expense under the extended insurance in the year at the beginning of which $(n+s+t-r)$ years remain.

Taking account of actual rates, denoted by double accent, to which the particular policyholder is subject, the present value of regular gross premiums to be received minus dividends and expenses from year to year if the policy is restored to full force is

$$\sum_{r=n+s}^{r=\infty} [\Pi_x - v''({}_{r+1}\Delta_x + {}_{r+1}E_x)] \frac{D''_{x+r}}{D''_{x+n+s}} .$$

Dividends and expenses are discounted to the beginning of the policy year because payment of the premium admits the policy to the dividend, which on failure to survive the year is allowed post mortem. In like manner, the present value of dividends and expenses if the policy is not reinstated but is continued as extended insurance is

$$\sum_{r=n+s}^{r=\infty} v''(\Delta^{1}_{x+r+1:\overline{n+s+t-r-1}|} + E^{1}_{x+r+1:\overline{n+s+t-r-1}|}) \frac{D''_{x+r}}{D''_{x+n+s}} .$$

terms for $r \gtrless n+s+t$ being equal to zero.

The reserve in hand for the unexpired term is $A_{x+\overline{n+s}:\overline{t}}^{\mathrm{I}}$.

The payment required because of back premiums less back dividends is usually determined by the policy. The payment automatically subjects the company to back expenses, being expense claims that were held in abeyance because the premiums were not paid, such as commissions and premium taxes. Having restored the policy, the company's position as to money in hand is substantially the same as if no default had occurred. The intention of the payment is either to establish this parity with policies that did not default or else to make a reduced charge giving a margin in favor of the policyholder as an inducement for him to reinstate. With such possible margin we are not now concerned. The assumption to be made is that parity is maintained with policies that did not default and, if desired, the effect of collecting any other amount on account of back payments can be determined with this as a basis. Inasmuch as a corresponding policy that did not lapse has ${}_{n+s}V_x$ to its credit in place of $A_{x+\overline{n+s}:\overline{t}|}^{\mathrm{I}}$ actually in hand, we have ${}_{n+s}V_x - A_{x+\overline{n+s}:\overline{t}}^{\mathrm{I}}$ as the payment required because of back premiums less back dividends, after allowing for back expenses and amounts incurred on the extended insurance.

Assembling the foregoing expressions we find that the increase that reinstatement causes in the present value of the company's net obligations is

$$A''_{x+n+s} - A''_{x+\overline{n+s}:\overline{t}|}{}^{\mathrm{I}} - \sum_{r=n+s}^{r=\infty} [\Pi_x - v''({}_{r+1}\Delta_x + {}_{r+1}E_x) + v''(\Delta_{x+r+1:\overline{n+s+t-r-1}|}^{\mathrm{I}} + E_{x+r+1:\overline{n+s+t-r-1}|})] \frac{D''_{x+r}}{D''_{x+n+s}} - ({}_{n+s}V_x - A_{x+\overline{n+s}:\overline{t}|}^{\mathrm{I}}) . \quad (1)$$

EXPRESSION (2): COMPANY'S POSITION IF REINSTATEMENT IS APPROVED

Of expression (1), the portion,

$$A''_{x+n+s} - \sum_{r=n+s}^{r=\infty} [\Pi_x - v''({}_{r+1}\Delta_x + {}_{r+1}E_x)] \frac{D''_{x+r}}{D''_{x+n+s}} - {}_{n+s}V_x ,$$

gives the present value of the cost to the company of restoring the policy to full force, in view of the actual rates to which the

insured is subject. Under the usual three-factor dividend formula, Δ (total dividend) $= M\Delta$ (mortality dividend) $+ I\Delta$ (interest dividend) $+ L\Delta$ (loading dividend), also $L\Delta = (\Pi - P)(1+i') - E$, so that $v'(\Delta + E) = v'(M\Delta + I\Delta) + (\Pi - P)$. Hence if i'' be assumed equal to i', the cost to the company if the policy is restored to full force is

$$A''_{x+n+s} - \sum_{r=n+s}^{r=\infty} [P_x - v'({}_{r+1}M\Delta_x + {}_{r+1}I\Delta_x)] \frac{D''_{x+r}}{D''_{x+m+s}} - {}_{n+s}V_x . \quad (2)$$

$M\Delta$ and $I\Delta$ are independent of the actual rates to which the reinstated policyholder is subject inasmuch as he is restored to the company's general group of policies, receiving dividends that refund the valuation rates down to the single-accented rates. If $'$ is written for $''$ in expression (2), the value is zero; this agrees with the fact that if the mortality rates of the restored policy are the same as the general mortality, the reinstatement does not involve any cost or strain for the company. As $'$ is changed to less and less favorable values of $''$, the value of expression (2) is positive and increasing.

EXPRESSION (3): COMPANY'S POSITION IF REINSTATEMENT IS DECLINED

The remainder of expression (1), viz.,

$$A''_{x+n+s:\overline{t}|} + \sum_{r=n+s}^{r=\infty} v''(\Delta_{x+r+1:\overline{n+s+t-r-1}|} + E_{x+r+1:\overline{n+s+t-r-1}|}) \frac{D''_{x+r}}{D''_{x+n+s}} - A_{x+n+s:\overline{t}|} ,$$

gives the present value of the strain on the company through treating the extension as subject to the average (single-accented) rates, as evidenced by the dividends allowed, whereas the individual risk is subject to double-accented rates. Expenses against paid-up insurance being a charge against the dividend earnings and the dividend allowed being the balance after such deduction, the sum of the bracketed quantities in the second term represents the dividend earnings of the year. If $'$ be substituted for $''$ the expression becomes

$$A'_{x+n+s:\overline{t}|} + \sum_{r=n+s}^{r=\infty} v'(\Delta_{x+r+1:\overline{n+s+t-r-1}|} + E_{x+r+1:\overline{n+s+t-r-1}|}) \frac{D'_{x+r}}{D'_{x+n+s}} - A_{x+n+s:\overline{t}|} ,$$

which is equal to zero because if discount is at the interest and mortality rates assumed in calculating dividends, then the present value of the benefit plus dividends equals the valuation initial reserve. Therefore, for single-primed rates the value of the second term is

$$A_{x+\overset{1}{n+s}:\overline{t}|}-A'_{x+\overset{1}{n+s}:\overline{t}|}\,.$$

For successive values of r during a limited range, the terms of the summation are substantially constant and therefore as rates ($''$) are applied different from ($'$) the second term varies substantially in the proportion $a''_{x+n+s:\overline{t}|} : a_{x+n+s:\overline{t}|}$. Hence a close approximation to the expression at the beginning of this paragraph is

$$A''_{x+\overset{1}{n+s}:\overline{t}|}+(A_{x+\overset{1}{n+s}:\overline{t}}-A'_{x+\overset{1}{n+s}:\overline{t}|})a''_{+n+s:\overline{t}}\div a'_{x+n+s:\overline{t}|}-A_{x+\overset{1}{n+s}:\overline{t}|}\,,$$

which may be written

$$(A''_{x+\overset{1}{n+s}:\overline{t}|}-A'_{x+\overset{1}{n+s}:\overline{t}|})-(1-\alpha)(A_{x+\overset{1}{n+s}:\overline{t}|}-A'_{x+\overset{1}{n+s}:\overline{t}|})\,, \qquad (3)$$

where α = ratio of the annuities, so that $(1-\alpha)$ is positive, less than 1 and usually small.

CRITERION

Assuming given actual rates as applying to the particular policy, then for a given age at issue and age attained on a certain plan the value of expression (2) is fixed; the value of expression (3), however, is not fixed but depends on the value of t, i.e., the number of years of unexpired extension, generally increasing as t increases. The criterion for reinstating impaired risks in force as extended insurance is that the value of expression (2) shall not be greater than the value of expression (3). In general, (2) is greater than (3) if the unexpired period is short and reinstatement in such cases would be declined as increasing the strain on the company. As the values of (3) increase with the length of the unexpired term and a length of term is reached where (3) = (2), then for unexpired terms equal to or greater than this figure reinstatement would be granted because the strain on the company would not be increased, i.e., the general body of policyholders would not be penalized by the reinstatement.

NUMERICAL CALCULATION

The foregoing analysis gives a means of reducing the problem to numerical calculation and calculations of (2) and (3) have been

made for representative cases on six different assumptions as to degree of impairment. It has been assumed that interest is earned

TABLE I

Duration	Value of Expression (2) Basis: $1,000					
	A	B	C	D	E	F
	Ordinary Life—Age 35 at Issue					
5	$29.84	$ 90.99	$139.65	$180.18	$120.23	$196.38
10	32.80	98.89	150.38	192.49	108.34	172.92
15	35.50	105.62	158.97	201.75	96.46	149.35
20	37.39	109.69	163.23	205.24	84.72	126.40
25	38.07	109.92	161.55	201.12	73.31	104.78
30	37.22	105.66	153.31	188.85	62.30	84.99
35	34.71	96.94	138.92	169.42	51.68	67.35
40	30.62	84.19	119.32	144.18	41.49	51.62
45	24.95	67.89	95.39	114.34	31.36	37.38
50	18.32	49.46	66.76	79.07	21.55	24.12
55	11.93	30.12	44.41	.....	11.95	10.10
	Twenty-Payment Life—Age 35 at Issue					
5	$20.36	$63.29	$ 98.71	$129.19	$ 91.76	$153.46
10	20.95	63.98	98.39	127.27	74.33	120.88
15	21.25	63.54	96.05	122.43	59.18	92.36
20	21.70	63.66	94.70	119.09	49.20	73.41
25	22.07	63.78	93.71	116.69	42.58	60.90
30	21.58	61.31	88.92	109.62	36.24	49.50
35	20.11	56.28	80.55	98.38	30.17	39.46
40	17.73	48.90	69.13	83.79	24.46	30.69
45	14.45	39.55	55.05	66.49	18.93	23.55
50	10.64	29.67	38.18	44.34	15.14	20.16
55	7.32	19.06	4.77	.....	20.56	37.12
	Endowment at 65—Age 35 at Issue					
5	$15.28	$ 51.22	$ 84.32	$114.87	$ 86.72	$149.72
10	14.08	47.25	77.87	106.13	66.05	112.95
15	11.69	39.33	65.00	88.82	45.09	75.99
20	7.74	26.24	43.68	60.09	24.75	40.93
25	2.77	9.50	16.02	22.37	7.43	12.00
	Twenty-Year Endowment—Age 35 at Issue					
5	$ 7.22	$ 24.98	$ 42.11	$ 58.69	$ 52.35	$ 94.08
10	4.50	15.66	26.50	37.08	27.23	48.85
15	1.50	5.31	9.05	12.75	7.54	13.45

at 5 per cent, that general mortality is 80 per cent A.M.[5] and that reserves are on the full level premium system, Amer. Exp. 3 per cent.

The respective assumptions as to the degree of impairment have been to take q_x at 100 per cent A.M.$^{(5)}q_x$, referred to as A; at 150 per cent A.M.$^{(5)}$ q_x referred to as B; at 200 per cent A.M.$^{(5)}$ q_x, referred to as C; at 250 per cent A.M.$^{(5)}$ q_x, referred to as D; at A.M.$^{(5)}$ q_x+.010, referred to as E; and at A.M.$^{(5)}q_x$+.020, referred to as F. General

TABLE II

Attained Age	Years of Unexpired Term	Value of Expression (3) Basis: $1,000					
		A	B	C	D	E	F
40	5	$ 5.49	$ 19.07	$ 32.48	$ 45.73	$46.47	$ 85.90
	10	10.97	37.77	63.72	88.81	78.76	141.11
	15	16.59	56.35	93.76	128.95	100.05	173.86
	20	22.10	73.59	120.12	162.34	112.12	189.02
	25	26.86	87.05	138.63	182.90	116.04	190.13
	30	29.93	93.47	144.31	185.22	112.72	180.26
45	5	7.63	26.46	44.97	63.13	48.21	87.24
	10	15.44	52.78	88.41	122.31	81.28	141.83
	15	23.14	77.46	127.12	172.43	101.84	171.36
	20	29.95	97.20	155.09	204.97	111.35	180.90
	25	34.37	107.39	165.46	211.99	110.77	174.42
50	5	11.13	38.46	65.08	90.96	51.03	89.37
	10	22.18	74.89	123.97	169.50	84.77	142.39
	15	31.98	104.65	168.00	223.19	103.40	166.48
	20	38.78	121.51	187.56	240.44	108.11	167.35
55	5	16.48	56.51	94.97	131.81	55.18	92.46
	10	31.39	104.04	169.01	227.05	89.10	142.22
	15	42.21	133.42	207.35	267.29	103.32	157.39
60	5	24.13	81.87	136.02	186.84	61.14	96.66
	10	42.58	137.09	217.01	283.32	93.44	139.96
65	5	34.34	114.55	187.20	252.75	68.80	101.63
70	5	46.67	151.72	241.61	317.99	77.33	106.70
75	5	59.65	186.21	284.98	360.70	85.39	110.15
80	5	69.02	202.01	290.60	346.64	88.70	107.80
85	5	66.48	173.57	230.06	.....	79.80	91.26

mortality being assumed at 80 per cent, the assumed impairments are correspondingly heavier than appear on their face; thus A is at 100÷80=125 per cent of the general mortality assumed, B at 188 per cent, C at 250 per cent, and D at 313 per cent.

Specimen values of expression (2) on the six assumptions as to degree of impairment appear in Table I, being the values at quinquennial durations on four plans of insurance issued at age 35. Also specimen values of expression (3) are shown at quinquennial intervals in Table II.

TABLE III

CRITICAL VALUE OF t, IN YEARS

DURATION	Limit	A	B	C	D	E	F	Limit	A	B	C	D	E	F
		Age 15 at Issue							Age 25 at Issue					
Ordinary Life														
5	4	50	52	....	....	....	....	5	39	36	35	34	....	....
10	9	41	39	37	36	....	....	12	33	30	27	26	26	24
15	15	34	32	30	29	26	24	17	26	24	22	21	19	16
20	20	29	26	25	24	20	17	19	20	19	17	16	14	12
25	22	24	21	20	19	16	14	19	15	14	12	11	11	10
30	23	19	17	15	14	12	11	etc.	11	10	9	8	8	7
35	etc.	14	12	11	10	10	9	....	8	7	6	5	6	5
40	....	10	9	8	7	7	7	....	5	4	4	3	4	4
45	....	7	6	5	5	5	5	....	4	3	3	2	3	3
50	....	5	4	4	3	4	4	....	2	2	2	1	2	2
55	....	3	3	2	2	3	3	....	2	1	1	1	2	2
60	....	2	2	1	1	2	2	....	1	1	1	1	1	1
65	....	2	1	1	1	2	1	....	1	1	1	1	1	1
70	....	1	1	1	1	1	1	....	....	....	....	....	....	....
75	....	1	1	1	1	1	....	....	....	....	....	....	....	....
Twenty-Payment Life														
5	12	34	31	29	27	20	18	13	26	24	22	21	16	15
10	29	27	25	24	22	15	14	26	21	19	17	17	12	11
15	etc.	24	21	20	19	12	11	etc.	16	15	14	13	9	8
20	....	21	18	16	15	10	10	....	13	11	10	10	7	6
25	....	16	15	13	12	9	8	....	10	9	8	7	6	5
Endowment at 65														
5	6	31	31	30	30	26	26	8	22	22	21	21	19	18
10	14	25	25	25	24	20	19	18	17	17	16	16	13	13
15	23	20	20	20	19	15	15	etc.	12	12	12	12	10	9
20	etc.	15	15	15	15	12	11	....	8	8	8	8	7	6
25	....	11	11	11	11	9	8	....	5	5	5	5	4	4
30	....	8	7	7	7	6	6	....	3	2	2	2	2	2
35	....	5	4	4	4	4	4	....	1	1	1	1	1	1
40	....	2	2	2	2	2	2	....	....	....	....	....	....	....
45	....	1	1	1	1	1	1	....	....	....	....	....	....	....
Twenty-Year Endowment														
5	15	6	6	6	6	6	6	15	7	7	7	7	6	6
10	etc.	3	3	3	3	3	3	etc.	3	3	3	3	3	3
15	....	1	1	1	1	1	1	....	1	1	1	1	1	1

TABLE III—*Continued*

Duration	Critical Value of t, in Years													
	Limit	A	B	C	D	E	F	Limit	A	B	C	D	E	F
		Age 35 at Issue							Age 45 at Issue					
	Ordinary Life													
5	7	30	29	26	25	....	....	7	....	21	20	19	....	....
10	13	24	21	20	19	19	16	10	16	15	13	11	14	11
15	15	18	16	14	13	14	11	11	11	10	9	7	9	8
20	15	13	11	10	9	10	9	11	7	6	5	5	6	5
25	etc.	9	8	7	6	7	6	etc.	5	4	3	3	4	4
30	....	6	5	4	4	5	4	....	3	2	2	2	3	3
35	....	4	3	3	3	3	3	....	2	1	1	1	2	2
40	....	3	2	2	2	2	2	....	2	1	1	....	1	1
45	....	2	1	1	1	2	2	....	1	1	1	....	1	1
50	....	1	1	1	....	1	1	....	....	....	....	....	....	....
55	....	1	1	1	....	1	1	....	....	....	....	....	....	....
	Twenty-Payment Life													
5	13	19	17	16	16	14	12	10	14	12	11	11	11	10
10	21	14	12	12	11	9	8	14	9	8	7	7	7	6
15	etc.	10	9	8	7	6	6	etc.	5	5	4	4	4	4
20	....	7	6	5	5	5	4	....	3	3	3	2	3	3
25	....	5	4	4	3	4	3	....	2	2	2	2	2	2
	Endowment at 65													
5	11	14	14	14	14	12	11	12	7	7	7	7	6	6
10	19	10	9	9	9	8	7	etc.	4	3	3	3	3	3
15	etc.	6	6	5	5	5	5	....	1	1	1	1	1	1
20	....	3	3	3	3	3	2	....	....	....	....	....	....	....
25	....	1	1	1	1	1	1	....	....	....	....	....	....	....
	Twenty-Year Endowment													
5	15	7	7	7	7	6	6	12	7	7	7	7	6	6
10	etc.	3	3	3	3	3	3	etc.	4	3	3	3	3	3
15	....	1	1	1	1	1	1	....	1	1	1	1	1	1

TABLE III—*Continued*

Duration	Critical Value of t, in Years													
	Limit	A	B	C	D	E	F	Limit	A	B	C	D	E	F
		Age 55 at Issue							Age 65 at Issue					
	Ordinary Life													
5	5	....	....	....	11	....	....	3	....	....	....	....	....	....
10	7	11	9	8	6	10	9	4	6	5	4	3	....	5
15	7	6	5	5	4	5	5	4	3	3	2	2	3	3
20	etc.	4	3	2	3	3	3	etc.	2	2	2	2	2	2
25	....	2	2	1	1	2	2	....	2	1	1	1	1	1
30	....	2	1	1	1	2	2	....	....	....	....	....	....	....
35	....	1	1	1	1	1	1	....	....	....	....	....	....	....
	Twenty-Payment Life													
5	6	10	9	9	8	9	9							
10	9	6	5	5	4	5	5							
15	etc.	3	2	2	2	3	2							
20	....	2	1	1	1	2	2							
25	....	1	1	1	1	1	1							
	Endowment at 65													
5	5	2	2	2	2	2	2							
	Twenty-Year Endowment													
5	7	7	7	7	7	7	6							
10	etc.	4	3	3	3	3	3							
15	....	1	1	1	1	1	1							

CRITICAL VALUE OF t

An example will illustrate the use made of Tables I and II.

For Ordinary Life in Table I duration 20 with impairment C the value of expression (2) is shown to be $163.23. The attained age is 55 and Table II in Column C shows that $163.23 is not greater than the value of expression (3) if there are at least 10 years of unexpired term. Hence in this case if the insurance after reinstatement will be for the same amount as the insurance in force as term extension, then reinstatement can be granted in spite of the impairment, if at least 10 years of the term remain. It is to be observed that no direct account needs to be taken of the length of time that premiums have been in default. Table III gives the critical values of t found from a series of tabulations similar to Tables I and II.

In Table III the columns headed "Limit" show the number of years extension purchased by the full reserve and are exhibited as being the maximum unexpired periods that it would be possible for an actual case to present. Critical values of t over the "limit" are shown in reduced type as being unexpired periods not arising in practice. The values in the A column are also shown in reduced type because the A mortality would not be treated as impaired. An applicant for reinstatement showing 100 per cent A.M.(5) mortality, being 125 per cent of the company's general 80 per cent A.M.(5) experience, would be reinstated without regard to term extension in accordance with the principle quoted in the first paragraph.

ELIMINATION OF EXTENT OF IMPAIRMENT

It is desirable that rules for reinstating impaired risks be independent of the extent of impairment, if possible. Probably C will represent the average of all the impaired applicants, erring perhaps on the side of heavy mortality, which is on the side of liberality as reducing the value of t. Having regard to the values printed in bold type, it is seen that the C values agree closely with the other columns in most cases. The use of the C values as representing all degrees of impairment seems a satisfactory basis and probably deviates from the facts in any individual case by less than the error that would be introduced if an attempt were made to estimate the risk. The few points at which the C values differ appreciably from the other columns apply to the less frequent cases in practice and may be ignored.

Table IV shows the result of drawing off the values in column C of Table III, the life plans according to age attained and the endowment plans according to the number of years that remain before the policy normally matures as an endowment. It may be noted that if the life plans are viewed as endowments maturing at age 96, the attained ages 25 to 90 could be replaced by years to maturity 71 to 6 without affecting the form of the table.

ELIMINATION OF AGE AT ISSUE

The arrangement of Table IV is such that there is only slight variation according to the age at issue, and this leads to further condensation. Referring to Ordinary Life, attained age 40 involves only issue age 15 and the value is 20; attained age 45 involves issue ages 15 and 25, giving an average of 16 for attained age 45; at-

tained age 50 involves issue ages 15, 25, and 35 of which the value for 25 may be taken as the average, being 12; in like fashion the values for attained ages 55, 60, 65, etc., may be taken as 9, 7, 5, etc. Similar reasoning may be followed for the other plans and Table V shows the result, including additional plans.

TABLE IV

	Critical Value of t, in Years										
Age at Issue	15	25	35	45	55	65	15	25	35	45	55
Age Attained	Ordinary Life						Twenty-Payment Life				
25	.....	.....	.....	.....	.....	.....	24	.....	.....	.....	.....
30	.....	.....	.....	.....	.....	.....	20	.....	.....	.....	.....
35	.....	.....	.....	.....	.....	.....	16	17	.....	.....	.....
40	20	.....	.....	.....	.....	.....	13	14	.....	.....	.....
45	15	17	.....	.....	.....	.....	10	10	12	.....	.....
50	11	12	14	.....	.....	.....	8	8	8	.....	.....
55	8	9	10	.....	.....	.....	5	5	5	7	.....
60	5	6	7	9	.....	.....	4	4	4	4	.....
65	4	4	4	5	.....	.....	3	3	3	3	5
70	2	3	3	3	5	.....	2	2	2	2	2
75	1	2	2	2	2	4	1	1	1	1	1
80	1	1	1	1	1	2	1	1	1	1	1
85	1	1	1	1	1	2	1	1	1	1	1
90	1	1	1	1	1	1	1	1	1	1	1
Years to Maturity	Endowment at 65						Twenty-Year Endowment				
35	20	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
30	15	16	.....	.....	.....	.....	.....	.....	.....	.....	.....
25	11	12	.....	.....	.....	.....	.....	.....	.....	.....	.....
20	7	8	9	.....	.....	.....	.....	.....	.....	.....	.....
15	4	5	5	.....	.....	.....	6	7	7	7	7
10	2	2	3	4	.....	.....	3	3	3	4	3
5	1	1	1	1	2	.....	1	1	1	1	1

INDEBTEDNESS

The case where indebtedness is canceled when granting the extended insurance requires special consideration if the insurance is extended at an altered amount because of the debt. Tables III–V have been constructed on assuming that the amount of the policy if in full force is the same as the amount if in force as extended insurance. If a is the altered amount extended per unit of the regular

amount insured by the policy then the values of Table II will need to be multiplied by a before being compared with Table I; by having been permitted to reduce the amount insured the company the value of expression (3) is correspondingly reduced.

On examining Table II it will be seen that if all the values are decreased in the ratio $a:1$ then the number of years at which a given value is reached is increased in approximately the inverse ratio. Therefore a practical rule will be to derive the critical value of t in the case of extension at a per unit, as the normal critical value of t divided by a.

TABLE V

CRITICAL VALUE OF t, IN YEARS

Age Attained	Ordinary Life	30-Pay't Life	20-Pay't Life	10-Pay't Life	Years to Maturity	End't at 65	40-Year End't	30-Year End't	20-Year End't	10-Year End't
20.....	.....	.....	.....	25	35.....	20	.....	.....	.....	.....
25.....	.....	.....	24	22	30.....	16	16	.....	.....	.....
30.....	.....	21	20	20	25.....	12	12	14	.....	.....
35.....	.....	17	17	16	20.....	8	8	9	.....	.....
40.....	20	15	14	14	15.....	5	5	5	7	.....
45.....	16	11	11	10	10.....	3	2	3	3	.....
50.....	12	10	8	8	5.....	1	1	1	1	2
55.....	9	7	6	5						
60.....	7	4	4	4						
65.....	5	3	3	3						
70.....	3	2	2	2						
75.....	2	1	1	1						
80.....	1	1	1	1						
85.....	1	1	1	1						
90.....	1	1	1	1						

The case of paid-up additions differs from that of indebtedness because, although the value of expression (3) is raised because the extension is at an amount in excess of the face of the policy, it is also true that the value of expression (2) is greater than normal because the regular policy is increased by paid-up insurance in the amount of the additions. The resulting change in the critical value of t will be insignificant.

These considerations lead to a practical rule covering all cases. Let R be the amount of insurance (face amount and additions, if any) if the policy is in full force, and let T be the amount at which the insurance is extended. Then the critical value of t is equal to R/T times the normal value.

EFFECT OF CHANGE IN ASSUMPTIONS

Table VI has been reached by taking the case of participating insurance with Amer. Exp. 3 per cent full level premium reserves. We are interested in the effect of assuming other conditions.

A change of $\frac{1}{2}$ per cent in the interest rate or a change to preliminary term valuation will not materially alter the results, so that Table VI may be taken as applicable. In expression (2),

$$A''_{x+n+s} - \sum_{r=n+s}^{r=\infty} [P_x - v'(_{r+1}M\Delta_x + _{r+1}I\Delta_x)] \frac{D''_{x+r}}{D''_{x+n+s}} - {}_{n+s}V_x ,$$

the "summation" term represents the net premium minus the mortality and interest dividend. A lighter reserve basis reduces both the net premium and the dividend, and deduction of the dividend from the premium year by year gives a decreasing series where the rate of decrease is not so rapid as with the heavier reserve basis. This change in slope of the series of payments with proportionately higher values in the later years increases the variation of the "summation" term with change in the basis—payments being proportionately later, the value suffers more heavily from discount. On any reserve basis expression (2) = 0 if $''='$; also the "summation" term has an increased value if the reserve is lighter, because the total is zero. We have seen that as $''$ is made more and more stringent than $'$, then the "summation" term decreases more rapidly in the case of light reserves than for heavier reserves. Therefore, for any particular $''$ the value of expression (2) is increased by using lighter reserves. But such change in the reserve basis also increases the value of expression (3),

$$(A''_{x+\overset{1}{n+s}:\overline{t}|} - A'_{x+\overset{1}{n+s}:\overline{t}|}) - (1-a)(A_{x+\overset{1}{n+s}:\overline{t}|} - A'_{x+\overset{1}{n+s}:\overline{t}|}) ,$$

where a decrease in $A_{x+\overset{1}{n+s}:\overline{t}|}$, the only quantity affected by the change in assumption, results in an increase of the total. Seeing that expressions (2) and (3) both vary in the same direction, a moderate change in the reserve basis will not make a material change in the value of t.

NON-PARTICIPATING

The problem has been studied from the point of view of participating insurance for two reasons. The first reason is that this meth-

od leads to more general expressions; having developed the theory by allowing for dividend payments, it is easy to pass to non-participating by eliminating the dividends but the inverse process of passing from non-participating to participating is not so evident. The second reason has been the fact that participating policies are essentially decreasing payment contracts as against uniform premium payments collected under non-participating policies. For the normal case equality is maintained, but the difference in the form of the payment series is important when we discount future payments with impaired life mortality rates.

For non-participating insurance the "summation" term referred to in the second preceding paragraph reduces to a level series, the dividend having disappeared, and expression (2) becomes

$$A''_{x+n+s} - P'_x a''_{x+n+s} - {}_{n+s}V'_x . \qquad (4)$$

If $'$ be written for $''$, expression (4) $= 0$. The annual payment is the net payment (P'_x) to carry the insurance according to the assumed general experience. In like manner, the assumed reserve is on the basis denoted by $'$; this is due to the difference in the theories underlying participating and non-participating insurance; the participating theory is to carry the contract on a year-by-year basis, whatever reserve the company chooses to carry being credited with earnings and charged with claims—the present value of the normal contract in any year being equal to the reserve carried; the non-participating theory takes the long-range view and stands to gain or lose according as the contract deviates from the record as estimated for it in advance—having fixed upon a normal mortality assumption and level premiums (P'_x) the normal policy represents to the company at all times the corresponding reserve (${}_{n+s}V'_x$), any more stringent value infringes on the company's expectation of expense margin, profits, etc., and any less stringent value fails to meet claims on the basis assumed. Also, for non-participating insurance expression (3) becomes

$$A''_{x+\overset{1}{n+s}:\overline{t}|} - A'_{x+\overset{1}{n+s}:\overline{t}|} . \qquad (5)$$

The critical value of t is therefore obtained by solving

$$A''_{x+n+s} - P'_x a''_{x+n+s} - {}_{n+s}V'_x = A''_{x+\overset{1}{n+s}:\overline{t}|} - A'_{x+\overset{1}{n+s}:\overline{t}|}$$

and, transforming the left-hand side into

$$1 - d\ddot{a}''_{x+n+s} - P'_x\ddot{a}''_{x+n+s} - 1 + a'_{x+n+s} \div a'_x = a''_{x+n+s}(-P'_x - d)$$
$$+ a'_{x+n+s} \div a'_x = (a'_{x+n+s} - a''_{x+n+s}) \div a'_x$$

we have as the equation to solve for t,

$$(a'_{x+n+s} - a''_{x+n+s}) \div a'_x = A''^{\,1}_{x+n+s:\overline{t}|} - A'^{\,1}_{x+n+s:\overline{t}|} .$$

Table VI gives the values of t resulting from this equation in the case of Ordinary Life at issue ages 25 and 45 and on assumed impair-

TABLE VI

Duration	Critical Value of t, in Years					
	A		C		E	
	N.P.	Par.	N.P.	Par.	N.P.	Par.
	Ordinary Life—Age 25 at Issue					
5	40	39	35	35	.....	.....
10	34	33	29	27	27	26
15	29	26	24	22	21	19
20	23	20	19	17	17	14
25	19	15	15	12	14	11
30	14	11	11	9	11	8
35	10	8	8	6	8	6
40	7	5	5	4	6	4
45	5	4	3	3	4	3
50	3	2	2	2	3	2
55	2	2	1	1	2	2
60	1	1	1	1	1	1
65	1	1	1	1	1	1
	Ordinary Life—Age 45 at Issue					
5	.....	.....	22	20	.....	.....
10	20	16	15	13	17	14
15	14	11	10	9	11	9
20	10	7	7	5	8	6
25	6	5	4	3	6	4
30	4	3	3	2	4	3
35	3	2	2	1	2	2
40	2	2	1	1	2	1
45	1	1	1	1	1	1

ment bases A, C, and E, as earlier defined. The values appear in the columns headed "N.P." The values in the columns headed "Par." are copied from Table III.

The "Par." values run lower than those in the "N.P." column. The "N.P." values are such as result from the usual form of investigation that makes the broad assumption that dividends under participating policies make the contract equivalent to one with level net premiums. Table VI shows that such an assumption is not valid when discussing other than normal risks. It also shows that the decreasing payments involved in participating policies warrant reinstatement in case of shorter unexpired term extensions than where the payments are level. The use of Table V for non-participating policies would therefore be on the side of liberality to the particular policyholder reinstated.

OFFICE TABLE

On examination of Table V it is clear that there is little distinction between the various limited payment plans, also among the various endowment plans, and this leads to further condensation. As a result, Table VII is offered for practical office use in determining if a policy in force as extended term insurance can be reinstated even though the risk is impaired.

The following will illustrate the use of Table VII:

Policy issued at age 40 on the Ordinary Life plan, continued as extended insurance on default at end of 10 years, and considered for reinstatement 2 years after default, the risk being found to have become impaired.

"Age Attained" is $40+10+2=52$; from Table VII the normal "Minimum Unexpired Term" is 11 years.

If the insurance when restored to full force is, say, $1,000, and the insurance in force as term extension is also $1,000, then $R/T=1{,}000/1{,}000$ and $11\times R/T=11$. The extension having been granted for, say, 12 years, 27 days, the actual unexpired term is 10 years, 27 days; as this is less than the 11 years' minimum, reinstatement is declined.

If the insurance when restored to full force is, say, $1,050, being $1,000 face+$50 paid-up additions, and the insurance in force as term extension is also $1,050, then $R/T=1{,}050/1{,}050$ and the "Minimum Unexpired Term" is $11\times 1{,}050/1{,}050=11$. The extension having been granted for, say, 13 years, 240 days, the actual unexpired term is 11 years, 240 days; as this is equal to or greater than the 11 years' minimum, reinstatement is approved.

TABLE VII

MINIMUM UNEXPIRED TERM*

Normal Number of Years Required for Reinstatement of Policy on Impaired Risk If in Force as Extended Term Insurance

Age Attained	Ordinary Life Plan	Limited Payment Life Plans	Years to Maturity	Endowment Plans
25	.....	24	40	24
26	.....	24	39	24
27	.....	23	38	23
28	.....	22	37	22
29	.....	21	36	21
30	.....	20	35	20
31	.....	20	34	20
32	.....	19	33	19
33	.....	19	32	18
34	.....	18	31	17
35	24	17	30	16
36	24	17	29	16
37	23	16	28	15
38	22	16	27	14
39	21	15	26	13
40	20	14	25	12
41	20	14	24	12
42	19	13	23	11
43	18	13	22	10
44	17	12	21	9
45	16	11	20	8
46	16	11	19	8
47	15	10	18	7
48	14	10	17	7
49	13	9	16	6
50	12	8	15	5
51	12	8	14	5
52	11	7	13	4
53	11	7	12	4
54	10	6	11	3
55	9	6	10	3
56	9	6	9	3
57	8	5	8	2
58	8	5	7	2
59	7	5	6	2
60	7	4	5	1
61	7	4	4	1
62	6	4	3	1
63	6	3	2	1
64	5	3	1	1

* If R (face amount and additions, if any) differs from T (amount at which the insurance is extended), multiply the normal value by R/T.

TABLE VII—*Continued*

Age Attained	Ordinary Life Plan	Limited Payment Life Plans	Years to Maturity	Endowment Plans
65.....	5	3	.....	.....
66.....	4	3	.....	.....
67.....	4	3	.....	.....
68.....	3	2	.....	.....
69.....	3	2	.....	.....
70–74.....	2	2	.....	.....
75 and over...	1	1	.....	.....

If the insurance when restored to full force is, say, $1,050, being $1,000 face +$50 paid-up additions, and the insurance in force as term extension is, say, $925, as the result of deducting $125 through canceling a loan, then $R/T = 1{,}050/925$ and the "Minimum Unexpired Term" is $11 \times 1{,}050/925 = 12$. The extension having been granted for, say, 5 years, 120 days, the actual unexpired term is 3 years, 120 days; as this is less than the 12 years' minimum, reinstatement is declined.

If, instead of being on the Ordinary Life plan, the policy were a Twenty-Year Endowment, we would consider the "Years to Maturity" instead of the "Age Attained." "Years to Maturity" is $20 - 10 - 2 = 8$; from Table VII the normal "Minimum Unexpired Term" is 2 years. The ratio, R/T, is derived and applied in the same way as in the foregoing illustrations and the comparison of minimum and actual is also made in the same manner. In all cases where there is a pure endowment the actual unexpired term cannot be less than the required minimum, so that in such cases it is unnecessary to consult the table; this is evident on observing that in Table VII the "Minimum" runs at a figure smaller than the "Years to Maturity."

The results shown in Table VII are probably in their simplest form from the standpoint of a guide for ready reference in connection with reinstatements. There is, however, a striking similarity of the columns. The first twenty values of each as printed are practically alike. Also the limited payment values are consistently two-thirds of those for ordinary life.

PRACTICAL APPLICATION

Although Table VII is in a form suitable for office use, it is not clear that it can be uniformly employed without injustice in certain cases.

This paper is a study of the problem of reinstatement of impaired lives under term extension, a problem which exists in consequence of the peculiar privilege granted by the practice of term extension. In applying the results of the analysis it should not be forgotten that there are elements in the problem which cannot be valued but to which weight should be given in practice. Would it, for example, be a good general rule under the conditions assumed to reject every impaired applicant for reinstatement on the ordinary life plan at attained age 55 where the remaining extension was less than 9 years? Or should the circumstances and motives, the duration before default, and the promptness of application for reinstatement also be considered?

The answers to these questions give scope for personal opinion and are not ventured at this time. It is hoped that there will be a discussion of Table VII from the practical standpoint in which its advantages and limitations will be brought to light and resulting in a sound basis for the reinstatement of policies in force as extended term insurance.

The results in this paper do not apply to policies containing a provision for disability benefits.

MR. MOIR'S FORMULA

Mr. Moir's formula,[1] $({}_{u}A)^{1}_{y\overline{t}|}+\Sigma P'-e={}_{n+m}(uV)'_{x}$, becomes, in the notation of this paper, $A''_{x+\overline{n}+s:\overline{t}}+$(Premiums to be received on reinstatement with interest$-$Expense of reinstatement to be deducted)$-A''_{x+n+s}+P'_{x}\,a''_{x+n+s}=0$. The bracketed portion is the net amount obtained by the company on reinstatement which, as earlier explained, is to be taken as ${}_{n+s}V'_{x}-A'_{x+\overline{n}+s:\overline{t}}$, the single prime appearing because Mr. Moir's formula assumes that valuation and normal mortality and interest are identical. Inserting this value, the formula becomes

$$A''_{x+\overline{n}+s:\overline{t}|}+{}_{n+s}V'_{x}-A_{1+\overline{n}+s:\overline{t}|}-A''_{x+n+s}+P'_{x}a''_{x+n+s}=0\ ,$$

which is the same as equating expressions (4) and (5).

MR. MURPHY'S FORMULA

Mr. Murphy's formula,[2] ${}_{t}A'_{x+n}=P_{x}a'_{x+n}$, becomes, in the notation of this paper, $A''_{x+n}-A''_{x+n:\overline{t}}-P'_{x}a''_{x+n}=0$; let the expression

[1] *TASA*, XI, 299. [2] *Ibid.*, XVIII, 254.

equated to zero be denoted by M. Expression (4) minus expression (5) may be written

$$A''_{x+n+s} - A''_{x+n+s:\overline{t}|}{}^{1} - P'_x a''_{x+n+s} - {}_{n+s}V'_x + A_{x+n+s:\overline{t}|}{}^{1} = N \text{ , say,}$$

and the critical value of t is that value (t_n) which makes $N=0$. Mr. Murphy's critical value of t is that value (t_m) which makes $M=0$.

When giving the formula, Mr. Murphy stated it to assume "that reinstatement is applied for soon after lapse." If therefore the expression, $M \equiv A''_{x+n} - A''_{x+n:\overline{t}|}{}^{1} - P'_x a''_{x+n}$, is written for any actual case the value of t is fixed because by hypothesis the case is considered as of the inception of the extension period; at this point of time the value of t is the period of extension actually granted and ${}_nV'_x - A'_{x+n:\overline{t}|}{}^{1} = 0$ as being the formula by which the extension period was determined. Hence no values of M that are legitimate by hypothesis are altered by the subtraction of ${}_nV'_x - A'_{x+n:\overline{t}|}{}^{1}$ $(=0)$; hence $M = M - {}_nV'_x + A'_{x+n:\overline{t}|}{}^{1}$

On the other hand N is a general expression in which t may have any value $\gtrless$ extension period. Mr. Murphy's expression (M) is therefore seen to be the particular form assumed by the general expression (N) when $S=0$ and $t=$extension period.

A policy having lapsed at duration n the equation $N=0$ will test the status at any time (s) after lapse to find when the existing value of t (t_m), the remaining portion of the original extension period, is such that $N=0$. This is the turning-point; hitherto reinstatement could have been granted but from this time on it is to be declined.

At the time of lapse $N \equiv M$, being usually a negative or positive quantity, indicating approval of reinstatement if negative and declination if positive. As expressing the conditions of any policy on lapse, in accordance with hypothesis, zero is only a particular value that may be assumed by M the same as any other value and t is not a variable but is a constant, being the extension period actually in effect in the case considered.

If we are concerned with the sign of M and not its absolute value we may introduce an artifice that violates the hypothesis but is valid for the particular purpose. If M is negative then we know from an examination of $M \equiv A''_{x+n} - A''_{x+n:\overline{t}|}{}^{1} - P'_x a''_{x+n}$ that the value of M would be greater if t were smaller and that at some

reduced value (t_m) we will have $M=0$. The sign of M will therefore be known if we determine t_m by solving $M=0$; if the actual $t>t_m$, sign of t is $-ve$; and vice versa.

If other uses are proposed for the general relation $M=0$ caution must be exercised to avoid erroneous conclusions because of the violation of the hypothesis. The results must be traced to the hypothesis to assure their validity.

Mr. Murphy's formula has been discussed at some length with the object of reconciling the statements made as to the effect of indebtedness. The present paper indicates a higher critical value of t if the amount of extended insurance is reduced by a loan. Mr. Murphy's paper draws the opposite conclusion based on the statement "the older the age at issue the less is the term of extended insurance necessary for reinstatement at a given attained age,[1]" which is inferred from the table on page 255 of his paper. The primary purpose of the values tabulated by him is in comparison with the guaranteed extension periods actually effective. Assuming full level net premium American Experience 3% reserves applied to purchase extended insurance on the same basis, it will be found that the actual extension on lapse is less than the tabulated period in every case shown. The table therefore gives the legitimate result that in each case the value of M is positive and reinstatement is to be declined. The opinion is ventured, however, that the conclusions to be drawn from the table as to the variation of the necessary term according to age are not clear and must be reached with caution. In this connection it is of interest to note that Table IV shows the opposite trend. This may be viewed as a verification of the present conclusions as to the effect of indebtedness which were reached by another line of reasoning.

IMPAIRED LIFE TABLES

Commutation columns especially calculated in connection with this paper are included at the end hereof. These are 5 per cent tables corresponding with the ultimate $3\frac{1}{2}$ per cent Impaired Life Tables that were published in Volume XI of *The Record*. The special values of q were obtained directly from the official A.M.[5] values and the functions were calculated by the formulas

$$D_{15}=100{,}000\ v^{15};\ D_x=vp_{x-1}\ D_{x-1};\ N_x=\sum_{x=x}^{x=\infty} D_x;\ M_x=vN_x-N_{x+1}.$$

[1] *Loc. cit.*, p. 256.

TABLE VIII

COMMUTATION TABLE
American Men (Modified), 5 Per Cent
q_x = 80 Per Cent A.M.$^{(5)}q_x$

Age x	D_x	N_x	M_x	Age x	D_x	N_x	M_x
15	48,101.7	895,064.2	5,479.59	60	4,005.6	43,212.1	1,947.88
16	45,684.3	846,962.5	5,352.75	61	3,733.4	39,206.5	1,866.42
17	43,386.0	801,278.2	5,229.89	62	3,473.0	35,473.1	1,783.80
18	41,200.0	757,892.2	5,109.89	63	3,224.1	32,000.1	1,700.29
19	39,121.6	716,692.2	4,993.40	64	2,986.1	28,776.0	1,615.81
20	37,145.1	677,570.6	4,879.83	65	2,758.9	25,789.9	1,530.81
21	35,265.3	640,425.5	4,768.85	66	2,542.1	23,031.0	1,445.39
22	33,478.0	605,160.2	4,660.85	67	2,335.5	20,488.9	1,359.84
23	31,778.7	571,682.2	4,555.74	68	2,138.8	18,153.4	1,274.35
24	30,164.2	539,903.5	4,454.51	69	1,952.0	16,014.6	1,189.40
25	28,630.1	509,739.3	4,356.80	70	1,774.8	14,062.6	1,105.15
26	27,172.7	481,109.2	4,262.74	71	1,607.2	12,287.8	1,022.07
27	25,788.7	453,936.5	4,172.68	72	1,449.0	10,680.6	940.40
28	24,474.4	428,147.8	4,086.41	73	1,300.1	9,231.6	860.50
29	23,226.7	403,673.4	4,004.16	74	1,160.5	7,931.5	782.81
30	22,042.3	380,446.7	3,925.79	75	1,030.2	6,771.0	707.77
31	20,917.8	358,404.4	3,850.92	76	909.0	5,740.8	635.63
32	19,850.3	337,486.6	3,779.51	77	796.8	4,831.8	566.71
33	18,836.8	317,636.3	3,711.26	78	693.5	4,035.0	501.36
34	17,873.9	298,799.5	3,645.35	79	599.0	3,341.5	439.88
35	16,959.0	280,925.6	3,581.59	80	513.1	2,742.5	382.50
36	16,089.7	263,966.6	3,519.86	81	435.6	2,229.4	329.44
37	15,263.0	247,876.9	3,459.34	82	366.3	1,793.8	280.88
38	14,476.7	232,613.9	3,399.85	83	304.8	1,427.5	236.82
39	13,728.7	218,137.2	3,341.21	84	250.8	1,122.7	197.34
40	13,016.8	204,408.5	3,283.06	85	203.9	871.9	162.38
41	12,339.0	191,391.7	3,225.11	86	163.6	668.0	131.79
42	11,693.5	179,052.7	3,167.18	87	129.4	504.4	105.38
43	11,078.4	167,359.2	3,108.91	88	100.8	375.0	82.94
44	10,492.3	156,280.8	3,050.36	89	77.3	274.2	64.24
45	9,933.4	145,788.5	2,991.09	90	58.2	196.9	48.82
46	9,400.3	135,855.1	2,931.01	91	43.0	138.7	36.40
47	8,891.6	126,454.8	2,869.94	92	31.1	95.7	26.54
48	8,406.0	117,563.2	2,807.75	93	22.0	64.6	18.92
49	7,942.4	109,157.2	2,744.44	94	15.2	42.6	13.17
50	7,499.4	101,214.8	2,679.65	95	10.3	27.4	9.00
51	7,076.1	93,715.4	2,613.46	96	6.8	17.1	5.99
52	6,671.5	86,639.3	2,545.82	97	4.3	10.3	3.81
53	6,284.6	79,967.8	2,476.61	98	2.6	6.0	2.31
54	5,914.6	73,683.2	2,405.88	99	1.6	3.4	1.44
55	5,560.5	67,768.6	2,333.42	100	.9	1.8	.81
56	5,221.7	62,208.1	2,259.41	101	.5	.9	.46
57	4,897.4	56,986.4	2,183.76	102	.3	.4	.28
58	4,587.0	52,089.0	2,106.57	103	.1	.1	.10
59	4,289.9	47,502.0	2,027.90				

TABLE VIII—*Continued*

q_x=150 Per Cent A.M.$^{(5)}q_x$

Age x	D_x	N_x	M_x	Age x	D_x	N_x	M_x
15	48,101.7	841,921.2	8,010.21	60	3,100.5	26,515.1	1,837.88
16	45,573.4	793,819.5	7,772.47	61	2,834.7	23,414.6	1,719.72
17	43,173.4	748,246.1	7,542.63	62	2,582.2	20,579.9	1,602.20
18	40,893.6	705,072.7	7,318.71	63	2,342.7	17,997.7	1,485.67
19	38,729.6	664,179.1	7,102.02	64	2,116.1	15,655.0	1,370.62
20	36,674.5	625,449.5	6,891.19	65	1,902.3	13,538.9	1,257.59
21	34,722.7	588,775.0	6,685.79	66	1,701.2	11,636.6	1,147.08
22	32,869.8	554,052.3	6,486.36	67	1,512.8	9,935.4	1,039.69
23	31,111.1	521,182.5	6,292.88	68	1,337.0	8,422.6	935.92
24	29,443.9	490,071.4	6,107.17	69	1,173.7	7,085.6	836.29
25	27,863.1	460,627.5	5,928.46	70	1,022.8	5,911.9	741.28
26	26,364.7	432,764.4	5,756.87	71	884.3	4,889.1	651.49
27	24,945.4	406,399.7	5,593.03	72	757.9	4,004.8	567.20
28	23,601.1	381,454.3	5,436.61	73	643.5	3,246.9	488.89
29	22,328.6	357,853.2	5,287.97	74	540.8	2,603.4	416.83
30	21,124.0	335,524.6	5,146.64	75	449.4	2,062.6	351.18
31	19,983.5	314,400.6	5,012.04	76	369.0	1,613.2	292.18
32	18,904.0	294,417.1	4,884.14	77	299.0	1,244.2	239.75
33	17,882.0	275,513.1	4,762.33	78	238.8	945.2	193.79
34	16,913.2	257,631.1	4,645.05	79	187.8	706.4	154.16
35	15,994.7	240,717.9	4,531.94	80	145.1	518.6	120.40
36	15,123.8	224,723.2	4,422.69	81	110.1	373.5	92.31
37	14,296.9	209,599.4	4,315.98	82	81.8	263.4	69.26
38	13,511.5	195,302.5	4,211.38	83	59.5	181.6	50.85
39	12,765.4	181,791.0	4,108.69	84	42.2	122.1	36.39
40	12,056.1	169,025.6	4,007.26	85	29.1	79.9	25.30
41	11,381.4	156,969.5	3,906.66	86	19.5	50.8	17.08
42	10,739.3	145,588.1	3,806.53	87	12.7	31.3	11.21
43	10,127.6	134,848.8	3,706.23	88	8.0	18.6	7.11
44	9,544.9	124,721.2	3,605.79	89	4.8	10.6	4.30
45	8,989.2	115,176.3	3,504.61	90	2.8	5.8	2.52
46	8,459.2	106,187.1	3,402.67	91	1.5	3.0	1.36
47	7,953.4	97,727.9	3,299.69	92	.8	1.5	.73
48	7,470.4	89,774.5	3,195.42	93	.4	.7	.37
49	7,009.1	82,304.1	3,089.86	94	.2	.3	.19
50	6,568.2	75,295.0	2,982.72	95	.1	.1	.10
51	6,146.8	68,726.8	2,874.10	96	.0		
52	5,744.0	62,580.0	2,764.00				
53	5,358.7	56,836.0	2,652.22				
54	4,990.4	51,477.3	2,539.10				
55	4,638.1	46,486.9	2,424.44				
56	4,301.5	41,848.8	2,308.70				
57	3,979.8	37,547.3	2,191.83				
58	3,672.7	33,567.5	2,074.25				
59	3,379.7	29,894.8	1,956.14				

TABLE VIII—*Continued*

$q_x = 200$ Per Cent A.M.(5) q_x

Age x	D_x	N_x	M_x	Age x	D_x	N_x	M_x
15	48,101.7	810,685.9	9,497.61	60	2,578.5	19,445.9	1,652.50
16	45,494.2	762,584.2	9,180.66	61	2,324.7	16,867.4	1,521.49
17	43,021.9	717,090.0	8,874.76	62	2,085.5	14,542.7	1,392.99
18	40,675.8	674,068.1	8,577.32	63	1,860.7	12,457.2	1,267.50
19	38,451.4	633,392.3	8,289.86	64	1,650.3	10,596.5	1,145.70
20	36,341.3	594,940.9	8,010.78	65	1,454.2	8,946.2	1,028.19
21	34,339.4	558,599.6	7,739.42	66	1,272.3	7,492.0	915.54
22	32,441.3	524,260.2	7,476.53	67	1,104.6	6,219.7	808.42
23	30,641.9	491,818.9	7,221.95	68	950.9	5,115.1	707.32
24	28,938.8	461,177.0	6,977.99	69	811.1	4,164.2	612.80
25	27,326.5	432,238.2	6,743.73	70	685.0	3,353.1	525.33
26	25,800.9	404,911.7	6,519.39	71	572.2	2,668.1	445.15
27	24,358.5	379,110.8	6,305.60	72	472.3	2,095.9	372.50
28	22,994.9	354,752.3	6,101.93	73	384.7	1,623.6	307.39
29	21,706.7	331,757.4	5,908.73	74	308.9	1,238.9	249.90
30	20,489.9	310,050.7	5,725.58	75	244.2	930.0	199.91
31	19,340.1	289,560.8	5,551.49	76	189.8	685.8	157.14
32	18,254.1	270,220.7	5,386.45	77	144.8	496.0	121.18
33	17,228.1	251,966.6	5,229.69	78	108.2	351.2	91.48
34	16,257.1	234,738.5	5,079.08	79	79.1	243.0	67.53
35	15,338.0	218,481.4	4,934.12	80	56.4	163.9	48.60
36	14,468.0	203,143.4	4,794.50	81	39.1	107.5	33.98
37	13,642.9	188,675.4	4,658.36	82	26.3	68.4	23.04
38	12,860.2	175,032.5	4,525.32	83	17.1	42.1	15.10
39	12,117.5	162,172.3	4,395.01	84	10.7	25.0	9.51
40	11,412.2	150,054.8	4,266.73	85	6.5	14.3	5.82
41	10,741.8	138,642.6	4,139.77	86	3.8	7.8	3.43
42	10,104.3	127,900.8	4,013.79	87	2.1	4.0	1.91
43	9,497.3	117,796.5	3,887.94	88	1.1	1.9	1.01
44	8,919.5	108,299.2	3,762.39	89	.5	.8	.46
45	8,368.7	99,379.7	3,636.33	90	.2	.3	.19
46	7,843.6	91,011.0	3,509.74	91	.1	.1	.10
47	7,342.8	83,167.4	3,382.45	92	.0		
48	6,864.7	75,824.6	3,254.00				
49	6,408.5	68,959.9	3,124.70				
50	5,972.7	62,551.4	2,994.06				
51	5,556.5	56,578.7	2,862.28				
52	5,159.2	51,022.2	2,729.57				
53	4,779.7	45,863.0	2,595.75				
54	4,417.5	41,083.3	2,461.15				
55	4,071.8	36,665.8	2,325.81				
56	3,742.4	32,594.0	2,190.30				
57	3,428.6	28,851.6	2,054.71				
58	3,130.2	25,423.0	1,919.58				
59	2,846.9	22,292.8	1,785.34				

TABLE VIII—*Continued*

q_x=250 Per Cent A.M.(5) q_x

Age x	D_x	N_x	M_x	Age x	D_x	N_x	M_x
15	48,101.7	783,005.4	10,815.73	60	2,142.2	14,524.1	1,450.58
16	45,414.9	734,903.7	10,419.48	61	1,904.1	12,381.9	1,314.49
17	42,870.6	689,488.8	10,037.80	62	1,681.8	10,477.8	1,182.86
18	40,458.6	646,618.2	9,667.26	63	1,475.3	8,796.0	1,056.44
19	38,174.6	606,159.6	9,309.86	64	1,284.3	7,320.7	935.70
20	36,010.5	567,985.0	8,963.59	65	1,108.8	6,036.4	821.35
21	33,959.6	531,974.5	8,627.48	66	948.7	4,927.6	714.05
22	32,017.5	498,014.9	8,302.50	67	803.7	3,978.9	614.23
23	30,178.8	465,997.4	7,988.45	68	673.5	3,175.2	522.30
24	28,441.4	435,818.6	7,688.13	69	557.8	2,501.7	438.67
25	26,799.3	407,377.2	7,400.38	70	456.0	1,943.9	363.43
26	25,248.1	380,577.9	7,125.34	71	367.5	1,487.9	296.65
27	23,784.3	355,329.8	6,863.83	72	291.6	1,120.4	238.25
28	22,403.1	331,545.5	6,615.22	73	227.5	828.8	188.03
29	21,101.1	309,142.4	6,380.03	74	174.2	601.3	145.57
30	19,873.7	288,041.3	6,157.45	75	130.7	427.1	110.36
31	18,716.3	268,167.6	5,946.41	76	95.9	296.4	81.79
32	17,625.4	249,451.3	5,746.77	77	68.6	200.5	59.05
33	16,596.8	231,825.9	5,557.47	78	47.8	131.9	41.52
34	15,625.1	215,229.1	5,376.09	79	32.3	84.1	28.30
35	14,706.9	199,604.0	5,201.95	80	21.1	51.8	18.63
36	13,839.2	184,897.1	5,034.58	81	13.3	30.7	11.84
37	13,017.4	171,057.9	4,871.79	82	8.0	17.4	7.17
38	12,238.8	158,040.5	4,713.06	83	4.6	9.4	4.15
39	11,501.0	145,801.7	4,558.06	84	2.5	4.8	2.27
40	10,801.1	134,300.7	4,405.83	85	1.3	2.3	1.19
41	10,136.6	123,499.6	4,255.67	86	.6	1.0	.55
42	9,505.2	113,363.0	4,106.96	87	.3	.4	.28
43	8,904.6	103,857.8	3,958.99	88	.1	.1	.10
44	8,333.4	94,953.2	3,811.82	89	.0		
45	7,789.3	86,619.8	3,664.55				
46	7,271.1	78,830.5	3,517.27				
47	6,777.4	71,559.4	3,369.81				
48	6,306.5	64,782.0	3,221.64				
49	5,857.7	58,475.5	3,073.15				
50	5,429.5	52,617.8	2,923.89				
51	5,021.3	47,188.3	2,774.24				
52	4,632.3	42,167.0	2,624.35				
53	4,261.5	37,534.7	2,474.13				
54	3,908.6	33,273.2	2,324.16				
55	3,572.8	29,364.6	2,174.49				
56	3,254.1	25,791.8	2,025.92				
57	2,951.8	22,537.7	1,878.58				
58	2,665.8	19,585.9	1,733.14				
59	2,396.0	16,920.1	1,590.28				

TABLE VIII—*Continued*

q_x=A.M.(5) q_x+.010

Age x	D_x	N_x	M_x	Age x	D_x	N_x	M_x
15	48,101.7	759,580.7	11,931.19	60	2,360.2	21,951.1	1,314.91
16	45,194.5	711,479.0	11,314.55	61	2,165.4	19,590.9	1,232.50
17	42,460.0	666,284.5	10,732.17	62	1,981.8	17,425.5	1,152.01
18	39,886.9	623,824.5	10,180.97	63	1,808.9	15,443.7	1,073.49
19	37,466.7	583,937.6	9,660.15	64	1,646.3	13,634.8	997.02
20	35,189.8	546,470.9	9,167.37	65	1,493.6	11,988.5	922.72
21	33,047.6	511,281.1	8,700.88	66	1,350.4	10,494.9	850.64
22	31,032.7	478,233.5	8,259.68	67	1,216.4	9,144.5	780.95
23	29,137.6	447,200.8	7,842.32	68	1,091.2	7,928.1	713.67
24	27,356.6	418,063.2	7,448.83	69	974.6	6,836.9	649.03
25	25,682.6	390,706.6	7,077.52	70	866.3	5,862.3	587.14
26	24,109.6	365,024.0	6,727.50	71	766.1	4,996.0	528.20
27	22,632.0	340,914.4	6,397.98	72	673.7	4,229.9	472.28
28	21,244.1	318,282.4	6,087.79	73	588.8	3,556.2	419.46
29	19,940.9	297,038.3	5,796.22	74	511.2	2,967.4	369.90
30	18,717.3	277,097.4	5,522.19	75	440.6	2,456.2	323.64
31	17,568.2	258,380.1	5,264.39	76	376.8	2,015.6	280.82
32	16,489.4	240,811.9	5,022.17	77	319.6	1,638.8	241.56
33	15,476.3	224,322.5	4,794.28	78	268.6	1,319.2	205.78
34	14,524.3	208,846.2	4,579.24	79	223.5	1,050.6	173.47
35	13,629.6	194,321.9	4,376.18	80	184.0	827.1	144.61
36	12,788.7	180,692.3	4,184.30	81	149.7	643.1	119.08
37	11,997.7	167,903.6	4,002.29	82	120.3	493.4	96.80
38	11,253.6	155,905.9	3,829.51	83	95.3	373.1	77.53
39	10,553.5	144,652.3	3,665.29	84	74.4	277.8	61.17
40	9,894.6	134,098.8	3,508.94	85	57.2	203.4	47.51
41	9,274.2	124,204.2	3,359.71	86	43.2	146.2	36.24
42	8,689.8	114,930.0	3,216.94	87	32.0	103.0	27.10
43	8,139.1	106,249.2	3,080.04	88	23.2	71.0	19.82
44	7,620.2	98,101.1	2,948.72	89	16.5	47.8	14.22
45	7,130.9	90,480.9	2,822.29	90	11.4	31.3	9.91
46	6,669.5	83,350.0	2,700.45	91	7.7	19.9	6.75
47	6,234.3	76,680.5	2,582.85	92	5.1	12.2	4.52
48	5,823.5	70,446.2	2,468.92	93	3.2	7.1	2.86
49	5,435.9	64,622.7	2,358.63	94	2.0	3.9	1.81
50	5,069.9	59,186.8	2,251.48	95	1.2	1.9	1.11
51	4,724.3	54,116.9	2,147.30	96	.7	.7	.67
52	4,397.9	49,392.6	2,045.87	97	.0		
53	4,089.5	44,994.7	1,946.90				
54	3,798.2	40,905.2	1,850.33				
55	3,523.0	37,107.0	1,756.00				
56	3,263.1	33,584.0	1,663.86				
57	3,017.5	30,320.9	1,573.65				
58	2,785.6	27,303.4	1,485.44				
59	2,566.7	24,517.8	1,399.19				

TABLE VIII—*Continued*

$q_x = \text{A.M.}(5)\ q_x + .020$

Age x	D_x	N_x	M_x	Age x	D_x	N_x	M_x
15	48,101.7	666,647.1	16,356.60	60	1,489.1	12,929.8	873.40
16	44,736.4	618,545.4	15,281.86	61	1,352.0	11,440.7	807.20
17	41,603.6	573,809.0	14,279.36	62	1,224.5	10,088.7	744.09
18	38,686.2	532,205.4	13,343.08	63	1,106.0	8,864.2	683.90
19	35,970.4	493,519.2	12,469.48	64	996.1	7,758.2	626.66
20	33,441.9	457,548.8	11,653.86	65	894.2	6,762.1	572.20
21	31,087.6	424,106.9	10,892.03	66	800.0	5,867.9	520.58
22	28,896.1	393,019.3	10,180.89	67	713.0	5,067.9	471.67
23	26,856.3	364,123.2	9,517.10	68	632.9	4,354.9	425.52
24	24,959.0	337,266.9	8,898.67	69	559.3	3,722.0	382.06
25	23,194.0	312,307.9	8,322.19	70	491.8	3,162.7	341.20
26	21,552.5	289,113.9	7,785.17	71	430.2	2,670.9	303.01
27	20,026.4	267,561.4	7,285.38	72	374.2	2,240.7	267.50
28	18,607.6	247,535.0	6,820.22	73	323.5	1,866.5	234.62
29	17,288.9	228,927.4	6,387.59	74	277.8	1,543.0	204.32
30	16,063.4	211,638.5	5,985.38	75	236.8	1,265.2	176.55
31	14,924.3	195,575.1	5,611.20	76	200.3	1,028.4	151.33
32	13,865.7	180,650.8	5,263.28	77	168.0	828.1	128.57
33	12,881.8	166,785.1	4,939.65	78	139.6	660.1	108.17
34	11,966.7	153,903.3	4,637.97	79	114.8	520.5	90.01
35	11,115.6	141,936.6	4,356.71	80	93.4	405.7	74.08
36	10,324.0	130,821.0	4,094.43	81	75.1	312.3	60.23
37	9,587.2	120,497.0	3,849.25	82	59.6	237.2	48.30
38	8,901.3	110,909.8	3,619.88	83	46.7	177.6	38.24
39	8,262.8	102,008.5	3,405.25	84	36.0	130.9	29.77
40	7,668.2	93,745.7	3,204.12	85	27.3	94.9	22.78
41	7,114.3	86,077.5	3,015.37	86	20.4	67.6	17.18
42	6,598.3	78,963.2	2,838.15	87	14.9	47.2	12.65
43	6,117.3	72,364.9	2,671.35	88	10.7	32.3	9.16
44	5,669.0	66,247.6	2,514.35	89	7.5	21.6	6.47
45	5,251.0	60,578.6	2,366.30	90	5.1	14.1	4.43
46	4,861.2	55,327.6	2,226.55	91	3.4	9.0	2.97
47	4,497.7	50,466.4	2,094.54	92	2.2	5.6	1.93
48	4,158.5	45,968.7	1,969.51	93	1.4	3.4	1.24
49	3,842.1	41,810.2	1,851.14	94	.9	2.0	.80
50	3,546.8	37,968.1	1,738.80	95	.5	1.1	.45
51	3,271.2	34,421.3	1,632.09	96	.3	.6	.27
52	3,014.1	31,150.1	1,530.76	97	.2	.3	.19
53	2,774.1	28,136.0	1,434.29	98	.1	.1	.10
54	2,550.1	25,361.9	1,342.39	99	.0		
55	2,341.0	22,811.8	1,254.72				
56	2,146.0	20,470.8	1,171.20				
57	1,964.1	18,324.8	1,091.49				
58	1,794.5	16,360.7	1,015.42				
59	1,636.4	14,566.2	942.77				

THE INCONTESTABLE CLAUSE

By H. W. BUTTOLPH

It has long been my opinion that the life insurance companies, in their general attitude toward the incontestable clause have been encouraging a very grave danger and neglecting a very important defense against a certain class of unjust and fraudulent claims. The sole purpose of this paper is to emphasize, if possible, the danger referred to and to direct attention to an advantage to be gained from the incontestable clause, which has hitherto been apparently overlooked.

While the incontestable clause had appeared in life insurance policies for many years before 1907, yet it was in no sense standardized prior to that year, and, as might naturally be expected, court decisions with reference to it were often conflicting.

For a number of years at least one prominent company had used a clause stating that the policy was incontestable from date. This the courts repeatedly held did not prevent a contest on the ground of fraud in securing the policy. At the same time, the courts in numerous instances upheld the validity of those clauses which prescribed a reasonable length of time within which any fraud practiced in securing the policy might be detected. The enactment of a law in New York providing for standard forms of policies, followed by standard provisions acts in other states, began the movement for the standardization of the incontestable clause, resulting in the form now current.

The New York form required this clause: "This policy shall be incontestable, except for non-payment of premiums (leave blank or insert 'one year' or 'two years') from its date. If the age of the insured has been understated, the amount payable hereunder shall be such as the premium paid would have purchased at the correct age." While this would have been better had it contained the more modern provision that to make the policy incontestable the specified period must end during the life time of the insured, yet, in my opinion, every other addition that has been made to the clause is detrimental. But such additions were almost immediately made. Illi-

nois, for example, at its next legislative session, added to the exception "and except for violations of the conditions of the policy relating to the naval or military service in time of war." Since then, other additions have been made, all of them of an illogical nature and increasing the dangers with which it menaces life insurance.

Beginning thus with provisions crudely drawn, followed by the incontestable-from-date clause repeatedly held to be contrary to public policy if literally construed, and by compulsory legislation of a questionable character, all with the silent acceptance if not active assistance of the insurance companies, the situation has become one in which it is small wonder if even the higher courts become confused and sometimes fail to agree, one with another, in the principles upon which their decisions are based.

It appears that two misconceptions of the meaning and intent of the law almost immediately arose and have been permitted to continue with little protest and even some encouragement on the part of life insurance companies:

First, that in making a policy incontestable, we in some way negative its plainly expressed provisions and in reality, to a certain extent, make claims incontestable; and

Second, that the incontestable clause is binding on the company but not on the insured.

The first of these two errors originated early and has been supported by many court decisions, though most recently there have been encouraging signs of a disposition on the part of a number of higher courts to give to the expression "this policy shall be incontestable" its true meaning.

Perhaps the early interpretation was encouraged by the inclusion in the clause of entirely unnecessary exceptions. If a contract is incontestable, why should it be necessary to say "except for violation of its provisions regarding military or naval service"? If it cannot be contested, all of its provisions are, of course, to be enforced and a contest of a claim based upon the violation of the terms of the contract is not a contest of the policy.

In *Ruling Case Law*, Volume XIV, page 1233, reference is made to cases which seem to the editor to warrant the statement that if a policy provides that it shall be incontestable after a certain period, except for certain causes, death by suicide not being one of them, the incontestable clause applies in case of the death of the insured by suicide,

although the policy contains another clause providing that death by suicide is not a risk assumed by the insurer.

However, there also seems to the same editor to be ground for this further statement:

But it would seem that a provision for reduced liability in the case of death by suicide is not affected by the incontestable clause.

If the policy referred to above had not contained in its incontestable clause the exceptions covering death from certain causes, the suicide clause might have been held to be valid and not affected by the incontestable clause. Upon this theory, I believe a removal of the exceptions is more to be desired than an increase in their number.

While there are many decisions holding that, in some way not clearly explained, the incontestable clause does modify the terms of the policy, yet it has seemed to me that of late the higher courts are gradually coming more and more to the other point of view. I will not attempt reference to all of the cases which have come to my attention, but will quote from a few illustrative of this attitude.

In *Travelers Insurance Company* vs. *Peake* (89 S. Rep. 418) the Supreme Court of Florida said

Policies of insurance are designed to secure indemnity to the insured for losses or injuries stated in the policy, and the terms used should be so construed as to effectuate the purpose designed, ambiguous provisions being fairly construed in favor of the insured, but parties sui juris are bound by their valid contracts, and where a particular risk is expressly and clearly excepted from the risks assumed by the insurer, the courts have no power to enforce indemnity for losses or injuries resulting from such excepted risks, as expressed by the indemnity contract contained in the policy.

In the case of *Lee* vs. *Southern Life and Health Insurance Company*, there was a clause in the policy excepting death caused by a malicious act of the beneficiary from the risks assumed by the company. There was also an incontestable clause. The Court of Appeals of Alabama held that they were not inconsistent, that the death of the insured by the malicious act of the beneficiary was not covered by the policy, and that a refusal to pay was not a contest of the policy but a carrying out of its terms.

In *Hearn* vs. *Standard Life Insurance Company*, suicide within

the contestable period was alleged, but suit was not brought until after the expiration of that period, and the plaintiff then claimed that the policy was incontestable. In the opinion of the Federal Court hearing the case, is found the following sound reasoning.

> If a risk is not assumed, how can there be any liability? If there is no liability, why must there be a denial thereof before the expiration of the year? If the risk is not assumed, certainly a bill in equity will not lie to rescind the contract after suicide and, of course, none could be executed before. To defend against suicide, there is a complete and adequate remedy at law.

In *Sanders* vs. *Jefferson Standard*, a double indemnity clause provided for an exception on the basis of which the case was defended, and the U.S. Circuit Court of Appeals for the Fifth Circuit held that it is as much the right of insurer to defend suit on the policy after the expiration of the contestable period on the basis of this exception as it would be to dispute the liability on the ground that the death of the insured had not occurred. The provision for incontestability does not have the effect of converting a promise to pay on the happening of a stated contingency into a promise to pay whether such contingency does or does not happen. The defense was not a "contest" within the meaning of the incontestable clause.

Certainly, these decisions and many more which might be quoted should encourage us, in spite of those of a contrary character, to continue to fight for a clean-cut distinction between what one supreme court defined as a question of coverage as opposed to a question of forfeiture.

The second of the two misconceptions as to the scope and intent of the clause seems to have been almost entirely overlooked. I do not remember to have seen a reference to it in any court decision, which, perhaps, may be taken to mean that the defendant companies have never brought it strongly to the attention of the courts.

The *American and English Encyclopedia of Law* states that, when a policy stipulates that it shall be incontestable for fraud, "the practical and intended effect is to create a short period of limitation within which the insurer must test, if ever, the validity of the contract." Why insurer and not also "insured"? If the incontestable clause prevents the company from alleging fraud in the securing of the insurance by the policyholder, why does it not also

prevent the policyholder or his beneficiary from raising the question of misrepresentation or an injustice in the terms of the policy?

The soundness of my contention has been recognized in some of the decisions, though apparently upon grounds not dependent upon the incontestable clause. For example, in *Clements* vs. *Life Insurance Company of Virginia* (70 S.E. Rep. 1076), the court said

> The rule that all prior parol agreements are merged in a subsequent written contract touching the same subject is now too well established to need the support of cited authority. . . . . Equity will not assist one whose condition is traceable only to that want of diligence which may fairly be expected from a reasonable and prudent person, and even when he is watchful and discovers a wrong practised upon him, a court of equity requires that he should be prompt in asserting his claim to relief against it, for it will not aid those who sleep on their rights, but only those who are vigilant.

In *Arndt* vs. *Jefferson Standard Life Insurance Co.* (97 S.E. Rep. 631), the Supreme Court of North Carolina said

> We are of the opinion that plaintiff was guilty of negligence in not reading his policy. . . . . There was no fraud, trick, or artifice resorted to at the time the policy was delivered in order to prevent the plaintiff from reading it, and he kept it for nine years without doing so.

With such decisions as these arrived at, apparently without reference to the incontestable clause, is it not reasonable to suppose that that clause would be held to be binding upon insured as well as insurer? And yet, when the Kansas City Court of Appeals in the case of *Johnson* vs. *American Central Life Insurance Company* decided that the anniversary of a policy which had been in force for eight years should have been the date when the policy was actually handed to the insured rather than the date requested in the application, written in the policy, and observed by the policyholder for many years, record shows no attempt on the part of the company to establish the validity of the contract as written because of the incontestable clause.

I believe that the true purpose of the clause is to discourage litigation based upon either an alleged fraud in securing the insurance on the one hand or misrepresentation of or injustice in the terms of the policy on the other, when such litigation probably will not be instituted until long after the contract was executed and after the insured and perhaps the agent are dead.

The case above referred to, in which the American Central was defendant, was held to be governed by the rule laid down in the case of Halsey vs. the same company. The incontestable clause could not have been invoked by the defense in the Halsey case, but, whether the reasoning there was right or wrong, the presence in the Johnson contract of the incontestable clause and the lapse of eight years should have furnished some grounds for differentiation between the two.

The danger involved in this entire question seems to me to be serious, particularly in view of the recent development in the disability and double indemnity benefits. This danger was apparently recognized at the time of a recent amendment to the Illinois law when, to the exceptions already in the clause covering the non-payment of premiums and violation of conditions relating to military or naval service in time of war, was added "and except as to provisions and conditions relating to benefits in the event of total and permanent disability and those granting additional insurance specifically against death by accidents"; but the wisdom of meeting it in this way may be questioned. If this last amendment was necessary, should there not be another reading "and except as to the provision requiring the furnishing of proofs of death?"

The continued amendment of the clause by the addition of exceptions is awkward and, in view of the oft-quoted legal maxim "*Expressio unius est exclusio alterius*," dangerous. If possible, the courts must be made to realize that there is a vast difference between contesting a policy and contesting a claim, between a question of forfeiture and a question of coverage, and also that the incontestable clause is quite as binding upon the insured as it is upon the company.

OSCULATORY INTERPOLATION: NEW DERIVATION AND FORMULAS

By W. A. JENKINS

The objects of this paper are: (1) to present a new generalized method of developing osculatory interpolation formulas;[1] (2) to show that this new method produces formulas of degrees lower, in a majority of cases, than those of formulas derived by the methods of either Sprague or Lidstone,[2] as generalized by Dr. Reilly;[3] (3) to derive by this new method the fifth-difference formulas of Sprague, Buchanan, and Henderson, and to demonstrate the redundancy in degree of the first two; (4) to compare the three methods of development as regards degrees of formulas produced.

STATEMENT OF PROCEDURE

We shall assume the formula to be of the following form:

$$\left.\begin{aligned} u_x = &\sum_{i=0}^{n-1} {}_{x+i}C_{2i+1}\Delta^{2i}u_{1-i} + \phi(x)\Delta^{2n}u_{1-n} \\ &+ \sum_{i=0}^{n-1} {}_{\xi+i}C_{2i+1}\Delta^{2i}u_{-i} + \phi(\xi)\Delta^{2n}u_{-n} \end{aligned}\right\} \qquad (1)$$

[1] The theory and practice of osculatory interpolation is quite fully set forth in the following papers: T. B. Sprague, *Journal of the Institute of Actuaries*, XXII (1880), 270–85; George King, *Journal of the Institute of Actuaries*, XLII (1908), 225–94; XLIII (1909), 109–51; G. J. Lidstone, *Journal of the Institute of Actuaries*, XLII (1908), 394–97; James Buchanan, *Journal of the Institute of Actuaries*, XLII (1908), 369–94; James W. Glover, *Quarterly Publication of the American Statistical Association*, XII, 85–109; Johannes Karup, *Transactions of the Second International Actuarial Congress*, pp. 78–109; James W. Glover, *The United States Life Tables* (1921), p. 344; Robert Henderson, *Transactions of the Actuarial Society of America*, XXII (1921), 175–95; John F. Reilly, *Record of the American Institute of Actuaries*, XIII (1924), 4–22; XIV (1925), 12–20.

[2] Henderson's "osculatory" formula is of degree lower than that of the corresponding formula derived by the proposed method, but it will be noticed that his formula is not truly osculatory. As he points out, it has a "break" in the first derivative of $\frac{1}{3}\frac{1}{6}\Delta^6 u$.

[3] John F. Reilly, *op. cit.*

where $\xi = 1 - x$ and ${}_{x+i}C_{2i+1} = \dfrac{(x+i)!}{(2i+1)!\,(x-i-1)!}$ and $\Delta^0 u_a = u_a$. Thus the terms up to and including those involving $\Delta^{2n-2}u$ are identically those of Everett's interpolation formula,[1] only those terms involving $\Delta^{2n}u$ being modified.

We shall assume the function $\phi(x)$ to be a polynomial, whose degree. together with the coefficients of the various powers of x, will be determined according to the following conditions: (I) When the formula is applied to any interval, the end ordinates of the interval will be reproduced. (II) When the formula is applied to any two adjacent intervals, the two curves determined will have contact of order p and their junction.

EXAMPLE OF PROCEDURE: FIFTH-DIFFERENCE FORMULAS

To illustrate the foregoing procedure, we shall derive the fifth-difference formula with contact of order 2, corresponding to the formula of Sprague[2] generally used. Let

$$u_x = xu_1 + {}_{x+1}C_3\Delta^2 u_0 + \phi(x)\Delta^4 u_{-1} + \xi u_0 + {}_{\xi+1}C_3\Delta^2 u_{-1} + \phi(\xi)\Delta^4 u_{-2} .$$

Condition (I) determines that

$$\phi(x) = x(x-1)\Psi(x) ,$$

where $\Psi(x)$ is a polynomial. Then the first derivative at $x=1$ becomes

$$u_1' = u_1 + \tfrac{1}{3}\Delta^2 u_0 + \Psi(1)\Delta^4 u_{-1} - u_0 + \tfrac{1}{6}\Delta^2 u_{-1} + \Psi(0)\Delta^4 u_{-2} .$$

When we apply the formula to the next interval, the first derivative becomes at the same point

$$\bar{u}_1' = u_2 - \tfrac{1}{6}\Delta^2 u_1 - \Psi(0)\Delta^4 u_0 - u_1 - \tfrac{1}{3}\Delta^2 u_0 - \Psi(1)\Delta^4 u_{-1} .$$

Whence

$$\bar{u}_1' - u_1' = -[2\Psi(1) + 2\Psi(0) + \tfrac{1}{6}]\Delta^4 u_{-1} - \Psi(0)\Delta^6 u_{-2} , \qquad (2)$$

[1] J. D. Everett, *Journal of the Institute of Actuaries*, XXXV (1901), 453.

[2] T. B. Sprague, *op. cit.*, p. 280.

and for the continuity of the first derivative

$$\Psi(1) = -\frac{1}{12} \qquad \Psi(0) = 0 \,. \tag{3}$$

Similarly, for the second derivative,

$$\begin{aligned} u_1'' &= \Delta^2 u_0 + 2[\Psi'(1) + \Psi(1)]\Delta^4 u_{-1} + 2[-\Psi'(0) + \Psi(0)]\Delta^4 u_{-2} \\ \bar{u}_1'' &= \Delta^2 u_0 + 2[\Psi'(1) + \Psi(1)]\Delta^4 u_{-1} + 2[-\Psi'(0) + \Psi(0)]\Delta^4 u_0 \\ \bar{u}_1'' - u_1'' &= 2[\Psi(0) - \Psi'(0)](\Delta^5 u_{-1} + \Delta^5 u_{-2}) \,, \end{aligned} \tag{4}$$

the condition for the continuity of the second derivative being

$$\Psi'(0) = \Psi(0) \,. \tag{5}$$

Equations (3) and (5) can be satisfied if $\Psi(x)$ is of second degree. If we let

$$\Psi(x) = a_0 + a_1 x + a_2 x^2 \,,$$

and apply these conditions, we see that $a_0 = a_1 = 0$, $a_2 = -\frac{1}{12}$, and

$$\Psi(x) = -\frac{x^2}{12} \qquad \phi(x) = -\frac{x^3(x-1)}{12} \,.$$

Thus we have the fifth-difference formula with contact of order 2:

$$\left.\begin{aligned} u_x = x u_1 + {}_{x+1}C_3 \Delta^2 u_0 \; &- \frac{1}{12} x^3 (x-1) \Delta^4 u_{-1} \\ + \xi u_0 + {}_{\xi+1}C_3 \Delta^2 u_{-1} &- \frac{1}{12} \xi^3 (\xi - 1) \Delta^4 u_{-2} \,. \end{aligned}\right\} \tag{6}$$

It is seen that the continuity of the second derivative places but one condition on $\Psi(x)$. In the derivations of both Sprague and Lidstone,[1] the use of the "partial curves" imposes two conditions to obtain the same continuity.

FIFTH-DIFFERENCE FORMULAS OF SPRAGUE, BUCHANAN, AND HENDERSON

If we let

$$\Psi(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \,,$$

[1] G. J. Lidstone, *op. cit.*, p. 397.

and apply the conditions of equations (3) and (5), we see that $a_0 = a_1 = 0$, $a_3 = -\frac{1}{12} - a_2$, and that a_2 can be assigned any arbitrary value at will.

If we wish to let $a_2 = -\frac{7}{24}$, then

$$\Psi(x) = \frac{5x^3 - 7x^2}{24} \qquad \phi(x) = \frac{x^3(x-1)(5x-7)}{24} ,$$

which leads to Sprague's formula:

$$u_x = xu_1 + {}_{x+1}C_3\Delta^2 u_0 + \frac{1}{24}x^3(x-1)(5x-7)\Delta^4 u_{-1}$$

$$+ \xi u_0 + {}_{\xi+1}C_3\Delta^2 u_{-1} + \frac{1}{24}\xi^3(\xi-1)(5\xi-7)\Delta^4 u_{-2} .$$

If we let $a_2 = -\frac{1}{3}$, then

$$\Psi(x) = \frac{3x^3 - 4x^2}{12} \qquad \phi(x) = \frac{x^3(x-1)(3x-4)}{12} ,$$

which determines Buchanan's formula.[1]

By (2) and (4), the "breaks" in the first and second derivatives are

$$D_1 = -[2\Psi(1) + 2\Psi(0) + \tfrac{1}{6}]\Delta^4 u_{-1} - \Psi(0)\Delta^6 u_{-2} ,$$

$$D_2 = 2[\Psi(0) - \Psi'(0)](\Delta^5 u_{-1} + \Delta^5 u_{-2}) .$$

If we let

$$\Psi(x) = a_0 + a_1 x ,$$

and demand that $D_2 = 0$, but let D_1 vary from zero but involve only $\Delta^6 u$, we obtain:

$$a_0 = a_1 = -\frac{1}{36}$$

$$\Psi(x) = -\frac{x+1}{36} \qquad \phi(x) = -\frac{(x+1)x(x-1)}{36} = -\tfrac{1}{6}({}_{x+1}C_3) ,$$

which determines Henderson's formula.[2] It is seen that

$$D_1 = \frac{1}{36}\Delta^6 u_{-2} .$$

[1] James Buchanan, *op. cit.*, p. 374.

[2] Robert Henderson, *op. cit.*, p. 191. See p. 87, n. 2.

From the foregoing derivations, it is obvious that the degree of Sprague's and Buchanan's formulas is redundant by 1. We shall see later, in fifth-difference formulas, that for each additional even derivative made continuous this redundancy of degree is increased by 1.

GENERALIZED DERIVATION

Assuming u_x to be of the form (1), condition (I) requires that

$$\phi(x) = x(x-1)\Psi(x).$$

If $\bar{u}$ is defined as before, condition (II) demands that $\bar{u}_1^{(p)} - u_1^{(p)}$ be made zero for $p = 1, 2, 3, \ldots, p$. It will be convenient to treat the even and odd derivatives separately.

$$u_1^{(2s)} = \sum_{i=s}^{n-1} \left(_{x+i}C_{2i+1}^{(2s)}\right)_{x=1} \Delta^{2i} u_{1-i} + \left(\phi^{(2s)}(x)\right)_{x=1} \Delta^{2n} u_{1-n}$$

$$+ \sum_{i=s}^{n-1} \left(_{\xi+i}C_{2i+1}^{(2s)}\right)_{\xi=0} \Delta^{2i} u_{-i} + \left(\phi^{(2s)}(\xi)\right)_{\xi=0} \Delta^{2n} u_{-n},$$

$$\bar{u}_1^{(2s)} = \sum_{i=s}^{n-1} \left(_{x+i}C_{2i+1}^{(2s)}\right)_{x=0} \Delta^{2i} u_{2-i} + \left(\phi^{(2s)}(x)\right)_{x=0} \Delta^{2n} u_{2-n}$$

$$+ \sum_{i=s}^{n-1} \left(_{\xi+i}C_{2i+1}^{(2s)}\right)_{\xi=1} \Delta^{2i} u_{1-i} + \left(\phi^{(2s)}(\xi)\right)_{\xi=1} \Delta^{2n} u_{1-n}.$$

As $_{x+i}C_{2i+1}$ is an odd function of x, and as

$$\Delta^k u_{M+1} = \Delta^k u_M + \Delta^{k+1} u_M,$$

$$\bar{u}_1^{(2s)} - u_1^{(2s)} = \left(\phi^{(2s)}(x)\right)_{x=0} \left(\Delta^{2n+1} u_{1-n} + \Delta^{2n+1} u_{-n}\right). \qquad (7)$$

For $\bar{u}_1^{(2s+1)}$ and $u_1^{(2s+1)}$ we obtain equations the same as the two above with $(2s)$ replaced by $(2s+1)$. We can write their difference as follows:

$$\bar{u}_1^{(2s+1)} - u_1^{(2s+1)} = \sum_{i=s}^{n-1} \left(_{x+i}C_{2i+1}^{(2s+1)}\right)_{x=0} \Delta^{2i} u_{2-i} + \left(\phi^{(2s+1)}(x)\right)_{x=0} \Delta^{2n} u_{2-n}$$

$$-2\sum_{i=s}^{n-1}\left({}_{x+i}C_{2i+1}^{(2s+1)}\right)_{x=1}\Delta^{2i}u_{1-i}-2\left(\phi^{(2s+1)}(x)\right)_{x=1}\Delta^{2n}u_{1-n}$$

$$+\sum_{i=s}^{n-1}\left({}_{x+i}C_{2i+1}^{(2s+1)}\right)_{x=0}\Delta^{2i}u_{-i}+\left(\phi^{(2s+1)}(x)\right)_{x=0}\Delta^{2n}u_{-n}\ ,$$

which can be transformed, by the relation between differences, into

$$\bar{u}_1^{(2s+1)}-u_1^{(2s+1)}=\alpha+\beta+\left(\phi^{(2s+1)}(x)\right)_{x=0}\Delta^{2n+2}u_{-n}+\left[2\left(\phi^{(2s+1)}(x)\right)_{x=0}\right.$$
$$\left.-2\left(\phi^{(2s+1)}(x)\right)_{x=1}+\left({}_{x+n-1}C_{2n-1}^{(2s+1)}\right)_{x=0}\right]\Delta^{2n}u_{1-n}\ ,$$

where

$$\alpha=2\left[\left({}_{x+s}C_{2s+1}^{(2s+1)}\right)_{x=0}-\left({}_{x+s}C_{2s+1}^{(2s+1)}\right)_{x=1}\right]\Delta^{2s}u_{1-s}$$

$$\beta=\sum_{i=s+1}^{n-1}\left[2\left({}_{x+i}C_{2i+1}^{(2s+1)}\right)_{x=0}-2\left({}_{x+i}C_{2i+1}^{(2s+1)}\right)_{x=1}+\left({}_{x+i-1}C_{2i-1}^{(2s+1)}\right)_{x=0}\right]\Delta^{2i}u_{1-i}\ .$$

From the nature of the function ${}_{x+i}C_{2i+1}$ it follows directly that $\alpha=\beta=0$. Whence

$$\left.\begin{aligned}\bar{u}_1^{(2s+1)}-u_1^{(2s+1)}&=\left(\phi^{(2s+1)}(x)\right)_{x=0}\Delta^{2n+2}u_{-n}+\left[2\left(\phi^{(2s+1)}(x)\right)_{x=0}\right.\\&\left.-2\left(\phi^{(2s+1)}(x)\right)_{x=1}+\left({}_{x+n-1}C_{2n-1}^{(2s+1)}\right)_{x=0}\right]\Delta^{2n}u_{1-n}\ .\end{aligned}\right\}\quad(8)$$

It is clear from equations (7) and (8) that the continuity of an even derivative imposes one restriction on $\phi(x)$, namely, that

$$\left(\phi^{(2s)}(x)\right)_{x=0}=0\ ,\qquad(9)$$

and that of an odd derivative two, i.e.,

$$\left.\begin{aligned}\left(\phi^{(2s+1)}(x)\right)_{x=0}&=0\\2\left(\phi^{(2s+1)}(x)\right)_{x=0}-2\left(\phi^{(2s+1)}(x)\right)_{x=1}+\left({}_{x+n-1}C_{2n-1}^{(2s+1)}\right)_{x=0}&=0,\end{aligned}\right\}\quad(10)$$

and that for contact of order $p=2k$, $\phi(x)$ must be of degree $3k+1$; and for contact of order $p=2k-1$, $\phi(x)$ must be of degree $3k$.

We obtain equations (9) and (10) in terms of $\Psi(x)$, remembering that

$$\phi(x)=x(x-1)\Psi(x)\ ,$$

as follows:

$$(2s-1)\Psi^{(2s-2)}(0)-\Psi^{(2s-1)}(0)=0 \tag{11}$$

$$\Psi^{(2s)}(0)+\Psi^{(2s)}(1)-2s\Psi^{(2s-1)}(0)+2s\Psi^{(2s-1)}(1) = \frac{\left({}_{x+n-1}C_{2n-1}^{(2s+1)}\right)_{x=0}}{2(2s+1)} \tag{12a}$$

$$2s\Psi^{(2s-1)}(0)-\Psi^{(2s)}(0)=0\,. \tag{12b}$$

For contact of order $p=2k$: equation (11) must hold for $s=1, 2, 3, \ldots, k$; equations (12a) and (12b) must hold for $s=0, 1, 2, \ldots, k-1$.

For contact of order $p=2k-1$: equation (11) must hold for $s=1, 2, 3, \ldots, k-1$; equations (12a) and (12b) must hold for $s=0, 1, 2, \ldots, k-1$.

For contact of order $p=2k$, we see, from equations (11) and (12b), if

$$\Psi(x)=a_0+a_1x+\ \ldots\ +a_{3k-1}x^{3k-1}\,,$$

that

$$a_0=a_1=\ \ldots\ =a_{2k-1}=0\,.$$

If

$$(h)^{(m)}=h(h-1)(h-2)\ \ldots\ (h-m+1)$$

and

$$C_n^s=\left({}_{x+n-1}C_{2n-1}^{(2s+1)}\right)_{x=0}\,,$$

equation (12a) reduces to

$$(2k+1)^{(2s)}a_{2k}+(2k+2)^{(2s)}a_{2k+1}+\ \ldots\ +(3k)^{(2s)}a_{3k-1}=\frac{C_n^s}{2(2s+1)}\,.$$

For contact of order $p=2k-1$, we find similarly, if

$$\Psi(x)=a_0+a_1x+\ \ldots\ +a_{3k-2}x^{3k-2}\,,$$

that

$$a_0=a_1=\ \ldots\ =a_{2k-2}=0\,.$$

Equation (12*a*) becomes

$$(2k)^{(2s)}a_{2k-1}+(2k+1)^{(2s)}a_{2k}+\ \ldots\ .\ +(3k-1)^{(2s)}a_{3k-2}=\frac{C_n^s}{2(2s+1)}\,.$$

CONDITIONS IMPOSED UPON $\Psi(x)$

The complete conditions for contact of order $p=2k$ are

$$\left.\begin{aligned}&\Psi(x)=a_{2k}x^{2k}+a_{2k+1}x^{2k+1}+\ \ldots\ .\ +a_{3k-1}x^{3k-1}\\&(2k+1)^{(2s)}a_{2k}+(2k+2)^{(2s)}a_{2k+1}+\ \ldots\ .\ +(3k)^{(2s)}a_{3k-1}\\&\qquad\qquad=\frac{C_n^s}{2(2s+1)}\end{aligned}\right\}\qquad(13a)$$

$$s=0,\ 1,\ 2,\ \ldots\ .\ ,\ k-1\ ,$$

and for contact of order $p=2k-1$ are

$$\left.\begin{aligned}&\Psi(x)=a_{2k-1}x^{2k-1}+a_{2k}x^{2k}+\ \ldots\ .\ +a_{3k-2}x^{3k-2}\\&(2k)^{(2s)}a_{2k-1}+(2k+1)^{(2s)}a_{2k}+\ \ldots\ .\ +(3k-1)^{(2s)}a_{3k-2}\\&\qquad\qquad=\frac{C_n^s}{2(2s+1)}\end{aligned}\right\}\qquad(13b)$$

$$s=0,\ 1,\ 2,\ \ldots\ .\ ,\ k-1\ .$$

Equations (13*a*) or (13*b*) yield k simultaneous linear equations from which the k coefficients of $\Psi(x)$ are determined. The general solution of these equations would possibly be of advantage.

It was found convenient to tabulate values of

$$C_n^s=\left(_{x+n-1}C_{2n-1}^{(2s+1)}\right)_{x=0}$$

for small values of s and n as shown in Table I.

TABLE I

n \ s	0	1	2	3
1	1	0	0	0
2	−1/6	1	0	0
3	1/3	−1/4	1	0
4	−1/140	7/120	−1/3	1

The following recursion formula was used:

$$C_n^s = -\frac{n-1}{2(2n-1)}C_{n-1}^s + \frac{s(2s+1)}{(n-1)(2n-1)}C_{n-1}^{s-1}.$$

APPLICATIONS OF GENERALIZED DEVELOPMENT

To illustrate the derivation of an osculatory formula by means of equations (13*a*) or (13*b*), we shall again develop the fifth-difference formula with contact of order 2.

Here $n=2$; $p=2k=2$; $k=1$;

$$\Psi(x) = a_2x^2$$

and $s=0$; $a_2=\frac{C_2^0}{2}=-\frac{1}{12}$. Therefore,

$$\Psi(x) = -\frac{x^2}{12} \qquad \phi(x) = -\frac{x^3(x-1)}{12}$$

and

$$u_x = xu_1 + {}_{x+1}C_3\Delta^2u_0 - \frac{1}{12}x^3(x-1)\Delta^4u_{-1}$$

$$+\xi u_0 + {}_{\xi+1}C_3\Delta^2u_{-1} - \frac{1}{12}\xi^3(\xi-1)\Delta^4u_{-2},$$

as before. Other formulas obtained from this generalized development are shown in Table II.

In Table III are tabulated the degrees of osculatory interpolation formulas developed by three methods: Sprague's[1] and Lidstone's,[2] as generalized by Dr. Reilly, and the proposed method. The first figure in each square is the degree obtained by Sprague's method, the second by Lidstone's, and the third by the proposed method.

TABLE OF COEFFICIENTS

Table IV gives the coefficients of the leading major differences (Δu_0, Δ^2u_0, ...) to calculate the leading minor differences (δu_2, δ^2u_2, ...) of the function when six ordinates ($u_0, u_1, \ldots, u_5$) are given and the fifth-difference formula ($n=2$, $p=2$, Table II)

[1] John F. Reilly, *op. cit.*, XIII, 21.

[2] *Ibid.*, XIV, 18.

TABLE II

$$u_x = xu_1 + {}_{x+1}C_3\Delta^2 u_0 + {}_{x+2}C_5\Delta^4 u_{-1} + \ldots + \phi(x)\Delta^{2n}u_{1-n}$$
$$+ \xi u_0 + {}_{\xi+1}C_3\Delta^2 u_{-1} + {}_{\xi+2}C_5\Delta^4 u_{-2} + \ldots + \phi(\xi)\Delta^{2n}u_{-n}$$

VALUES OF $\phi(x)$

Order of Contact	$n=1$	$n=2$	$n=3$	$n=4$
$p=1$	$\frac{x^2(x-1)}{2}$*	$-\frac{x^2(x-1)}{12}$	$\frac{x^2(x-1)}{60}$	$-\frac{x^2(x-1)}{280}$
$p=2$	$\frac{x^3(x-1)}{2}$	$-\frac{x^3(x-1)}{12}$	$\frac{x^3(x-1)}{60}$	$-\frac{x^3(x-1)}{280}$
$p=3$	$-\frac{x^4(x-1)(3x-5)}{4}$	$\frac{x^4(x-1)(7x-11)}{48}$	$-\frac{x^4(x-1)(29x-45)}{960}$	$\frac{x^4(x-1)(265x-409)}{40,320}$
$p=4$	$-\frac{x^5(x-1)(2x-3)}{2}$	$\frac{x^5(x-1)(11x-16)}{60}$	$-\frac{x^5(x-1)(9x-13)}{240}$	$\frac{x^5(x-1)(409x-589)}{50,400}$

* This formula, commonly known as Karup's formula, is the only one in Table II published heretofore (cf. Johannes Karup, *loc. cit.*)

is used to subdivide the middle major interval $(u_2 - u_3)$ into five minor intervals and to interpolate four ordinates $(u_{2.2}, u_{2.4}, \ldots u_{2.8})$ at these subdivisions.

TABLE III

DEGREES OF OSCULATORY INTERPOLATION FORMULAS DEVELOPED BY DIFFERENT METHODS

n / p	1	2	3	4	5	6
1	3, 3, 3	3, 4, 3	3, 6, 5	3, 8, 7	3, 10, 9	3, 12, 11
2	5, 5, 4	5, 5, 4	5, 6, 5	5, 8, 7	5, 10, 9	5, 12, 11
3	7, 7, 6	7, 7, 6	7, 7, 6	7, 8, 7	7, 10, 9	7, 12, 11
4	9, 9, 7	9, 9, 7	9, 9, 7	9, 9, 7	9, 10, 9	9, 12, 11
5	11, 11, 9	11, 11, 9	11, 11, 9	11, 11, 9	11, 11, 9	11, 12, 11
6	13, 13, 10	13, 13, 10	13, 13, 10	13, 13, 10	13, 13, 10	13, 13, 11

It is obvious from Table IV, inasmuch as the minor fourth differences are constant, that the numerical computation of interpolation is somewhat simpler by this formula than by Sprague's formula in which the minor fifth differences are constant.

TABLE IV

$$\delta^k u_2 = a\Delta u_0 + b\Delta^2 u_0 + c\Delta^3 u_0 + d\Delta^4 u_0 + e\Delta^5 u_0$$

k	a	b	c	d	e
1	.2	.32	.088	$-.02293\dot{3}$	$.00053\dot{3}$
2	.....	.04	.048	$.00026\dot{6}$	$.00213\dot{3}$
3	.....	.....	.008	.0144	$-.0008$
4	.....	.....	.....	$-.0064$	$-.0032$
5	.....	.....	.....	.....	.0000

CONCLUSIONS

1. Under the condition usually demanded by actuaries, namely, that $n = p$, the degrees of the formulas given in Table II are *less* than those of corresponding formulas derived by any other method, save when $n = p = 1$. Then all methods produce the same result.

2. The proposed method embodies two new features: (*a*) By dispensing with the "partial curves" of Sprague's and Lidstone's

methods, the author finds that a saving of one is effected in the degree of the resulting modified terms, for each even derivative made continuous. (*b*) The last two coefficients of Everett's interpolation formula are assumed to be modified. This amounts to assuming the modification of the last two coefficients of Newton's interpolation formula. Lidstone's method modifies the last term only; Sprague's, a variable number of terms.

3. Feature (*a*) decreases the degree of the modified terms whenever $p>1$.

4. Feature (*b*) proves a decided advantage in simplicity when the order of contact is high in comparison with the order of differences, but a decided disadvantage when the order of contact is low and the order of the differences high, if the resulting formulas are considered in the light of Sprague's formulas. The reason for this disadvantage is of course that the degree of the formula derived by the proposed method is forced up, not by the modified terms, but by the last unmodified terms.

5. As shown in Tables II and III, the method presented produces the formula of minimum degree in each case below the upper heavy diagonal line; that degree being actually less than the degree obtained by Sprague's method in each case below the lower heavy diagonal line.

6. Lidstone's method of development produces the formula of minimum degree in but one case, namely, when $n=p=1$. The proposed method yields a lower degree than Lidstone's in every other case.

7. It seems probable that a minimum degree curve over the whole range of values of n and p, superseding the proposed method when $n\geqq p+1$ (nearly) and Sprague's and Lidstone's methods when $p>1$, could be obtained by dispensing with the "partial curves" and assuming that a *variable* number of terms are modified.

DISCUSSION OF THE PAPER READ AT THE PREVIOUS MEETING

RETURNS UNDER DISTRICT-AGENCY CONTRACTS

By M. A. LINTON

MR. A. T. MACLEAN:

(Written discussion presented by Mr. Richard Little)

In accepting the invitation to discuss Mr. Linton's paper I did so with considerable temerity for much the same reasons as indicated in my discussion of his previous paper. His essay on "Returns under General Agency Contracts" was, to my mind, a courageous undertaking, but scarcely to be compared with the present paper in its effort to evolve a standard for dealing with the further problem of district agencies and agency contracts.

In the first place, I would again like to emphasize that to my mind the company is not at all concerned with the profit made by the general agent out of his agency. If he develops a sufficient and satisfactory volume of business, and if his agency is conducted in accordance with the policy of the company, I think we may leave it at that. I take it, however, that the object of the present paper is to render some guide to the general agent as to how far he may go in the matter of his district agents, and with such desire to assist the general agent one can have no other feeling than sympathy.

Just as is the case between the company and the general agent, so it is between the general agent and the subordinate agent. His contract will in almost every case be a matter of "trading." Furthermore, his contract will of necessity depend on the kind of contracts offered in that particular territory; the ability of competing general agents to make attractive contracts to new men; and the degree of assistance that the general agent can render to his sub-agent.

The sales strength of the same company varies in different localities. Company A may have a general agent in one city with great organizing ability and with capital behind him. The same company may have in another district a general agent who is less able in the

matter of organization and who is limited financially. The problem of these two agents will vary considerably just on account of this factor alone, and the problem will be further complicated by the local conditions under which they must obtain their new agents.

While statistics and comparisons may be helpful, they are not always the full answer to questions of field development, as there must always be considered in this business of ours that human element, that much sought for and desired quality—*personality*. Two general agents may have equal ability so far as knowledge of the business is concerned, but the one who with such "personality" views his problems from the human standpoint is bound to succeed where the other may fail. Such an agent will attract men to him and hold them, even against those of his competitors who offer more attractive contracts. So I repeat, there is more involved in the consideration of this subject than mere figures, which represent only the cold facts, or underlying working model, to which some consideration has, of course, to be given.

The question of the sub-agent is not by any means limited to a matter of obtaining such sub-agents and figuring how much they will write, and if they do write a certain amount, what will be the "profit." That would be the case only if "all other things were equal." But as has often been stated, such a condition seldom holds. If there be a variety of conditions under which the company itself has to seek its general agents and deal with them, there is certainly an infinitely greater variety of conditions facing the general agent who is looking for a profitable sub-agency business. This is still further complicated when we take into account the factor that Mr. Linton has introduced in the person of the "supervisor" who is himself part developer and part solicitor.

It seems to me that the attitude of the general agent who came to such a paper as this for advice would be to accept the figures shown in the tables without any very great consideration of what was behind them, and it is just in this particular that the publication of such tables seems to me undesirable. In my own somewhat limited experience, and in the opinion of those associated with me in the office who have had considerable experience along agency lines, there is no such thing as any particular regular production by sub-agents as a class over the long period of years outlined in the paper. It is true, of course, that there are many individual sub-

agents who are steady producers, but such men are apt to graduate into other classes, either with the company in which they start or in other companies, that as a result of the record of the sub-agent make him attractive offers. In fact, the figures shown by the tables seem to me to be of the same type as those mortality tables which, based on a very small number of lives, develop wonderfully complete tables of expectation, annuities, and insurance values. The completed table seems to bear the hall mark of authority, and a smooth system of graduation may produce wonderful results, but what does it all amount to if the basic data are insufficient?

In a problem such as Mr. Linton has discussed, the trouble is that the basic data are of such a heterogeneous type that the combination thereof presents little significance as far as the individual case, or even as far as the individual agency, is concerned, and while in the following remarks I offer some discussion of the paper as it stands, I would like it to be clearly understood that as far as I am personally concerned I should not care to offer these tables to any inquiring general agent, with the indication that they represent what he might expect as a result of his experience with sub-agents.

In the first place, I think it is a fair criticism to make that none of the three district-agency contracts suggested by Mr. Linton can be sufficiently profitable to the general agent unless his company is willing to contribute something in the way of an expense allowance for district-agency development, and in fact Mr. Linton appears to arrive at the same conclusion, as indicated in the last paragraph on page 222. Then again, it would seem the most practical course for a district agent in the early years of his career to devote his time to writing personal business. By this means he may become well established financially and build up a good income before entering upon organization work, which is always expensive and under which the profits are deferred.

In the concluding paragraph of his paper Mr. Linton indicates the need for supervisors as related to the subject of district-agency development, and while this is probably a very desirable, if not a necessary, condition to create, it is, none the less, a very expensive proposition for the general agent, and would mean the deferring of the profits on the business for a considerable period. Then again, if I have read the paper aright, Mr. Linton appears to avoid the question of operating expenses for the district agency. If, therefore,

the general agent finds it difficult to operate sub-agencies under the conditions outlined by Mr. Linton, I think that when all of the expenses are taken into account the problem would be still more acute.

There is no question in my mind but that there are many general agents who are simply losing money through district-agency development. It is an exceedingly hard thing to prove this in many cases, because there is so much service given the sub-agent in the way of time, agency meetings, letters of advice, help on particular cases—the financial value of which it is almost impossible to estimate, but which, none the less, is an actual financial outlay to the general agent, either directly or indirectly.

In addition to this there are certain other details that would affect the underlying basis of Mr. Linton's tables. In actual practice there is a considerable variation in the amount of the production of sub-agents, and tables based on regular production are difficult to apply to the actual conditions of the case. Mr. Linton assumes an average policy of $3,500, but I think this is too large for the type of business he has under consideration. The reduction in this amount to something like $2,000, which I am inclined to think is more nearly the case, would make a considerable difference in the figures submitted.

Perhaps one of the most important questions in connection with the possible profit under district-agency contracts is the matter of advances. Mr. Linton deals with this matter somewhat briefly, and with his general conclusion I think everyone must be in agreement. This, however, does not alter the fact that it is the almost universal practice to make such advances to new agents, and that there is a very great deal of money lost by reason thereof.

A friend of mine who is a very successful agent has stated that he has advanced money "against nothing in particular except a hope and confidence in the ability of a young man to make good," and that is, after all, fundamentally the position in which most general agents find themselves.

I think that the losses on account of such advances would make a material difference in the figures shown in the paper, and it is to my mind too important a subject to be altogether ignored or to be passed over simply by the statement that "it is much better to reduce or eliminate advances." That, of course, is the ideal situa-

tion, but the extent to which it can be actually carried out in practice is certainly most difficult to determine, and would vary tremendously in different agencies. The problem of judging men worthy of advances, and how long to carry them is one that must be decided by the general agent himself, and it will be as a result of his ability in this direction, coupled with a very large element of good or bad luck, that a large part of the question of profit or loss is going to depend.

I take it that the object of the paper is to endeavor to indicate a method of creating profitable district agencies by means of systematic development and expense allowances. Mr. Linton would have the district agent appoint sub-agents to increase the production of the sub-agency and at the same time allow some margin of profit for both the district agent and himself. If the district agent appoints a sub-agent and gives him renewals, there is very little left for the general agent. In many cases the sub-agent under a district agent will receive no renewals at all. If he proves that he can write life insurance, then he is very shortly going to be dissatisfied, because he is bound to be approached by other companies offering him a better contract, and the final result is that the district agent has trained that man for somebody else. This, I really believe, is the common case, and the best that the general agent can do if he wants to hold that particular individual is to make some readjustment of his contract that practically eliminates all possibilities of profit.

In my humble opinion, the only way for the district agent to make money is to write personal business. If he leaves sub-agents alone he will be happier, less worried, make more money, and have a better future. The theory of splitting up down the line may be very good, but again acknowledging my limited experience I do not believe that it would work out successfully in practice.

In conclusion I would not desire that the foregoing remarks be taken to indicate that I do not believe that the development of general agents' territory cannot be a profitable thing. My opinion is exactly the opposite of that, because I believe there can be no real profit to a general agent without expansion and increased development. The trouble with most of the general agents and branch office managers is that they really do not know how much money they are making out of their agencies. There are too many "leaks"

in the system and too many places where money is spent without any real account thereof being charged to the profits of the agency. The problem of the sub-agent is only one phase of the larger problem of the successful operation of the agency as a whole, and if Mr. Linton's paper serves to make the general agent or the branch manager think a little more of what is really going on in his agency then he will have accomplished something very much worth while.

MR. GEORGE GRAHAM:

I am not going into a detailed discussion of Mr. Linton's paper. I am quite sure he will agree, however, with what Mr. Maclean said, that there are other factors to be considered in these problems of agency development than those that can be put in tabular form.

I was interested in the statement made by Mr. Linton in the last paragraph of his paper. He says: "It is obviously in the interest of economy to have the premium collections and the other work of a general agency centralized in a relatively large office where the work can be standardized and well supervised."

Of necessity, money spent for agency development for the acquiring of new business must come in the main from the renewal premium income of the company; that is, the expenses outside of the direct acquisition costs, commissions, and medical and inspection fees. But why should the general agent be expected to supervise the collection of renewal premiums? Is he the man best fitted to do so? I do not think he is. I think that a man, expert in office detail, could do this work much more efficiently and much more economically, who would relieve the general agent so that he could devote his time to the work that he is best fitted for—the organization and care of his agency.

The reason I take it that it is customary for some companies to place the responsibility of renewal premium collections on the general agency is to justify the premium collection fee that is paid to the general agent and out of which he meets the expenses of maintaining his office. Then if there is any balance left over out of his collection fees he is expected to use it in the development of his agency. It occurs to me that there is little restraint on him if he should elect to put that money in his own bank account and allow his agency to run along without that development work that would be possible from the use of the balance of these collection fees.

If there is a large collection fee attached to a general agency, that indicates that the general agency is a well-established one, one that has been running successfully and doubtless is now being conducted successfully; so that the need in that particular agency for the expenditure of much money in further development and organization work is not so great as in the case of the new agency, the agency which has no renewal premium collections or no substantial amount of renewal premium collections, and therefore nothing in the way of moneys available for the thing that is very necessary to insure success, its development, and the building up of a new organization.

That raises the question then: Is the company not better off to take directly under its own charge the collection of renewal premiums and frankly appropriate out of its renewal premium income and its other revenues the proper amount in its judgment that it can afford and feel should be spent on agency development work, and apportion that money where it can do the most good?

Naturally, that money would then be sent to the new agencies, to the agencies that are small where the company wishes further development to be made; it would then be paying out its money directly for the purpose that it has in mind. It could, therefore, exercise a much more effective supervision over the manner in which that money should be expended, and altogether I think would get more beneficial results.

It occurs to me that this practice of placing the responsibility of renewal premium collections on the general agent and allowing the premium collection fees arises, in part at least, from the so-called expense limitations of the New York statutes and has compelled those companies that operate under these statutes to make a virtue of necessity.

In reading Mr. Linton's paper the thought occurred to me, as one of those who have constantly before us the problems of the younger companies, that we might with advantage change the form of our general agent's contract.

I have in mind those companies that make contract with the general agent, that specify, as a percentage of the premium, the compensation that they are willing to pay the general agent for the acquiring of new business and out of which he is also expected to do development work and organization work in his agency.

It seems to me that we might get better results if we were to distinguish between that part of the compensation we pay the general agent for the actual securing of new business and that part that we pay him for the purpose of developing his agency. It would clarify the situation I think for the general agent. It would make him see his problems more clearly. It would strengthen his hands at all events in making contracts with his soliciting agents. He would be in the position of paying his soliciting agent the maximum allowed by the company, if he will. He himself would receive exactly the same compensation, according to his contract, as his agents would receive for the writing of new business; and it would help him in seeing just how much he has available for the organization of his territory.

Anybody that has had experience in a matter of advances to agents will endorse everything that Mr. Linton says as to the grief and harmfulness of that practice. Yet it is one from which, while none of us like it, we cannot wholly get away. It has occurred to me that the evil can be at least reduced if, in making a contract with a man whom the company thinks well enough of to feel justified in investing some money in, it were clearly understood that there was a limit to the amount of money that he could get and that limit were set when the advances are arranged for.

It is not going to cure the evil; it might lessen it a little.

I believe if the company is willing to put money into an agent, a new agent doubtless, it would be better served if it would frankly pay that agent that money in the form of a salary or an allowance. It would make the responsibility of the agent to the company more direct and I believe would be productive of better results.

MR. W. H. McBRIDE:

It may be interesting to Mr. Graham at least, if not to others, to know that the company with which I am connected is experimenting along lines that are remarkably like those suggested by Mr. Graham in the remarks he has just made.

Our thought was that a distinction should be made, not only in the mind of the company but in the mind of the general agent or district agent who is doing the development work, as to the source of the funds used for development and the manner in which they were to be used. Our distinction, therefore, is as to the contract

year in which the business is produced. This is an experiment along very limited lines that we are undertaking; and our provision is a stated sum per thousand appropriated for development work which goes to the general agent for each thousand of business produced by his appointees during their first contract years.

To be a little more specific, we are allowing him $12.50 per thousand for the business which his appointees produce during their first contract year.

MR. M. A. LINTON:

(Written discussion submitted after the meeting)

When two persons base their thinking upon widely differing conceptions they are likely to arrive at different conclusions. Mr. Maclean proceeds on the assumption that it is not necessary for a life insurance company to interest itself in the profits of its general agents and that in like manner it is unnecessary for a general agent to interest himself in the profits of his district agents. It is assumed that competitive conditions in the field will enable the contracting parties to agree upon an arrangement that will be satisfactory to all concerned. Proceeding from a different conception, I take the ground that the company is interested in the prosperity of its general agents and is fully justified in helping them to determine what their profits from a given volume of business are likely to be. Likewise it is entirely reasonable for the company to assist the general agent and the district agent in arriving at a clear understanding of the probable profits that each will receive from a given type of contract.

The more I study the factors that go to make up a general agent's profit and in particular the relation of the time element to the problem, the more convinced I become that actuarial assistance is needed if an intelligible forecast of a general agent's probable profits is to be made. In the last paragraph of the discussion, Mr. Maclean says, "The trouble with most of the general agents and branch office managers is that they really do not know how much money they are making out of their agency. There are too many 'leaks' in the system and too many places where money is spent without any real account thereof being charged to the profits of the agency." It is not only the "leaks" which obscure the profits. It is frequently the time element. A general agent almost invariably

has to invest money in the early years to obtain an organization. For several years he may have a recurring deficit and it is not until the later years of the contract that he begins to reap a return. I have known general agents who have taken their contracts without fully realizing this fact and in consequence have become much discouraged. An analysis of the returns in later years has been reassuring.

Returning to the subject of district-agency contracts, I may say that I was led to investigate it because of questions asked by our general agents. They wished to establish district agencies in charge of men who would not only produce personal business but who would also gather around them a number of producers to assist in covering the territory of the district agency. We were asked what type of contract we would recommend for the district agent and his sub-agents and also how the expenses of the district agency should be borne. As indicated in the paper and as recognized by Mr. Maclean we found that under a conservative commission contract, such as would meet the requirements of the New York law, the expense allowance from the home office played an important part. If the general agent does not receive a substantial overriding commission there is only a small margin available for the district agent and for the expenses of the district agency unless there is help from the home office.

I thoroughly unite with Mr. Maclean in what he says about "personality." Nevertheless, it seems to me there is as much need for knowing the probable profits when contracting with an able man as when contracting with a mediocre man. I cannot help feeling that we are more likely to achieve success if the factors are thoroughly well known in advance than if they are left to guesswork. Of course if a district agent is to be nothing more than a personal producer the situation is a simple one. However, the purpose of the paper was to investigate the possibilities of developing district agencies involving a reasonable amount of organization. Several agencies of our own company have district agencies of this type.

Mr. Maclean believes that it would be a mistake to hand the paper on "District-Agency Contracts" to a general agent with the thought that he would be able to use it intelligently in making contracts. As a general proposition I thoroughly unite with that belief. The paper was presented to a body of actuaries and it was my

thought that it would be used primarily by the actuaries in conjunction with agency departments in assisting the latter to give intelligent answers to questions from the field. The paper outlined a *method* which could be applied in the light of the circumstances peculiar to the problem under consideration. Mr. Maclean states that I avoided the question of district-agency expenses. In the sense that I did not state categorically who should pay them, I plead guilty. However, the whole intention of the paper was to point out the various margins available, show the importance of the home-office expense allowance and then to suggest that the problem of allocating the district-agency expenses be solved in the light of the given conditions. The problem has no fixed solution. Each case requires individual treatment.

With Mr. Maclean's remarks about uniform production of so much business each year, I also unite. Anyone who works with tables such as those presented in the paper will find that the greatest value is to be derived from the tables relating to a single year's production. Assuming that a district agent has produced half a million of business with given characteristics we can estimate what that business is worth in future profits on the assumption of a continuance of certain termination and expense rates. This figure will give an indication as to whether or not the operations of the year were satisfactory. The tables of periodical production were introduced simply to show what would happen under the assumed conditions. There was also a set of tables relating to a given program of production starting at a given figure and increasing to two and one-half times that amount in five years. Again, the purpose was to illustrate a method rather than to present a set of figures which could be generally applied without change.

It is quite possible that the assumption of an average policy of $3,500 is somewhat high for district-agency business. Twenty-five hundred dollars might have been closer to the average experience. Any actuary in working over the problem will have no difficulty in reproducing figures on the basis of any average policy that may be desired.

Both Mr. Graham and Mr. Maclean refer to the subject of advances to agents. With their conclusions I heartily agree. We shall probably never get away from the system of making advances. However, there are entirely too many general agents who make ad-

vances without having any adequate system of training their new men and thus increasing the probability of their success. The purpose of my remarks was to emphasize the importance of this part of a general agent's organization. It is much better to spend money upon a system which will increase the probability that a new man will make good than to lose the same amount of money making advances to numbers of new men among whom there is a heavy termination rate. In too many instances the easy thing to do is to sit down and draw a check for a $100 advance. The more difficult thing is to help the new man make good. It takes time, patience, intelligence, and, in a large agency, a definite training organization. The agency with an efficient system of training new men is bound in the long run to attract the substantial type of man who is likely to be able to finance himself while he is learning the business.

Mr. Graham comments upon the system so generally used, of intrusting premium collections to general agents. Where agencies are sufficiently large a company can afford to have a separate cashier whose duties are those of premium collection and conservation of old business, thus releasing the general agent's energies for production and organization work. In many moderate-sized agencies an organization of this kind would be too expensive, and probably for that reason more than any other, the functions of agency development and premium collection are centered in the same individual. It is obviously useful to have the two functions located physically close together as they are continually overlapping.

Of course the wise general agent will always employ as a chief clerk or cashier a person who can be trained to handle most of the routine work that premium collections and normal service to old policyholders involve. In a sense, therefore, this achieves the subdivision of labor which Mr. Graham feels is advantageous. It does not, however, touch the question of the collection fee on old business. I am inclined to think that the best way to handle that question is to limit the collection fee to what is a reasonable percentage necessary to take care of old business. Many companies find that the average cost of collection does not exceed $1\frac{1}{2}$ per cent of the premiums. The company can limit, to a relatively small amount, the unused margin from the funds intended for collection expenses, by limiting the collection fee to this percentage. On old business from previous régimes this is all the succeeding general agent would

receive. The renewal commission on the business of the incumbent general agent should not be so large that the amount remaining after deducting $1\frac{1}{2}$ per cent would be an excessive compensation for the general agent's organizing work. Of course the manager basis in any event gives the home office a greater control of the purposes for which money is spent. That is one answer to Mr. Graham's question as to an effective way of insuring that funds are used for agency development.

In conclusion I would express to the gentlemen who have participated in the discussion, my appreciation of the points they have contributed.

INFORMAL DISCUSSION

DISABILITY BENEFITS

A. What is a proper basis for computing premium rates for total and permanent disability clause in a life policy?
   1. With a ninety-day provision
   2. Without a ninety-day provision

B. In view of past experience what types or risks usually acceptable for standard life insurance should be refused the disability benefits?

MR. EDWARD W. MARSHALL:

Since the Actuarial Society Committee on Disability Experience made its recent report, many actuaries have been considering the question whether their companies should increase premiums for the disability provision. The company with which I am connected, in common with others, has given this matter careful study in the past few weeks, and our conclusions to date may be of interest.

Mr. J. D. Craig, in *Transactions of the Actuarial Society of America* for May, 1926, presented premiums for the total and permanent disability provision without the ninety-day clause. As the disability provision of our company, however, contains a ninety-day clause, a complete investigation of the whole subject had to be made.

Much care must be used in preparing premiums for the ninety-day disability provision and in the handling of the disability claims resulting. It may be deduced from the report of the Actuarial Society Committee that the probability of a policyholder aged 25 becoming entitled to disability benefits before age 60 under the so-called "Class 3" with a ninety-day clause is about 31 per cent, as compared with a probability of death before age 60 of only 22 per cent according to Provident Mutual Standard Ultimate experience. From this and other experiences it appears that the chance of disability before age 60 under the ninety-day clause is at least as great as the chance of death before that age. Thus the ninety-day disability provision is no minor matter and premiums and claim administration must be given most careful consideration at all times.

The report of the Actuarial Society Committee indicated clearly

that a company's disability experience depends on a number of factors, several of which are purely managerial in their nature. Of particular importance are the initial selection of risks and the proper administration of disability claims. In order for a sound interpretation to be made of the Actuarial Society Committee report, it is almost vital to have a knowledge of the underwriting and claim-handling methods which produced that experience. As these are not available, the best that an actuary of a given company can do is to compare the experience of his own company with that of the Joint Disability Experience and to consider concurrently the probable relative effectiveness of the method of selection of risks and claim-handling employed in his company.

Accordingly, there is presented below an analysis of: (*a*) the experience of the Provident Mutual under its ninety-day disability provision covering both the rate at which disability occurred and the termination rate among disabled policyholders after the claim was approved; and (*b*) the method of selecting applicants for the disability provision, claim-handling, and other factors in the Provident Mutual which may have had an effect on the disability claim experience.

With these two phases of the matter before us, we are in a better position to deduce intelligently what would be a proper basis of premium for the ninety-day disability provision as administered by the Provident Mutual. Whether or not these premiums would be at all suitable for other companies would be a matter for each company's actuary to decide after considering his company's methods of selecting risks and handling disability claims.

Rate of disability.—Table I shows the Provident Mutual disability experience by policy years and all ages at issue combined, according to both policies and amounts of insurance.

Several interesting facts are revealed by this table. Although the experience is comparatively small the figures seem to confirm each other. The Provident Mutual experience by amounts seems less favorable than by policies for each policy year, which is not in accord with the Joint Experience where there was "no significant deviation" between the experience by policies and amounts. From published commercial health and accident insurance experience we would expect the experience by amounts to be decidedly worse than by policies.

The Provident Mutual actual claims were compared with those expected according to three different tables. The first two tables are based on the Joint Disability Experience for Class 3 contained in the Actuarial Society Committee report. The Joint Experience "Aggregate" table presented in that report is based on the experience of the second, third, and fourth policy years combined, and represents what might be considered an ultimate table. In order to ob-

TABLE I

PERCENTAGE OF ACTUAL TO EXPECTED DISABILITY CLAIMS 1922–25

BASIS OF EXPECTED CLAIMS	1ST POLICY YR.		2d POLICY YR.		3d POLICY YR.	
	Policies	Amounts	Policies	Amounts	Policies	Amounts
Joint Exp. 1926 Aggregate.	47%	51%	75%	97%	150%	182%
Joint Exp. 1926 Select.....	73	80	78	101	138	167
M. U. (Cammack).....	21	22	32	40	61	72
Actual Claims Involved..	56	$166,900	56	$200,900	39	$133,900

tain the corresponding "Select" table, to make a better comparison by policy years, the Aggregate disability rate for all ages combined was adjusted approximately to a select basis by applying the percentages which the ungraduated experience for the first, second, and third policy years respectively bore to the rate for the Aggregate experience, namely 64 per cent for the first policy year, 96 per cent for the second policy year, and 109 per cent for the third policy year. The third table is the Manchester Unity A. H. J. 1893–97 modified by Mr. Cammack (*T.C.A.S.*, Vol. VII).

It will be seen from the table that the Provident disability claim experience was more favorable in the first policy year, about the same in the second policy year, and considerably less favorable in the third policy year, than this Joint Select Experience. For the three policy years combined, the total Provident experience was 86 per cent of the Joint Select Experience by policies and 103 per cent by amounts. The results by age at issue groups were computed but are not shown because the groups were too small to furnish significant results.

One interesting by-product of this table is the comparison of the Manchester Unity Table and the Joint Experience. The rate of disability is decidedly higher in the Manchester Unity experience at all

ages. Apparently there were more temporary claims included in the Manchester Unity data. Perhaps this may indicate that as the American public becomes more familiar with the possibilities of the disability provision, many more temporary total disability claims may be presented. As will be seen later, the Provident Mutual disability claims are of slightly shorter average duration than those in the Joint Experience, which is apparently due to the greater proportion of Provident Mutual temporary total disability claims received.

The foregoing actual claims have been increased by all 1925 incurred claims reported since the end of 1925 or estimated to be reported hereafter. Incidentally, an analysis of the 138 disability claims approved to date indicates that they were made with satisfactory promptness in most cases. Sixty-three per cent were reported within one month after the earliest date on which the insured could have been eligible for benefits; 75 per cent within two months; 84 per cent within three months; 88 per cent within four months, and 93 per cent within five months, the average delay being one and three-quarters months. The reason for this excellent showing is that most of the cases of disability were known to exist during the ninety-day waiting period and at the end of that time claims were promptly made in many instances. Also, many of our agents keep in fairly close contact with their policyholders and claims are unearthed and their presentation facilitated.

Termination rates among disabled lives.—Table II shows the ratio of actual to expected terminations for all ages at disability combined.

This table shows that the rate of termination among disabled Provident Mutual policyholders was considerably greater than in the Joint Experience. It also confirms the point made above that there were relatively more temporary total disability claims presented to the Provident Mutual than were contained in the Joint Experience. It will be noted that the termination rate is much higher in the Manchester Unity than in the Joint Experience.

The next step is to combine the Provident Mutual disability claim experience with the termination experience on disabled lives and compare the resulting figures with those according to the Joint Experience Class 3. In order to do this, however, certain assumptions must be made. The first question is whether the ultimate rate

of disability for the Provident Mutual will be represented by the Joint Experience Aggregate table for Class 3. Assuming the normal law of error, the chance is less than 1 in 1000 that the percentage of actual to expected claims of 150 per cent by policies or 182 per cent by amounts for the third policy year is due only to a chance fluctuation and that the correct figure is 100 per cent. Thus some weight

TABLE II

PROVIDENT MUTUAL DISABLED LIFE EXPERIENCE 1922–25 PERCENTAGE OF ACTUAL TO EXPECTED TERMINATIONS

BASIS OF EXPECTED TERMINATIONS	1ST CLAIM YR.		2D CLAIM YR.		1ST 2 CLAIM YRS.	
	Policies	Amounts	Policies	Amounts	Policies	Amounts
Joint Exp. 1926	110%	126%	151%	118%	113%	124%
M. U. (Cammack)	80	84	.....	.....	.....	.....
Actual Terminations Involved	74	$570,100	8	$18,500	82	$588,600

must be given to the Provident third-year experience. Also, the disability experience may become more unfavorable in the future. The trend in disability costs has been decidedly upward in the past, as shown by the various experiences published. Consequently, it is tentatively assumed that the ultimate rate of disability will be 150 per cent of the Joint Experience Class 3 figures at all ages.

TABLE III

CLAIM VALUES

Age at Disability	Joint Exp. Class 3	Provident Mutual	M. U.
20	$1.98	$1.50	$ .46
30	2.86	2.17	.63
40	3.44	2.63	.74
50	3.61	2.73	.76
59	3.35	2.55	1.13

The Provident Mutual disabled life experience by policies has been assumed to hold for the first two years after disability and thereafter the Joint Experience Class 3 results were used. The graduated claim values of a monthly annuity on the $3\frac{1}{2}$ per cent basis according to this assumption are shown in Table III.

The net one year term single premiums for $10.00 monthly income resulting from these assumptions are shown in Table IV on a $3\frac{1}{2}$ per cent basis.

TABLE IV

NET SINGLE PREMIUM ONE YEAR TERM

Attained Age	Joint Experience	Provident Mutual	M. U.
20.....	$ .85	$ .96	$ .26
30.....	1.14	1.30	.60
40.....	1.60	1.84	1.21
50.....	2.70	3.06	2.19
59.....	4.65	5.31	5.31

These figures provide for benefits under the ninety-day disability provision with income commencing immediately if disability is total and permanent, or at the end of ninety days if total but not obviously permanent.

The assumed Provident Mutual single premiums are about 14 per cent greater than those of the Joint Experience at all ages. This practically constant relation arose from the method used to graduate the disabled life annuities. Lack of time has prevented the preparation of commutation columns and the exact calculation of the corresponding net annual premiums for the disability provision on Ordinary Life policies. In the meantime it is apparent that these net annual premiums would be roughly 15 per cent greater than those according to the Joint Experience.

The loading for disability premiums should take into account the cost of handling claims, in addition to the usual expense items covered. The cost of claim handling is high until there are enough claims to reduce the overhead expense involved. Even then the item is important particularly under the ninety-day provision. A margin to cover part of the cost of claim handling is sometimes derived by basing premiums on the assumption that *gross* premiums are waived during disability. As no premium taxes are payable out of these waived premiums, and no agents' commissions in many companies, there is a saving which may be applied toward the cost of handling disability claims. If this margin is not available, possibly an extra loading of at least 5 per cent to 10 per cent should be included for the purpose.

It is evident that gross premiums determined on the foregoing basis would be considerably in excess of the current rates of most companies with a ninety-day clause. If those companies are having a disability experience at all similar to that of the Provident Mutual (or in fact, of Class 3 of the Joint Experience) the question of increase in premium is likely to become one of immediate interest to them.

Selection of risks.—The Provident Mutual is very careful in its selection of risks for the disability provision. This provision is granted only to risks acceptable for standard insurance, but not to the following types of risks: persons under age thirty irrespective of weight if one parent or two brothers or sisters have suffered from or died of tuberculosis; persons aged thirty or over who are under standard weight with such a family history of tuberculosis; persons without a family history of tuberculosis who are under age thirty and 15 per cent or more under weight, or who are aged thirty or over and at least 20 per cent under weight; persons more than 20 per cent over weight; persons who have had a nervous breakdown or a tendency toward mental disturbances as indicated by personal history of two or more cases of nervous or mental trouble in the family history; persons who are paralyzed or who have suffered from paralysis or who are maimed, crippled, or deformed, or who are compelled to use a crutch or cane; persons who are deaf, or blind, or who suffer from disease of the eye or ear, or from pronounced impairment of eyesight or hearing; persons in an unfavorable occupation; persons with an unfavorable past record of sickness or accident; persons intending to reside outside of Continental United States.

Careful consideration has also been given to the relation of the outstanding disability income coverage of the insured to his personal earnings apart from income on investments. The possibility of over-insurance of disability provision constitutes a real hazard. Our company endeavors to minimize this risk by asking in the application what disability income protection the applicant has in life insurance or accident and health insurance companies. Also, the agent is required to state definitely his estimate of the insured's financial worth and his annual earnings derived from his own efforts. This is compared with the corresponding figures in the inspection report. The limit of disability income insurance issued is $250 per month for men, with the further provision that the maximum yearly disability

income from all sources including the insurance applied for shall not exceed a fixed percentage of the insured's personal yearly earnings excluding income from investments. This limit is fixed at 80 per cent for yearly earnjngs of $1,500 or under and is graded down to 50 per cent for yearly earnings of $10,000 or over. Where the total disability income in all companies does not exceed the percentage stated, but is unduly large in absolute amount, the company scrutinizes the application with great care. Single women and widows are granted not more than $100 per month income under a modified disability provision which provides benefits of about two-thirds of the value granted to men for the same premium, thus, in effect, charging women 50 per cent more than standard rates. For women, the limit of disability income from all sources including insurance applied for is 50 per cent of personal yearly earnings of $1,200 or less, graded down to $33\frac{1}{3}$ per cent of personal yearly earnings of $3,600.

Incidentally, we have noticed that a number of other companies are not observing these precautions at this time, and already in our claim experience we have had cases where the insured subsequently obtained from other companies disability income protection which, in addition to our own, actually furnished disability income in excess of the insured's total earnings at time of disability. The temptation to malinger in such cases is decidedly great and the incentive to return to work is so much reduced that claims are likely to be lengthened appreciably. It is to be hoped that all companies will take steps to protect themselves and other companies against these dangers of over-insurance of the disability income provision.

The reason for modifying the disability provision for women and establishing lower limits in their case was based on the assumption that the disability claim rate among women would be distinctly higher than among men. This assumption has recently been confirmed by the actual experience of a number of companies, which indicated that the disability rate on women has been from 150 per cent to 200 per cent of the claim rate for men. One such experience is that of the Mutual Life of New York presented by Mr. J. S. Thompson in *T.A.S.A.* for May, 1926.

Summarizing, it appears that the Provident Mutual selection of risks for the disability provision is exceedingly careful, and therefore the resulting rates of disability are as low as could be expected so far as they are effected by the initial selection of risks.

Procedure when disability claim is received.—When notice of a possible disability claim is received, the agent is requested to forward to the home office all correspondence regarding the claim, together with a written report on a special form, setting forth all information regarding the insured's disability which he can secure from the employer and other independent sources. On the strength of this report, a tentative decision is made whether total disability probably exists and if so whether it is likely to be permanent. If seemingly total and permanent, claim blanks are sent at once to the insured. If seemingly not permanent, the insured is instructed to notify the company after disability has continued for ninety days at which time blanks will be sent to him. These claim blanks consist of a statement to be executed by the insured and another by his attending physician. A special inspection report is also obtained in nearly all cases. Where facts seem doubtful or there is a large disability benefit involved a report by a company medical examiner is secured although this latter form is required in only a small percentage of the cases. Certain questions on most of these blanks have been found particularly helpful in developing the required information. The first is as follows: "If the insured is unable to perform his regular duties, could he engage in lighter work of some sort, such as light clerical or shop work, light housework, light outdoor work, 'chores' etc.? State what." This brings out the difference between total and partial disability. There are three separate definitions of disability (A, B, and C) in the Provident Mutual contract, under any one of which the insured may qualify for disability benefits. The following question is designed to cause the insured, doctor, or agent to read carefully the definitions of disability so that he can give a more reliable opinion of the insured's claim: "(*a*) In your opinion does the insured's present condition clearly come within the accompanying definitions of total and permanent disability? (*b*) If so, state under which definition, A, B, or C."

The insured is asked on the initial blank, "State all other insurance carried providing for disability benefits, including life insurance, fraternal insurance, accident and health insurance, pension fund, sickness benefits, etc. State company or organization and amount of benefit." On the follow-up blank we ask, "What disability benefits are you receiving from any other source?" The value of these two questions is to discover what other disability

benefits the insured is receiving so that the incentive to malingering may be evaluated. Then, too, the action of other companies in terminating the claim might be gauged from the answer to the second of the two questions.

If the papers are satisfactory and the disability claim committee approves the case, a follow-up date is set when further blanks are required, the date being far enough ahead so that unnecessary follow-ups may be avoided as much as possible, without running a considerable risk that the insured's recovery may occur in the meantime. Carelessness at this point may result in several monthly payments to which the insured is not entitled. In our follow-ups we secure a statement from the insured and his attending physician, also from our agent if available, an inspection report when needed, and at times a report from one of our own examiners.

All initial and follow-up blanks require an estimate of the probable date when the total disability will cease. In this connection the summary of the accuracy of these estimated recovery dates on the cases terminated by recovery shown in Table V may be of interest.

TABLE V

Comparison of Estimated and Actual Recovery Dates

Report	Overestimate (+) Underestimate (−) in Months	Standard Deviation
Initial Statement of the Insured	+1.0	.6
Initial Statement of the Attending Physician	+0.9	3.2
Agent's Report	+1.0	2.4
Inspection Report	+0.2	3.4
Medical Examiner's Report	−0.2	1.8
Insured's Follow-up Statement	+0.3	1.6
Attending Physician's Follow-up Statement	+0.5	1.6

It will be seen that the insured's statements contain the most accurate estimates when the standard deviation is taken into account, although in all the blanks except that of the medical examiner, there is a tendency to overestimate slightly the remaining period of disability. We find that in most cases the insured is optimistic regarding his chances of early recovery and only in the occasional case overestimates greatly the period of his disability.

The inspection report has been the least reliable in this respect because of widely incorrect estimates made.

It may be assumed that the care used by the Provident Mutual in following up the claim has been one factor in the favorable experience after disability. The approval of new claims has been based on a fair, reasonable interpretation of the contract provision, neither unduly strict and technical nor, on the other hand, careless or influenced by expediency. Our feeling is that it is a wise policy to attempt to charge an adequate premium for the disability provision, with sufficient margin to cover a fair and reasonable treatment of disability claims.

MR. A. W. LARSEN:

It has been our boast as actuaries that life insurance is a mathematical achievement, that it is grounded upon a basis of scientific truth such as no other business enterprise can approach. On the other hand, since our shortcomings would not reflect credit upon our achievements, we have, I believe, been inclined to minimize our practical failures. And is not our inability to arrive at a more satisfactory basis for the granting of disability benefits a failure for which we are not altogether blameless? Has not competition rather than truth guided us into the path we have traveled? A few weeks ago, the long sought report of the committee of the Actuarial Society appointed to investigate disability experience was made public. The committee which undertook this difficult task deserves the highest praise for the information they were able to glean from the more or less heterogeneous mass of material presented. The report, while not all that the committee itself desired, should be of no inconsiderable help to us in correcting past errors in the administration of the disability benefits.

The paucity of material is of course one thing for which there was no help, and lack of past experience is, I suppose, our legitimate apology for the loss of surplus due to excessive disability claims. I doubt, however, whether the extension of the disability benefits, when we were reasonably sure that such increased liberality would render premiums inadequate and make the compilation of future statistics doubly difficult, was justified by competition. In a few words I should like to plead for more uniformity if it is possible to secure it, in the underwriting of disability benefits and in the settle-

ment of disability claims. In a few years the investigation just made will be resumed to obtain the additional experience which will then be available and which will in part represent an experience based upon a better knowledge of the subject. If the data to be submitted are reasonably homogeneous, the results obtained from the investigation will be of immense value. On the other hand because of the comparatively small amount of material that will be available for several years to come, a division of the data on account of lack of homogeneity would again present difficulties and impair somewhat the value to be derived from the investigation. Cannot these difficulties be minimized by more active co-operation on the part of all of us? The disability benefits undoubtedly cannot for many years to come be subject to the exact analysis characteristic of life insurance so far as future experience is concerned. The benefit is small in comparison and consequently the need and desire to arrive at accurate results is not so much in evidence. However, a real approach to exactitude should be made, and can only be made, it seems to me, if we can secure a reasonable degree of uniformity in underwriting, in our contracts, and in settlement of claims. The next investigation will then yield results not only which will be a suitable basis for the calculation of premiums but, what is equally important, which will justify us in the preparation of new reserve tables if Hunter's tables now in use do not accord with facts. For this purpose, is it not possible to secure more homogeneous material and still leave to each company sufficient latitude for the exercise of competitive endeavor?

In the meantime we are confronted with the problem of revision of rates. The first step that probably occurs to many of us is to look up the revised rates of other companies having similar clauses, and either adopt these rates *in toto* or modify them slightly to make the results look original. Or we may continue to use Hunter's Net Premiums, loading them sufficiently to bring the rates substantially to those which will be met in competition. Neither of these methods is to my mind altogether satisfactory. The company must be its own judge as to the degree of liberality it exercises, and unless it has an analysis of its own experience to guide it, its rates will be determined largely by that judgement. The most liberal type of clause in use today is, I believe, that providing for the payment of benefits automatically after total disability has been in

existence for ninety days. Under this clause, if selection is only moderately rigid, if the contract is interpreted liberally, and claims are to be retroactive to the occurrence of disability, certainly the net Class 3 rates given in the report of the committee should be substantially loaded, especially when it is remembered that the report covers only an experience of limited duration, and little is known as to the effect which the degenerative diseases of old age will have upon later experience.

If the same clause is to be used but under rigid rules of selection, and with the letter of the contract adhered to, so far as is possible, Class 1 and Class 2 rates will serve as a guide, the amount of loading depending upon the good judgment of the actuary. It must be borne in mind that Class 2 and Class 3 rates are based upon the experience of the 2d, 3d, and 4th policy years, and there seems to be evidence of a tendency for the rate of disability to increase to the 5th and 6th years at least.

Class 1 rates cover the more general type of clause in use where there is no presumption of permanence in the usual sense but where permanence must be reasonably established. But the fact that the tendency of the courts and also the practice of companies is toward liberality of interpretation may require that there be a reasonable loading even upon the Class 1 rates based upon the experience of the 6th year.

The simplest way to arrive at the final rates would probably be to use Hunter's tables or the company's present rates as the basis, being guided in the choice of loading factors as stated by Class 1, 2, and 3 rates of the report.

In the past we may sometimes have felt that the matter of inadequate disability rates was not a serious one, as the loss could easily be absorbed by the life business. Participating companies have probably been less inclined to this view than have non-participating companies. For the benefit of life insurance in general certainly this view should not be entertained. Now that more light has been thrown upon the subject, an effort should be made to place the disability benefits upon a proper footing as to rates.

We are in a period of transition with regard to the disability benefit. Many of us are undoubtedly about to revise our rates in the light of the new information now made available. At the same time, for the purpose of future investigation, is it not possible by more

active co-operation on the part of all of us, to arrive at a more common basis fundamentally in the administration of the benefits without wholly sacrificing the individual desire for minor variations of a competitive nature?

MR. WENDELL M. STRONG:

We have had a recent investigation and several papers in regard to the subject of disability benefits, which give very considerable information about the experience on these benefits. There is one important point known to all of you which nevertheless I wish to speak about and emphasize. It is that disability statistics in their very nature differ from life insurance statistics and this is due to the difference between the event insured against in the two cases.

Life insurance depends upon the death of the insured, a definite thing in which there are no degrees, and in regard to whether the event has occurred or not there is no doubt except in an infinitesimal number of cases.

In regard to disability we have the question of two events instead of one; namely, the beginning of the disability and its termination by death or recovery, and we have with regard to the disability itself the question whether it is total or not, and the question whether it is permanent or not. The question of permanency has been largely eliminated by many companies by providing in the contract that total disability continuing for ninety days is thereafter presumed to be permanent during its continuance. While such a change eliminates one of the very difficult questions, namely, that of determining probable permanency, the one left in regard to totality is an extremely difficult one. The reason is that there is no hard and fast line separating total disability from a state other than total disability, such as there is between life and death. Total disability depends first, on the definition, second, on the legal interpretation of that definition as made by the courts, and third, on the interpretation given by the company itself. This last, with life insurance companies, is usually at least as liberal as that of the courts.

Thus, in comparing disability benefits and life insurance we have selection in each in the beginning but very little similarity thereafter. Life insurance selection has become pretty well standardized for policies at standard rates. There is, of course, some difference between the companies, but there is no radical difference between

them in what they will accept and what they will not, for standard life insurance. Unfortunately one hardly can regard the selection of life insurance companies for disability benefits as being standardized, so that at the beginning of the contract we start out with some difference.

Next we come to the question of what constitutes total disability. The differences here in interpretation may perhaps be thought of in comparison with the difference as to what constitutes intoxication between the strict prohibitionist who regards a slight increase in talkativeness from alcohol as extreme intoxication and the often quoted sailor's definition "Bill isn't drunk; see, he moved his finger"! The difference in definition and in interpretation makes very considerable difference in what the various companies regard as constituting disability that is total. This affects, first, the allowance of disability claims when presented, there being many claims that fall within the "no man's land" between liberal and strict interpretation even though the strict interpretation is no stricter than the courts would uphold. This same question of what constitutes totality of disability comes in again on the question of whether the insured has recovered so as to no longer be entitled to disability benefits. If we add to this the difficulty of the question whether disability is permanent or not which comes in for a great many of the cases where it is unquestionably total, we have another question at least as difficult applying to the statistics of disability, except such as are founded on the ninety-day clause.

All of this has a very great bearing on the question of disability experience and on the meaning of the statistics derived therefrom. It is as if we had a foot measure made of rubber and in each case we must determine how much it was stretched for the particular data concerned. This difficulty met the committee of the Actuarial Society on Disability Experience and is set forth in the report that they made.

Thus it would seem that we cannot properly speak of statistics of total and permanent disability as such as we can speak of mortality statistics. We must also have regard in some degree to the standards of selection and full regard to the standards used for determining totality, both in the definition by the contract and the attitude of the company in the allowance of claims and determination of recoveries.

Thus an experience can be applied directly only where it is a company's own experience and where the company has not charged its contract or its practice in the features concerning totality and permanency, or where it is that of other companies using similar standards. If the experience is that of a different interpretation or otherwise different standard, it cannot be used without allowance (a very difficult thing to make properly) for these differences.

I think we may assume as one result that actuaries will not be at a loss for future problems.

MR. JAMES W. GLOVER:

Dr. Strong has struck a vital point in connection with disability rates. I have been impressed throughout the discussion with much vagueness as to what was meant by the various classes of disability. We speak of "liberal interpretation." Now, what does "liberal interpretation" mean? It certainly varies greatly from one company to another.

Disability rates have no meaning until the disability is quite closely defined, and an important part of the work of the committee should consist in defining very carefully what is meant by a given disability range. This means going into more detail in connection with each set or class of disability tables. If, by way of illustration, tables were prepared based upon a very close interpretation of disability, a more favorable interpretation, a liberal interpretation, and a very liberal interpretation, each of these classes being fully defined, the companies themselves would be in a position to use these tables. Each company knows fully what its own interpretation of disability is and with this knowledge it would be able to select the proper table in the above set or a combination of these tables which would describe their experience.

MR. O. W. PERRIN:

The company with which I am connected adopted the Disability Income Benefit in 1916, and in 1922 incorporated the three months' clause, the schedule of disability rates which was put into effect in each of these years being higher at the time than the disability rates of practically any other company granting similar benefits. With the exception of two years (1916 and 1923) the Penn Mutual has not experienced any loss from Disability Benefits until 1925 in which year it was considerably greater than either of the years

1916 or 1923 even after allowance was made for the additional extra reserves which were put up in 1925. Although over the entire period the gains considerably more than offset the losses, the latter, I believe, reflect the changes in the company's administrative policy, the effect of which is carefully emphasized by the report of the disability committee, also increases in administrative expenses and a better education as to the value of these benefits on the part of the insuring public. We are engaged at present in making an analysis of our experience along the lines of the report of the disability committee which is expected to throw more definite light upon these features of our experience.

The new disability experience, so far as I have been able to digest it, appears to me to indicate that the rates now generally in effect need a larger relative increase at the younger ages than at the older and perhaps this is more likely to be justified in the future in view of the tendency to take at standard rates certain classes of risks under which some companies have already reported an unfavorable experience, or at any rate, an experience which shows considerable uncertainty as to what the future trend will be.

While none of the three classes of the new disability experience can be regarded as providing a completely satisfactory basis for premiums, it may be mentioned that Class 1 gives the broadest basis. If to these rates (Class 1) are added, say, double the Manchester Unity correction for three to six months, the result will be a rate for a three months' clause which will be about as stringent as Class 3 at the younger ages and between Classes 2 and 3 at the older. The necessity for a very substantial loading at all ages to take care of the heavy administrative expenses must also be kept in mind.

MR. JAMES S. ELSTON:

We are investigating this problem more or less continuously. Last September we increased our rates. I have no new experience to give except that we just have taken out our rate of disability and not only is the second-year rate greater than the first and the third greater than the second and so forth, but each year increases up to the seventh which is the oldest we have on a clause anything like the present benefit. That clause was much less liberal than the present disability provision, though our claim practice is practically the same. As to whether we will leave our rates unchanged

or increase them again I cannot say. The investigation has not gone that far.

There is one phase of the problem though that I would like to mention. It seems to me under the usual method of working out the value of the claim and combining it with the probability of disability that we do not get the full benefit of such experience as we do have. I doubt if there is any company that can accurately determine the average value of a disability claim at disability with full allowance for the fact that such claims as have been incurred have been influenced by recent selection of the risk. Neither can companies obtain the probability of disability that will correctly reflect future experience.

Yet companies who are too small or have too immature an experience to determine these factors can in some cases determine the cost per thousand insurance of the first three months of benefit. Some can also determine the cost for the second three months of payment. What is found from their own experience can then be combined with Hunter's rate. Determining the cost of separate portions of the benefit per thousand insurance is practically the method used by Mr. Hezlett as well as in the Manchester Unity and by accident and health companies. Your own experience may be worth considerable for the expensive part of the benefit and you will have to resort to uncertain values only for a limited part of the benefit. Having determined One Year Term net premiums for the component parts you can conbine them and commute them to obtain Ordinary Life and other premiums. It may be of interest to add that in some of our computations it made practically no difference in Ordinary Life premiums whether the One Year Term premiums were commuted on Hunter's active life table, or on our own mortality table which does not distinguish between active and disabled lives, but does allow for modern mortality. In other words, you can substitute something that is known for an appreciable part of the cost of the benefit, using estimates for a part only, and as your experience accumulates you can substitute a larger and larger amount of known costs in place of estimates.

MR. V. R. SMITH:

Mr. Larsen's remarks about co-operation made me think that perhaps you might be interested in what we have recently been doing in Toronto on this subject.

Most of us have too small an experience to be able to determine, to any exact degree, the premiums our companies should charge for the Total Disability Benefit, if we base the charge on our own experience, so the Actuaries' Club of Toronto appointed a committee to consider the subject and to submit their recommendations, both as to rates to be charged and the type of clause to be used.

The clause that we adopted is one that eliminates the use of the word "permanent" entirely. It is frankly a Total Disability clause with a three-months' waiting period; no payment of income disability for the first three months of total disability, and no waiver of any premium that falls due within the first three months of total disability.

We also endeavored to obtain a little more uniformity in the practice of the companies in paying claims and inserted a provision in the contract that, if the insured did not make claim for the benefit at the earliest possible moment, the benefit would not be dated back more than six months. In other words, the claim must be made within the first nine months of continuous total disability, otherwise the policyholder suffers a forfeiture of disability income for the delay over and above the nine months.

With regard to the premiums, we adopted the Hunter's Disability rate, providing for the waiver of the $O^{M(5)}$ 3 per cent net premium, loaded 40 per cent, plus a constant of $1.00 per thousand of insurance. I think the most of us felt that such a premium would be adequate, bearing in mind that the companies who contributed to the experience were placed in the—shall we say—conservative group, rather than the liberal group. However, there were one or two of us who felt that the rate, perhaps, was not adequate in view of the fact that the experience was somewhat short and that we had not reached the ultimate rate of disability, nor that we had reached the ultimate rate of expense in handling claims once the public in general has learned how to present claims and the proper method of malingering.

This clause and schedule of rates has not been adopted by all the companies, but probably will be adopted by practically all, if not all, of them. Several companies are taking steps to put them in force almost immediately.

MR. R. A. HOHAUS:

Mr. Craig presented a paper at the spring meeting of the Actuarial Society which was a practical adaptation of the report on

disability made by the Committee of Actuaries. That paper outlined the basis of a new scale of total and permanent disability premiums which probably will be adopted in the very near future by the Metropolitan.

The company does not use the automatic ninety-day clause and its insuring clause belongs in the group called Class 1 in the report. Moreover, the claim payments do not commence until the insured has been totally and permanently disabled—not just disabled—for at least three months, at which time payments will be retroactive to the date of disability up to a maximum of six months.

The experience of the Metropolitan was studied in connection with the tables of disability probabilities as shown for Class 1 in the report. The rates developed in the report from ultimate experience of the sixth policy year seemed to be a satisfactory basis for the new premiums. Based on these rates and excluding the first policy year, the ratio of the actual to the expected experience by the company was 81 per cent, and these sixth policy year ultimate rates with a modification from ages 42 to 59, were used as the premium basis. For ages 42 to 59 the rates were increased empirically so that at age 59 the rate of disability used was 10 per cent higher than that shown in the report, the reason being to make some provision for the possible contingency that there will be in the future an increase in claims arising from degenerative diseases.

The study of the company's mortality experience on disabled lives indicated that disabled life annuity values given in the report were satisfactory and hence these values were used for the premium calculations.

The interest basis was $3\frac{1}{2}$ per cent and the American Men Ultimate Table was used for the active life annuities. It was not thought necessary to develop a combined table as tests showed that satisfactory results would be produced by using the American Men D_x column and multiplying it by the disability rates and disabled life annuity values, this method being in effect a modification of that used for varying assurances.

Two loadings were used to determine the gross premium. The first was a loading of 5 per cent of the net premium to take care of the expenses incurred in the payment of claims. For a total monthly benefit of $10.00 a month (including the gross premium waived), this produces a charge for an additional annuity of $.50 per month which will be reserved both for active and disabled lives. The second

loading was 25 per cent of the net premium increased by the 5 per cent, making a total loading of $31\frac{1}{4}$ per cent of the net premium. This second loading is added to make provision for expenses, contingencies, etc.

A comparison at certain ages of the present premiums with the proposed new premiums for whole life, twenty-payment life, and twenty-year endowment shows the effect of the new basis and it is interesting to note that the percentage decreases as the age increases.

For whole life policies the new premiums will be 150 per cent of the present one at age twenty; 134 per cent at age thirty; 119 per cent at age forty; 105 per cent at age fifty; and 99 per cent at age fifty-five.

For twenty-payment life policies the comparison is quite similar. It is 156 per cent at age twenty; 138 per cent at age thirty; 120 per cent at age forty; 105 per cent at age fifty; and 99 per cent at age fifty-five.

For twenty-year endowment, the comparison differs for the younger ages and is 191 per cent at age twenty; 158 per cent at age thirty; 120 per cent at age forty; 106 per cent at age fifty; and 99 per cent at age fifty-five.

For the better class of sub-standard business, a type of policy and rates known as "Intermediate" is used by the company and this is the only sub-standard business for which disability coverage is granted. The net premiums for this business will be 150 per cent of the regular Ordinary disability premiums.

MR. W. A. P. WOOD:

I do not wish to add anything except just by way of illustration of what Mr. Smith has said. He has referred to the basis of the new premium rates. Might I say that for age twenty on the Ordinary Life plan I think the premium comes to $2.30. That is just an illustration so that you will be able to carry it away, $2.30 for Ordinary Life at age twenty. Of course, it is graded up for higher ages.

MR. J. G. PARKER:

The first few words in Section B are rather discouraging and it is only by eliminating them that I would say anything about Section B. These words are "In view of past experience." I do not believe many of us have any past experience on sub-standard disability risks.

The Canadian group has endeavored to put forward some rulings for the taking of disability risks at increased premiums. Take for instance, the simplest impairments; say, blindness in one eye. I think we have endeavored to cover such an impairment as that by two and a half times or two times the regular rate. Take another impairment such as the amputation of the leg below the knee, where a good artificial limb is worn. We have many such cases coming before us in Canada. We have endeavored to cover this impairment by an increased rate varying from two to two and a half times the regular rate depending on the degree of impairment.

Again, we have endeavored to cover amputation of the arm, as long as it is not too near the shoulder joint, at the rate of approximately twice the regular rate.

A family history of insanity, where there is one case of insanity in the family history, we endeavor to cover by a rating of two and a half times the regular rate.

In our endeavor to cover certain impairments we drew up three tables. One, which was possibly the most severe table, was meant to handle such impairments as personal history of pleurisy. The premium for the disability benefit varied with the numerical rating assigned to the impairment. For instance, a rating of 110 would call for a disability rate of one and one-half times the normal. For a risk that was rated 120, that is, 20 points more than normal for personal history of pleurisy, we gave the maximum disability rate which we used, namely, two and one-half times the normal.

The next table was a little more lenient and applied to "family history of tuberculosis," also to light weights without any family history of tuberculosis. This started with one and one-half times the normal rate for a risk which was rated 110, going up to two and one-half times for a risk rated 125.

The third table was very lenient and ran up to a rating around 160 per cent. A risk which showed 150 to 160 per cent would be charged the maximum rate of two and one-half times the regular disability rate. I cannot think of many groups to which this last table was applied except one which was a history of albumen and we take some of these risks at two to two and one-half times the regular rate, depending upon the mortality rate at which we assessed the risk.

Mr. James Fairlie: Mr. Parker referred to the special basis of premiums adopted by Canadian companies for disability benefits

where a case of single dismemberment already exists. I would like to ask if the disability provision used corresponds with that in ordinary use in the United States, which provides for allowance of the benefit in event of double dismemberment.

MR. PARKER: Yes.

MR. G. W. THOMSON:

I think an important test of this disability question, if we are going to give a ninety-day total disability clause, is to have a common-sense practical clause.

My company has the ninety-day clause and frequently we have found that the policyholders have delayed the sending in of their proofs. With our clause at the present time the payments commence from date of receipt of proof. That might be three months after the ninety days had expired and if we strictly interpreted our clause we would pay, say, six months after the commencement of disability.

On the other hand, we have found, as a practical proposition, that some companies go back to the end of the ninety days with a limitation of six months dating back. And we have had policyholders who have had a policy with us and a policy with another company and we have found that the other company will go back and pay that claim, commencing from the end of the ninety days. If we strictly interpret our clause we would not do so.

It seems to me that the public are entitled to some sort of a common-sense clause that they can readily understand. Our claim department finds that if we do not go back to the end of the ninety days and commence our payments from the end of the ninety days there is usually a good deal of friction on the part of the claimant. They say they were delayed in sending in the proof, that they were ill and could not attend to it, and they cannot understand a policy of that kind. They seem to think it unfair. Our claim department tells me that they have found a good deal of misunderstanding when we desire to adhere strictly to the terms of our present clause.

I believe that in granting permanent and total disability, we should have a clause that the public can well understand—a practical clause, that will eliminate friction, and charge the proper premium for it, if we are going to grant it at all.

I feel, with regard to the report of the disability committee, that there is sufficient evidence there to show that a very substantial premium is necessary, at least in the interests of precaution at the present time, until the experience is larger and we can draw more reliable conclusions from it. Our field men prefer a practical clause and they tell me that a substantial premium will not diminish its popularity. My company has been considering this and the companies in the West have been considering it to the best of our ability. As has been said here we have been largely guessing. We do not know. I do not think any of us know what the accurate premiums are yet. Under those circumstances I believe it behooves all of us as conservative actuaries to compute a substantial premium.

I think the companies in the West, a number of them, have expressed themselves very strongly to the effect that they are going to decidedly increase their rates. At the same time, I feel that we should draw the clause as so to give it a common-sense practical meaning. I like a clause going back six months. I think that is a very reasonable benefit. The public will be able to understand a clause of that kind and ought to feel that they are being treated perfectly fairly.

Take a case of insanity. If a man is insane and he has no relatives he cannot file his claim. How can a provision requiring proof to be filed by him be complied with, where he is unable to file a claim owing to insanity? In that case I would say that an exception should be made, by dating back the benefit a reasonable time. If an exception is made in that case and it is not in the clause and a situation is created that forms a precedent, I think it should be applied in every similar case of insanity.

I strongly feel that we should have a good substantial premium and the companies should co-operate with one another in establishing a premium that is at least sufficient and give a reasonable common-sense benefit.

MR. F. S. WITHINGTON:

I would like to ask for information regarding the experience and practice among the companies here represented, in the granting of disability benefits to women. The company I represent will not grant disability benefits to married women. It does grant disability

benefits in cases of single women and widows who are self-supporting. I would like to hear some expression as to the practice and experience in that particular.

MR. G. B. PATTISON:

In response to a request for information with reference to disability experience on the lives of married women, I may say that the company with which I am connected, ever since we have been issuing the income disability clause, has granted this benefit almost as freely to married women as to men.

The reason we started doing this in the first place was because of the location and the nature of our business, our business being all rural business, located in the Central West. Our agents solicited largely farmers and they were instructed to solicit the farmer and his wife together and write them both. When we added the income disability clause, we made it part of our policy and urged our agents to write this benefit as much as possible. Such being the case, the agent who went into a farmer's house and sold the famer a life insurance policy with income disability benefits was under a considerable handicap, if he could not sell these same benefits to the farmer's wife, so we gave our agents the privilege of writing the income disability benefits on both the farmer and his wife. We were perhaps a little more careful in our underwriting on married women then we were on men, but we made very little distinction. We tried to avoid issuing policies for too large an amount of income disability on married women and we tried to avoid issuing this benefit to married women in poor districts. By poor districts I mean, where the general community was not up to the average standard of the Middle West.

We have had a very favorable experience with the income disability benefit on married women and by favorable I mean that our experience with this benefit on women has been better than with the same benefit on men. I am compelled to confine myself to a general statement because I have no figures at my disposal as to our company's experience, except only an impression I have received from observing our disability claims.

I believe this favorable experience has been occasioned by the fact that our agents have actively solicited married women for life insurance and in this way have offset, to a large extent, any possible selection against the company.

It has been my observation that women as a whole are much more loath to make claim for total and permanent disability benefits than men are. In a great many cases the women are totally disabled for a year or more before they will present a claim to the company.

On the other hand, quite a fair sprinkling of the disability claims on women are what is known as "neurotics" and these are hard cases to handle because, while the claimant has a great many symptoms pertaining to every known disease, yet the examining physician is unable to find anything whatever the matter with her.

A word with reference to the total volume of this business: I would estimate that we have about fifteen million of income disability on female lives on our books at the present time and we have been writing this business since January 1, 1920. Of course, this is not a very large experience, but it does seem to me to be large enough to give us an inkling as to what we can expect on this particular character of business in the future. I, myself, feel confident, if we always confine our underwriting to the same type of women, we will always have a more favorable experience on women than we will on men.

EXPENSE BUDGETS

What plans have been used in budgeting the expenses of a life insurance company and what are the advantages liely to accrue in using such plans?

MR. A. A. RYDGREN:

(Written discussion presented by Mr. E. G. Fassel)

The company with which I am connected, the Continental Life Insurance Company, Wilmington, Delaware, has been using the budget system for more than five years.

We budget first-year expenses by requiring that the sum of the first-year mean reserves (on the New Jersey modified preliminary term basis), plus first-year commissions to agents, supervisors, and managers, plus increase in debit balances, plus branch office expenses in excess of collection fees, if any, plus the cost of any other direct first-year expense such as the cost of conventions, prizes, traveling expenses, etc., must not exceed the total first-year premiums. We make an account each month so that our agency manager knows just where he stands month by month.

Likewise, our home-office expenses are budgeted. At the beginning of the year we make an estimate of what we think home-office costs should be for the current year and it is the home-office manager's job to see that the actual expenditures do not exceed it. We divide the home-office budget up into different items, so much for salaries, so much for printing and stationery, and various other items, and finally so much for miscellaneous expenses, and also a certain amount for extraordinary expenses. The manager must keep within the budget and has always succeeded in doing so. I do not mean that we keep within the budget for each particular item, but that our total actual expenses, exclusive of extraordinary expenses, will not exceed the total budget, exclusive of the amount provided for extraordinary expenses. The only items which we put into the item of extraordinary expenses are, for instance, if we revise our rate book in a certain year, or if we get out a complete new set of policy forms, those expenses are considered extraordinary; those are jobs which do not arise each year and are not contemplated in the budget, except in so far as an amount is set aside for extraordinary expenses; consequently, in some years we spend nothing for extraordinary expenses but, on the other hand, we may some year spend almost twice the amount provided by the budget for extraordinary expenses.

Our budget system is not perfect. Its chief defect is in fixing at the beginning of the year what the budget for the current year should be. For some years we had a set formula reducing the cost per policy by a fixed amount each year. That particular system was not flexible enough and finally it broke down when we prepared our budget for 1925, and again in 1926. We contemplate formulating a more flexible formula that would fix the total amount of the budget for the current year, having in mind that the formula should not contemplate any diminution in the expense rate in the event the number of policies in force increases less than by a certain percentage (which I vaguely have in mind as being about 5 to 6 per cent) but that the increase in the number of policies in force above such minimum should be accompanied by a very small increase in the total cost; so that if the total number of policies in force should increase by, say, 15 per cent each year then we should be able to make a substantial reduction in our expense rate.

I do think that merely having a budget does result in the re-

sponsible heads of the office being more economical. We always, when contemplating a change of any kind whatsoever, take into consideration the cost involved. We might not do that to the same extent if we were not limited in our espenses by the budget. There is one further advantage of the budget system, as in the case of this particular company, the home-office manager is limited in the total expenses to the amount provided by the budget; therefore, neither the president nor the board of directors is so much concerned over what the money is spent for so long as the total does not exceed the budget; the responsibility being thus concentrated, does I think, save time that might be otherwise spent in conferences and explanations over matters they could not be in intimate touch with. Conversely it has the disadvantage of sometimes postponing, until the following year, the purchase of equipment that would make for efficiency, because there is not enough margin in the budget to absorb the expense; but we have never been seriously inconvenienced thereby.

I realize that we have just made a beginning in the matter of budgeting our expenses and that we shall likely expand our system and refine it as the company grows in size and we become more experienced with it. At any rate, we like our budget system sufficiently that we would under no circumstances do without it.

MR. S. E. ALLISON:

We have had a budget system in our company since the organization of the company. It has been subject to change. I think the subject came up for discussion because one of the members of the Institute was in our office and saw the system we use. We do not claim that it is perfect but we do claim, however, that it gives us control over our agency expenses mainly and serves as a warning to us of the other expenses in which extraordinary leaks could occur.

In regard to our agency expenditures we limit our agency department, or try to, rather, to a certain percentage of the first year's premium. That includes first-year commissions, other compensation of agents, any agency salaries, all home-office salaries that are directly chargeable to agency expenses, 25 per cent of printing, stationery, and advertising, all general advertising, all agency advertising, 25 per cent of postage, telegraph, and express.

Now, not always does our agency department keep within that

certain percentage, but if they do not they will have some very good explanations to make.

We found that we formerly had them up to 10 per cent higher, reduced by 5 per cent and then reduced it another 5 per cent. We hope to gradually reduce that percentage.

It has the effect of making each and every one of us, in the preparation of our budget in the beginning of the year, study just exactly what has happened in the past to enable us to try and profit by the mistakes which we have made in the past, and every year we find that we have made mistakes and that we can correct different items. One year we found an item of home-office expenditure that was proving rather a heavy one and which had been forcibly brought to our attention by this budget.

In the first place, we divide our budget by 12. It is not absolutely accurate in dividing it by 12 and keeping it monthly. It is in certain items. In certain other items we know that the one-twelfth division does not work at all. But if we find during the year that we are departing from that budget to any extent then we want to know why we are doing so.

We have found that it gives us better control of the situation. It entails a more thorough study of all conditions in the company and especially by those whom it is rather hard and difficult to make study them. That is the advantage. We find that we get better control of our company and are improving conditions by the use of the budgeting system.

We divide our budget into the various items and accounts according to the annual statement we file. I know we would not for a minute do away with the budget system in our company.

MR. RICHARD BOISSARD:

Expense accounting and expense budgets have probably been more important office functions in manufacturing concerns, especially those in a highly competitive industry, than in financial institutions, under which class life insurance companies would naturally fall, which have been rather slow in development along this line. It may be further due to the fact that of the income dollar of an average life insurance company, as stated by Walter L. Crocker, President of the John Hancock Mutual Life Insurance Company, 81 cents goes to payments to policyholders and beneficiaries and for legal and special reserves while only 19 cents for

expenses and taxes. Of this 19 cents, 2 cents represents taxes, 10 cents compensations to agents and representatives in the field, leaving only 7 cents or 7 per cent of the income going for controllable expenditures. But if the large percentage of the life insurance company's income dollar is absorbed in reserves and disbursed to policyholders, the fact still remains that the controllable disbursements, especially in a small company where they are a larger percentage, are important.

The company I represent has been operating under a budget plan for over five years. Such a plan seems especially valuable to a young and small company. If a rather detailed study be made of younger companies it is evident that a close relation exists between the character of growth and the type of management. It is also evident from such study that the type of management of young companies was indicated by the expenditures made under the controllable items. A small, young company with a small capital can, with safety, spend but a certain amount for development and new business. It likewise should limit its agency expenses as well as other general expenses.

The budget used by our company is a complete monthly statement on which the actual income and disbursement figures are used and actual figures for all items such as reserves, mortality, dividends, and accrual items are based on the end of the year figures properly prorated. The budget is usually prepared in the early part of January and submitted to the board of directors on the third Tuesday in January and when approved by them is used generally as a controlling account for such expenditures as may be controlled. The budget is first laid out on an annual basis and after approval is subsequently apportioned for each month on the basis of the monthly experience of the previous year. In laying out the budget, various schemes are used for various items. The estimate for the renewal premium income item, for example, is based on the previous year's renewal income plus first-year premium income, allowance being made for not only actual terminations that have taken place in the year just ended but for expected terminations during the coming year. Mortality is figured at 45 per cent of the expected, leaving a margin of 10 per cent for safety. Some items are so closely related that they are estimated in a group, such as increase in policy reserves and reserve payments to policyholders.

After the first draft has been made, the figures are used in making out a gain-and-loss exhibit modeled on the present annual statement report. From this gain-and-loss exhibit, actual to expected mortality, per cent earned on mean ledger assets, and similar factors are established, and if these are found comparable to the figures actually assumed in laying out the budget it is evident that no gross errors have been made nor have any important items been omitted. We have through experience found it advisable to make minor changes in our budget from time to time, and have this year a separate detailed budget for the agency department, the aggregate of which is used in the budget proper.

Aside from the advantages that every budget properly used will give, it is possible to set up a goal for the year and by following the progress each month be reasonably sure of attaining this goal. By knowing the company's true financial condition at the end of each month instead of yearly it is possible to consider changes in dividend scale, changes in retention, and similar matters at any time as we have pretty definite knowledge as to how they will affect the company's statement on December 31 following.

I might add this detailed budget plan which is readily applicable to a small company would probably not be so valuable to a large company. We keep all of our reserves, accruals, and accounts of that nature on Hollerith cards and have not found it either very difficult, on requiring a great amount of work, to draw off these figures monthly. We use accrued interest figures as of the end of the year and we actually make some error on that account, but we found when the insurance department made a mid-year examination that our uncollected, due, and deferred premiums, and items of that character, were but slightly different from the budget figures. This is easily explained because the amount of business written in various portions of the year and subsequently due in future years in those months, varies in the different parts of the year, but the variation is not as great as you might think.

POLICY CHANGES

What is a practical rule for changing a policy from a higher to a lower priced plan before the policy has acquired a cash surrender value?

What adjustments, if any, should be made in the Agency Commission and on account of taxes and other expenses which have been incurred?

MR. J. P. BOWERMAN:

Mr. William Young in Volume XI and pages 480-82 of *T.A.S.A.* has given a practical method of dealing with this subject.

Mr. John S. Thompson in Volume XXIII and pages 209 and 210 of *T.A.A.S.* has also discussed the same subject.

MR. J. G. PARKER:

In our company, where a policy has one premium paid, we find the net amount which we would receive, taking into account the agent's commission on both plans, and make an allowance in the first place of the difference between those two amounts. Then we charge to the policyholder 15 per cent of the difference in gross premiums. We call it an expense charge or a charge for making the change. This is actually (in some of our branch contracts) the amount of the branch manager's salary based on that particular premium. This gives us an allowance of some kind in regard to making this change.

We actually make no adjustment in that way in the Agency Commission. We do not feel that we could force any adjustment if we wished. It is true that many agents come and say, "Now, if you make this change I am quite willing to accept a smaller commission, the smaller commission which would be coming to me if you put it through," but we put no changes through in that way.

In regard to the taxes, that automatically adjusts itself with us, in that we put through our books an adjusting entry in first-year premiums, or renewal premiums if there is a renewal premium involved. In regard to the other expenses which have been incurred we make no attempt to make any adjustment.

MR. A. C. WASHBURNE:

For a number of years, the company which I represent has been changing from one form of policy to another simply on the difference in reserve plan. Irrespective of the cash values, if exchange is from a lower to a higher premium form, the difference in reserve is required; if from a higher to a lower premium form, the difference is credited, either given in cash or credited to future premiums. The status of the changed policy, from date of exchange—dividends, commissions, cash values, etc.—is the same as if it has been issued originally on that form. No adjustment of prior commissions or dividends is made.

We have found this plan very satisfactory to all concerned—to the company, to the policyholder, and to the agent.

MR. E. O. DUNLAP:

The method used by the company with which I am connected is a simple one. From the premium received on the higher-priced plan we deduct the medical fee, premium at the net one-year-term rate for the time the policy has been in force, and 5 per cent of the premium received. In the same manner we take the premium for the plan to which the change is to be made and deduct the medical fee, or an adjustment where necessary, the net one-year-term premium, and 5 per cent of the premium on the lower-priced policy. The credit on the new policy is then deducted from the credit to be allowed on the higher priced policy, and the difference in commission on the two plans is deducted from the net credit. The credit allowed may be applied only in the payment of premium on the new policy. In practice we have found this method generally satisfactory.

After a policy has acquired a cash value the change is made on the basis of the difference in reserves, and if the change is from a higher- to a lower-priced policy, the difference is paid in cash.

MR. E. G. FASSEL:

The company with which I am connected changes policies from higher to lower priced plans by a very simple rule. If the new policy is of the same amount, the allowance is the difference in the gross premiums of the current policy year according to the portion of the premium that was paid, plus the difference in the terminal reserve at the beginning of the policy year. In the first year this rule reduces to the difference in gross premiums. The dividend for the current year will be on the new basis but there is no adjustment of previous dividends. The agent's commission is adjusted.

There is no automatic adjustment of the premium tax along the lines that Mr. Parker referred to because our allowance, although calculated in part as a difference in gross premiums, goes through the accounts as surrender value.

If the new policy is of reduced amount, the excess is treated as if the amount of reduction had been originally issued in a separate policy and the allowance on the change is the surrender value, if any, of such separate policy, plus the foregoing allowance in respect of the amount continued in force.

Policies acquire a surrender value on payment of two full years' premiums in our contracts. The presence or absence of surrender values has no bearing on the change calculation but before making an allowance in cash the company applies whatever portion may be necessary to bring the premium payments up to two full years.

MR. PARKER: I would like to ask Mr. Fassel what he would do if the agent had left the company.

MR. FASSEL: The commission refund is charged to the general agent who is held responsible for it without regard to his arrangements with his sub-agents.

MR. W. H. McBRIDE:

Our practice in reference to change is based on the reserve difference, with a limitation that if the premium has been paid for less than two full years the change is made on premium difference. We make no adjustment of expenses which would involve a refund from the agent.

I think it may be worth while to consider the reasons for time limitation. As far as I know the state insurance departments are pretty generally willing to let the companies arrange these matters to suit themselves, but one state I have in mind (Virginia) is a little particular about that thing and is not willing to approve policy forms without a limitation on the time in which changes can be made on reserve difference. Where the policy is on a full or modified preliminary term basis the reasons are quite obvious.

MR. E A. PORTER:

We should bear in mind in this discussion that after all there is no contractual obligation on the part of the company to make these changes. Such changes are made as a matter of service to policyholders genuinely wishing to adjust life insurance carried in a way to better suit insurance needs. They should not, however, be made in order to make available surrender values to policyholders otherwise not entitled to such.

I believe that one of the first requirement, and a requirement which our company has always made in such cases, should be that any credit balance due the insured must be applied in payment or prepayment on premiums under the new policy and that on all such changes some arrangement be made for a cash payment of some sort as an evidence of good faith on the part of the policyholder.

Within the first year, we have treated such changes quite differently than after renewal. During the first year we have regarded the matter as being largely one to be handled by the agent and changes which are made are generally made at the suggestion of the agent and the commission is adjusted with the agent. Under such circumstances the calculation is very simple, being simply the difference in the premiums on the two contracts.

After the first year, however, keeping in mind the limitations stated, it has been our custom to allow the difference in reserves. So far as I recall now there are no special circumstances where we have found it necessary to depart from that. We do not try to bring the agent's commission into the calculation. I very seriously doubt if it is any of the policyholder's affair just what compensation we pay our agents. I doubt very seriously if it is of any advantage to try to explain to a policyholder just how much commission we pay an agent or why and how he should be debited or credited on account of it. The difference in reserve, however, is quite easily stated and fairly well understood by anyone who is involved in the transaction.

Changes appear to me not to be subject to any very sweeping rules. Every so often comes a request for an entirely unheard-of change. It is necessary to give a certain amount of study and attention to individual cases which arise. There are, however, fortunately, a rather large number which fall into groups, and can be handled according to a set plan.

It is entirely probable that there is very little to choose among the different practical plans which have been suggested here today, but it certainly is a great advantage in office routine to get as many of these changes from the class requiring special consideration and study to a group which can be handled more or less automatically without any great amount of skilled attention.

MR. L. A. ANDERSON:

I have been very much interested in a question that is not on the program, but it hooks up very closely with what the last speaker stated, and if I am not out of order I would like to bring it up.

It is the matter of allowing discount on premiums paid in advance. If you allow the difference in reserve toward paying future premiums, you have in a measure the same problem. I found that

some companies allow 3 per cent and some $3\frac{1}{2}$ and some a higher rate.

In one case a policyholder sold some property and had some ready cash and no particular place to put it. He wanted to pay his premium in advance to get it out of the way. It always seemed to me that it would be good policy to allow a fairly good rate of interest on such payment, first because it reduces the cost of collection and second because it practically insures the policy against lapse, for that many years at least, and third, it insures the commission to the agent.

I was wondering what the practice was among other companies in allowing a discount on premiums paid in advance.

MR. A. J. KOEPPE:

In the Union Central we discount premiums paid in advance at $3\frac{1}{2}$ per cent but at the end of every policy year when the premium falls due, we allow interest on the money that has been in the hands of the company for one year or less at the rate of $1\frac{1}{2}$ per cent, making a total interest return of 5 per cent to the insured who pays his premium in advance.

Our practice in regard to policy changes in the first year, if the change is from a higher to a lower priced plan, is to consider the first premium on the new policy paid and give no further return to the insured. Our object is to give him no better treatment than we would give the policyholder who surrenders his policy. In later years we make such changes on the basis of the difference in reserves, returning to the policyholder the excess reserve less a proportionate surrender charge. That is, the proportion is based upon the reserve surrendered.

Very often we make changes from a higher to a lower priced plan by increasing the amount of insurance to such an amount as the premium paid would have purchased on the lower priced plan. In that way the insured loses nothing and gets full credit for the amount he pays.

We make the change, however, to the nearest $500, so in some cases he might have to pay a few dollars of additional premium for the slightly increased amount to bring it up to that point.

MR. ALLISON: Does that change date from the original date of the policy?

MR. KOEPPE: That change dates back to the original date of the policy.

MR. ALLISON: Do you do that in every state of the Union?

MR. KOEPPE: In every state. We have an agreement with the state of Ohio, permitting us to make such changes and recently, as I think you know, there was a law passed in the State of Massachusetts providing for just such changes.

MR. ALLISON: In Virginia?

MR. KOEPPE: We do it in every state. We have never had any trouble. We had a little trouble in Ohio but we straightened it out with them.

MR. RICHARD LITTLE:

We generally follow the plan outlined by Mr. Fassel, always making sure that the policyholder cannot get any value indirectly on account of the change that he could not get directly.

Changes in plan of insurance from higher to lower reserve are made as follows:

I. When premiums have been paid for less than one year

We would apply the gross premium already paid to the gross premiums on the lower reserve basis bringing the insurance to the point where a payment is required on the part of the policyholder. At the same time, we would adjust the commission payable to the agent.

II. When premiums have been paid for one year or more, but less than two years

Under this condition it is our practice to refund the difference in reserve contingent on its being applied in part payment of the renewal premium. If the reserve allowance is more than sufficient to pay the renewal premium for the second policy year, we pay the excess to the insured. No adjustment of commission is made in connection with this change.

III. When premiums have been paid for two years or more

Our contracts have at this time an established cash value and it is our practice to refund to the policyholder the difference in reserves provided this difference does not exceed the difference in cash values.

All of the foregoing changes are, of course, subject to evidence of insurability.

MR. E. A. PORTER:

When I was on my feet a moment ago I forgot something I wanted to ask of the other speakers. It is of great interest to me to know exactly what sort of evidence of insurability is required on changes from a higher to a lower priced form where there is no increase in the volume of the insurance.

I might say that our own practice has been, except in a very few cases where it is waived, to require full medical examination after the first policy year, but I am very doubtful that there is any reason for going to so much expense. I would like to hear from other speakers as to the practice in this regard.

MR. H. H. JACKSON:

The company with which I am familiar requires, at the expense of the insured, precisely the same proof of insurability that would be required for new insurance of the same amount. On a small policy, he pays for his medical examination; on a large one, he pays the medical fee and also the microscopic. This is hardly so much to protect the company as to discourage such changes which, whatever rule you adopt, must necessarily be a loss to the company. It is one of those boasted services that we give to dissatisfied policyholders and let our satisfied policyholders pay for.

I, for one, have grown more and more regretful at the many services the calm premium-paying policyholder takes care of for the erratic purchaser, the latter often being stirred to action not by his needs at all, but by some attractive program, if that is the latest word of salesmanship, presented to him by some supersalesman.

Even after cash values are available, the payment of the difference in reserves to the changing policyholder frequently involves a technical injustice.

If you will compare the commissions which the agent has received on the higher premiums plan with those which he would have enjoyed had he sold the life plan in the first place, you will see that he has often been inequitably highly rewarded.

This is a problem of which I do not quite see the solution unless it be that all the companies must say to all their agents, "We cannot let you twist our old business in any form within the company, and we will not, under any circumstances, permit the twisting of the insurance of another company to a lower premium form in ours."

MR. C. H. BECKETT:

Mr. Anderson's question has been lost in the discussion. It seems to me that a fair basis for handling premiums paid in advance is to discount the premiums at the rate of interest used in calculating reserves and then if it is a participating policy, compensate the insured each year in the way of an extra dividend. A company allowing 3 per cent compound interest on premiums paid in advance and earning $4\frac{3}{4}$ per cent would increase the annual dividend by $1\frac{3}{4}$ per cent of premiums to the credit of the policy as paid in advance.

MR. L. A. ANDERSON:

That corresponds very closely to my own idea. It brings it practically up to the rate of interest allowed in computing dividends; does it not? And it seems to me that it is advisable to make that rate high enough so as to be an inducement to the policyholder to pay in advance.

MR. A. W. LARSEN: I would like to ask Mr. Anderson a question. You stated that the prepayment of premiums insures a policy against lapse. What would you do if a policyholder prepays the premiums for five years, and then a year later demands the surrender value of the policy?

MR. ANDERSON: There is a slight contingency there. He has a right to do that, but as a matter of practice those who demand the cash are so extremely few that they are negligible. I never had one of that kind. You would have to refund premiums if he demands it, but those who pay their premiums in advance are those who want to keep their policies in force and I think the cases where they would ask for refund of premiums once paid would be very few indeed.

MR. SMITH: May I ask what happens in case a policyholder dies and the policy becomes a claim after he has paid in advance but before his credits in that regard are all used up?

MR. ANDERSON: Pay the unused part with the policy.

MR. SMITH: Would it not be more advisable to accept that money on account and transfer that money to the premium account when and as the premium becomes due and to pay the commission when and as it becomes earned? You avoid the difficulty of having to make a refund if the man asks for it. You avoid the difficulty in paying cash surrender values and in making a refund in the event of death. Moreover, the necessary excess interest calculations may be made without difficulty.

MR. H. W. BUTTOLPH:

It seems to me there is a very real objection to the allowance of discount on premiums paid in advance, and that is, where are you going to draw the line? If you say a year in advance, then I say the man that pays a premium 365 days in advance is not entitled to a full year's discount unless the man who pays 364 days in advance gets some discount. Of course, it is entirely impractical to allow a discount on every premium that is paid in advance, and it seems to me if we allow discount on any premium paid in advance then we are getting into a matter of discrimination to which an insurance department might very seriously object.

MR. ANDERSON: May I just answer that? There is a principle of law that seems to me to cover Mr. Buttolph's objection. The law does not concern itself with trifles. That is, the minimum time is six months. In case of premiums paid six months or more in advance the company that I am speaking about allows a discount.

BOOK REVIEWS AND NOTICES

Population Statistics and Their Compilation. Actuarial Studies, No. 3. Principal contributor: HUGH H. WOLFENDEN. New York: Actuarial Society of America, 1925. Pp. viii+144.

This book is the latest addition to the series of "Actuarial Studies." It reviews the census and the registration of vital facts, as practiced by the various governmental agencies, and the application of statistical methods to the study of such data. Following a brief statement of the value and uses of information of this nature, the different methods and the organization used in collecting the data are discussed. The reliability of the statistics and the nature of the errors therein are then stated. The more important errors in census statistics are deficiency in the number of infants, "heaping" at certain digits of age, and the overstatement and understatement of ages at certain periods of life. The principal errors in the registration of births are incompleteness, legitimacy, and still-births, and in the death reports are occupation, cause, age, and residence. Preliminary adjustments for errors of age and the methods used to estimate population at a time other than the date of the census are described.

The chapter on "Mathematical Relationships between Births, Deaths, and Populations; and the Formulas for the Rates of Mortality" is followed by a résumé of the various methods used to construct mortality tables from population statistics. Difficulties met at infantile, adult, and the oldest ages and the corrective methods used are shown.

Other topics included are the construction of abridged tables, comparison of mortalities of different communities, mortality by cause and by occupation, and the procedure followed for the construction from census and registration data of tables relating to marriages, births, orphanhood, sickness, etc.

In a book discussing the foregoing subjects in a limited number of pages, it is quite likely that the treatment of some phases of the subject suffers. In this case it was the mathematical demonstrations of the several formulas. The description and the methods employed in the collection of the data, the nature of the errors, and the bases of the necessary adjustments are easily followed and understood. However, the various mathematical demonstrations are somewhat concise, and may be difficult to follow for a student who does not have a thorough knowledge of the mathematics employed. As an illustration, the explanation of the employment

of the osculatory interpolation is confined to less than eight pages. Although many formulas are quoted, the student must study the numerous references given in order to understand their derivation and the methods necessary for their practical use. The task of the student would be somewhat easier if he could find the actual derivation and application of the more important formulas in one volume, and would not have to refer to other articles and books in which the mode of attack varies with the author.

One of the difficulties of the acturial student who is reading references on a subject concerning which he has little practical knowledge is to differentiate between the more and less important material. Some may find such a difficulty with this volume, and a differentiation by means of heavier and lighter type or distinctive arrangement of the material might have been of assistance to such students.

The Table of Contents is very useful, and serves admirably both as a summary of the book and as an index of the material included.

References, when appropriate, are given in the text itself. This arrangement is a very satisfactory one, but the addition of a bibliography would have been an improvement, so that one who was seeking a specific reference would not have to go through the text. In view of the completeness of the Table of Contents, this is not a serious failing.

This book should not be regarded as a substitute for other readings on the subject or as a textbook in itself, for it covers a broad field and is necessarily very compact. The student will probably find it most valuable as a basis for his study and as a review, since it brings into one volume the many phases and practices of the subject. It is difficult to make a book of this type interesting, and, at the same time, to keep the topics in logical order. This Mr. Wolfenden has accomplished very ably.

R. A. H.

Life Insurance Accounting. By James W. Glover and Earl C. Wightman. Ann Arbor, Michigan: Edwards Brothers, 1925. Pp. iv+174.

This work has made its appearance at a time when there is a real need for a complete text on this subject. It covers very comprehensively the principal items of insurance accounting, including departmental requirements, especial emphasis being given to extensive criticism of the Convention Blank.

The book opens with a brief treatise on general principles of accounting, and points out the differences existing between insurance accounting and general commercial accounting. Subsequent chapters take up in

detail the manner of handling of the various items of income, disbursements, assets, liabilities, policy exhibit, and gain-and-loss exhibit. In their discussion of these items the authors connect their explanations in each case with the requirements of the Convention Blank and call attention to many places in which they consider the Blank illogical and not in accord with correct accounting principles.

At some points, issue is also taken with methods of accounting set forth in acturial papers by actuaries of recognized standing. Other actuaries have also expressed themselves as differing with some of the opinions advanced in this book. It would, of course, be difficult to publish a book on this subject that would not differ from the ideas of other actuaries as to some points. These differences should not cause much difficulty as the work shows clearly what are merely opinions of the authors.

The authors have purposely avoided the giving of sample forms for the books of accounts, because of the necessary modification for various companies. The headings of columns of the different books of one company are, however, given by way of illustration.

While students will necessarily bear in mind the fact that the opinions expressed therein are not all accepted by all actuaries, and allowance will need to be made for the fact that the last Convention Blank contains several modifications which make some of the discussions inapplicable, the book will be of real value in setting forth clearly and thoroughly the fundamental principles of this subject.

E. L. M.

OTHER RECENT PUBLICATIONS

A Manual of Problems and Tables in Statistics with Notes on Statistical Procedure. By Frederick C. Mills and Donald H. Davenport. Newark, N.J.: Henry Holt & Co. Pp. 203.

The Economic Problem. By R. G. Howtry. New York: Longmans, Green & Co., 1925. Pp. 395.

Statistical Analysis. By Edmund E. Day, Professor of Economics and Dean of the School of Business Administration, University of Michigan. New York: The Macmillan Co., 1925. Pp. 454.

Report of Committee on Joint Investigation of Experience of American and Canadian Companies with Reference to Total and Permanent Disability Benefits. New York: The Actuarial Society of America, 1926. Pp. 48.

OBITUARY

PERCY CHARLES HERBERT PAPPS

Born, 1876 Died, 1926

PERCY CHARLES HERBERT PAPPS

Percy Charles Herbert Papps, a Fellow of the Institute, died May 18, 1926. Mr. Papps was born in Hamilton, Ontario, Canada, on May 12, 1876. He started in the work of his profession at an early age with the Canada Life Assurance Company, and rose to be its Assistant Actuary in 1902. A year later he became Actuary of the Manufactures Life Insurance Company, which position he held until he went to Newark, New Jersey, in 1907 to become Actuary of the Mutual Benefit Life Insurance Company. In 1919 he was appointed Mathematician of that company.

He was an Associate of the Institute of Actuaries of Great Britain, a Fellow of the Actuarial Society of America, and a member of the Mathematical Society, as well as having been, since 1918, a Fellow of our Institute.

Mr. Papps made numerous contributions to life insurance literature, which were published in *The Record* of the Institute and elsewhere. He was an outstanding figure in actuarial circles and consistently exerted an influence that was constructive and sound.

ANNUAL REPORT OF THE TREASURER

of the

AMERICAN INSTITUTE *of* ACTUARIES

FOR THE YEAR ENDING MAY 31, 1926

INCOME

Cash on hand June 1, 1925	$ 252.36	
Dues	5,681.00	
Records	1,011.10	
Examinations	1,679.10	
Interest	367.73	$8,991.29

DISBURSEMENTS

Printing	$3,217.70	
Postage	307.23	
Library	63.04	
Insurance	120.32	
Express	12.39	
Clerical Expense	472.70	
Traveling Expense, Board Members	60.93	
Stenographic Expense	325.97	
Examination Expense	179.99	
Miscellaneous Expense	199.61	
Stationery	37.74	
Exchange	23.40	
Entertainment	226.05	
Telegrams	6.51	
Invested during year	1,596.30	$6,849.88
Cash on hand May 31, 1926		2,141.41
		$8,991.29

Assets:		
Cash	$2,141.41	
U.S. Third Liberty 4¼ per cent Bonds	2,970.94	
U.S. 4¼ per cent Notes	1,531.50	
U.S. Fourth Liberty Loan 4¼ per cent	2,064.80	$8,708.65
Liabilities		None
Surplus		$8,708.65

Audited and found correct.

S. E. Allison
E. R. Carter
E. L. Marshall
Auditing Committee

MINUTES OF THE ANNUAL MEETING
of the
AMERICAN INSTITUTE *of* ACTUARIES

HELD AT THE EDGEWATER BEACH HOTEL, CHICAGO
JUNE 3 AND 4, 1926

The Institue was called to order by the President at 11 A.M., June 3 and the following Fellows, Associates, and Nominees of Contributing Members were present:

FELLOWS

S. E. Allison
L. A. Anderson
W. N. Bagley
J. P. Bowerman
J. A. Budinger
R. C. Burton
Arthur Coburn
W. P. Coler
H. R. Corbett
H. W. Curjel
M. E. Davis
F. A. Draper
E. O. Dunlap
J. S. Elston
P. H. Evans
James Fairlie
E. G. Fassel
J. W. Glover
A. C. Good
C. R. Goodrich
George Graham
Marcus Gunn
R. A. Hohaus
R. G. Hunter
H. H. Jackson
W. M. Johnson
F. D. Kineke
J. M. Laird
A. W. Larsen
A. T. Lehman
A. J. McAndless
R. C. McCankie
James McIntosh
Henry Moir
P. V. Montgomery
W. E. Napier
J. H. Nitchie
J. G. Parker
S. L. Phelps
W. F. Poorman
E. A. Porter
H. L. Rietz
J. C. Rietz
J. C. Seitz
H. G. Sellman
V. R. Smith
C. F. Stein
B. J. Stookey
W. M. Strong
R. M. Webb
F. S. Withington
W. A. P. Wood

ASSOCIATES

T. L. Anderson
G. W. Bishop
C. F. Cross
A. C. Eastlack
J. H. Evans
C. H. Folz
J. S. Hale
J. V. Hanna
W. H. Huehl
F. R. Jordan
L. J. Kalmbach
J. H. Lithgow
W. H. McBride
Merlin Oates
J. F. Reilly
L. O. Shudde
F. D. Strudell
R. B. Sturtevant
H. G. Walton

NOMINEES OF CONTRIBUTING MEMBERS

G. D. Babbitt . . . . . Bankers Reserve Life Company
O. M. Barnes . . . . . Franklin Life Insurance Company
W. J. Barr . . . . . Louisiana State Life Insurance Company
H. R. Bassford . . . . . Metropolitan Life Insurance Company
G. S. Bere . . . . . London Life Insurance Company
A. C. Bigger . . . . . American Life Reinsurance Company
R. Boissard . . . . . National Guardian Life Insurance Company
F. R. Brunner . . . . . Security Mutual Life Insurance Company
O. J. Burian . . . . . Missouri State Life Insurance Company
L. C. Cox . . . . . National Life & Accident Insurance Company
James Craw . . . . . Springfield Life Insurance Company
T. H. Galbraith . . . . . Central Life Insurance Company of Illinois
A. O. Graeser . . . . . Ohio National Life Insurance Company
W. R. Halliday . . . . . Southern States Life Insurance Company
M. N. Hanson . . . . . Wisconsin Life Insurance Company
F. J. Hogan . . . . . Lincoln National Life Insurance Company
G. L. Holmes . . . . . Manufacturers Life Insurance Company
H. J. Hornberger . . . . . Great Northern Life Insurance Company
W. B. Kieft . . . . . Springfield Life Insurance Company
A. J. Koeppe . . . . . Union Central Life Insurance Company
H. W. Lane . . . . . LaFayette Life Insurance Company
J. R. Larus, Jr. . . . . Phoenix Mutual Life Insurance Company
Richard Little . . . . Massachusetts Mutual Life Insurance Company
J. V. S. Longenecker . . Equitable Life Insurance Company of Iowa
E. A. Lundgren . . . Prudential Life Insurance Company of America
R. M. Malpas . . . . . Reinsurance Life Company of America
P. N. Mantz . . . . . Lincoln National Life Insurance Company
E. W. Marshall . . . . . Provident Mutual Life Insurance Company
Carl Mitcheltree . . . . . Columbus Mutual Life Insurance Company
T. M. Mott . . . . . Liberty Life Insurance Company
E. H. Nicholson . . . . . Lincoln National Life Insurance Company
F. J. Owen . . . . . Berkshire Life Insurance Company
J. G. Phipps . . . . . Peoples Life Insurance Company
R. F. Preston . . . . Northwestern National Life Insurance Company
T. C. Rafferty . . . . . International Life Insurance Company
G. E. Reilly . . . . . Midland Mutual Life Insurance Company
J. B. Russel . . . . . Peoria Life Insurance Company
C. D. Rutherford . . . . . Sun Life Assurance Company of Canada

H. M. Sarason ..... Missouri State Life Insurance Company
E. M. Thomas ... John Hancock Mutual Life Insurance Company
V. V. Veatch ..... Merchants Life Insurance Company
D. N. Warters ..... Bankers Life Company
A. C. Washburne ..... Berkshire Life Insurance Company
J. F. Williams ..... Illinois Life Insurance Company

The Minutes of the meeting held November 5 and 6, 1925, as printed in No. 30 of *The Record*, were approved.

The Secretary gave a summary of the business transacted by the Board since the last meeting of the Institute and announced that Mr. Arthur Barton Wood had been admitted to Fellowship without examination; that Mr. Maximilian Richard Hollenberg, Mr. Frank D. Kineke, Mr. Leslie R. Martin, Mr. Hudson J. Stowe, had been admitted to Associateship without examination and that The Dominion Life Assurance Company of Canada, The Mutual Life Assurance Company of Canada, The Sun Life Assurance Company of Canada, The North American Life Assurance Company of Canada, The Liberty Life Insurance Company, The Louisiana State Life Insurance Company, The Springfield Life Insurance Company, had all been admitted to Contributing Membership.

It was also announced that as a result of the examinations held in April, the following have been admitted to Fellowship by examination: Joseph Aloysius Budinger, Wendell Phillips Coler, Malvin Edward Davis, Alexander Caldwell Good, Clarence Robertson Goodrich, Frank D. Kineke, Eloise Bender Koch (Miss), A. Thomas Lehman, Alva John McAndless, Leslie R. Martin, Ross Emerson Moyer; and that the following had been admitted to Associateship by examination: Theodore Loyal Anderson, Harry M. Atrubin, Elgin Robertson Batho, James Robertson Beveridge, David Ernest Buckner, Stuart Fulton Conrod, John Henry Evans, Walter Harold Huehl, Philip C. Irwin, Walter Otto Menge, Otto C. Richter, Simon Shannon, Alexander C. Wellman, Floyd Eugene Young.

The President announced the death of Mr. Percy Charles Herbert Papps, a Fellow of the Institute, and a resolution expressing the loss experienced by the Institute in the death of Mr. Papps and the high esteem in which he was held by the members was moved by Mr. J. G. Parker and seconded by Mr. Lawrence M. Cathles.

The resolution was adopted by a standing vote of the Institute. An obituary notice embodying the facts contained in the resolution appears elsewhere in *The Record*.

The President reported that the Board appointed E. R. Carter as Librarian and H. H. Jackson as Editor of *The Record* for the ensuing year. Upon motion duly seconded, the Institute confirmed the appointment.

The Treasurer submitted his report which was accepted subject to approval by a duly appointed Auditing Committee.

Election of officers was, by motion, deferred until 11 A.M. June 4.

Formal papers were then presented.

The morning session adjourned at 12:30 P.M.

The Institute reconvened at 2:30 P.M. and a resolution thanking the Examination Committee was submitted by E. L. Marshall and George Graham and unanimously approved by the members.

The paper presented at the November meeting was then discussed.

The meeting adjourned at 4:20 P.M.

Friday, June 4, the Institute reconvened at 10:20 A.M. and continued the Informal discussion until 11 A.M., at which time the election of officers took place. This resulted as follows:

President	J. G. PARKER
Vice-President	ROBERTSON G. HUNTER
Secretary	ELGIN G. FASSEL
Treasurer	BERT J. STOOKEY
Governors for three-year term	R. C. MCCANKIE O. W. PERRIN

The meeting then continued with the Informal Discussion and adjourned at 12 Noon.

EXAMINATIONS OF THE INSTITUTE

HELD ON APRIL 13 AND 14, 1926

EXAMINATION FOR ADMISSION AS ASSOCIATE

PART 1

1. Simplify (*a*) $\dfrac{x^{\frac{2}{3}}-4\sqrt[3]{x^{-2}}}{\sqrt[3]{x^2}+4+4x^{-\frac{2}{3}}}$;

(*b*) $$\frac{1+\frac{1}{m}}{\frac{1}{m}} \times \frac{\frac{1}{m}}{m^2+\frac{1}{m}} \div \frac{\frac{1}{m}}{m-1+\frac{1}{m}}$$

2. Resolve x^4+9x^2+81 into two factors, and x^8+81x^4+6561 into three factors.

3. It is between two and three o'clock, and in ten minutes the minute-hand will be as much before the hour-hand as it is now behind it. What is the time?

4. Employing synthetic division find the quotient of $3x^5+11x^4+90x^2-19x+53$ by $x+5$; also the quotient of $15x^5+22x^4-21x^3-25x^2+10x+4$ by $5x^3+4x^2-3x-1$.

5. A, B, and C start from the same place at the rates c, $c+d$, $c+2d$ miles an hour respectively; B starts k hours after A; how long after B must C start in order that they may overtake A at the same instant, and how far will they then have walked?

6. (*a*) Find the coefficient of x^{18} in $\left(x^2+\frac{3a}{x}\right)^{15}$;

(*b*) Find the simplest form of the $(r+1)$th term of $(1-nx)^{\frac{1}{n}}$.

7. Show that if x is real the expression $\dfrac{x^2-15}{2x-8}$ cannot lie between 3 and 5.

8. Prove that if from a point without a circle a tangent and a secant be drawn, the tangent is the mean proportional between the whole secant and its external segment.

9. Prove that the sum of two opposite sides of a quadrilateral circumscribed about a circle is equal to the sum of the other two sides.

10. Prove that if from a point within an equilateral triangle perpendiculars to the three sides are drawn, the sum of these lines is equal to the perpendicular from any vertex to the opposite side.

PART 2

1. Prove the identity $\frac{1}{1+\sin^2 a}+\frac{1}{1+\csc^2 a}=1$.

2. From the top of a cliff 150 feet high the angles of depression of two boats which are due south of the observer are 15° and 75°; find their distance apart, having given cot $15°=2+\sqrt{3}$, and cot $75°=2-\sqrt{3}$.

3. Given $\sin 18°=\frac{\sqrt{5}-1}{4}$, find cos 36°, tan 54°, sec 72°.

4. Find the value of cos 20°+cos 100°+cos 140°.

5. The sides of a triangle are 35, 49, 63; find the greatest angle, given log 2 = .3010300, log 3 = .4771213, log cos 47°53′ = 9.8264910, difference for 60″ = 1397.

6. Find the co-ordinates of the center and the radius of the circle $x^2+y^2+Dx+Ey+F=0$.

7. A point moves so that the square of its distance from a fixed point is proportional to its distance from a fixed line. Show that the locus is a circle.

8. Find the equations of the bisectors of the angles between the lines $5x-12y=0$, and $5x+12y+60=0$.

9. Find the equation of the line parallel to the line $3x+4y-15=0$, such that the point (2,−4) shall lie midway between the two lines.

10. Derive formulas for the slope and length of the line joining two points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$.

PART 3

1. Prove that the arithmetic, geometric, and harmonic means between any two positive quantities are in descending order of magnitude.

2. (*a*) Express $\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ with a rational denominator.

(*b*) Simplify $(28-10\sqrt{3})^{\frac{1}{2}}-(7+4\sqrt{3})^{-\frac{1}{2}}$.

3. Find the sum of $2n$ terms of a series of which every even term is a times the term before it, and every odd term c times the term before it, the first term being unity.

4. Solve the following equations:
(*a*) $x^4+1-3(x^3+x)=2x^2$;
(*b*) $\sqrt{3x^2-2x+9}+\sqrt{3x^2-2x-4}=13$.

5. Find the number of ways in which $m_1+m_2+\cdots+m_{2r}$ things can be divided into $2r$ groups containing $m_1, m_2, \cdots, m_{2r}$ things severally, when $m_1=m_3=m_5=\cdots$, and $m_2=m_4=m_6=\cdots$.

6. Prove that the number of combinations of n things r at a time when repetitions are allowed is equal to the number of combinations of $n+r-1$ things r at a time when repetitions are excluded.

7. Sum the following series to n terms and to infinity:

$$\frac{1}{3!}+\frac{5}{4!}+\frac{11}{5!}+\frac{19}{6!}+\cdots.$$

8. Find the sum of n terms of the series:
$1+3x+6x^2+10x^3+15x^4+\cdots$.

9. A and B play for a prize. A is to throw a die first, and is to win if he throws 6. If he fails B is to throw, and to win if he throws 6 or 5. If he fails A is to throw again, and to win with 6, 5, or 4, and so on. Find the chance of each player.

10. If ten things are distributed among three persons, find the chance of a particular person having more than five of them.

PART 4

1. State the steps to be taken (*a*) in finding the derivative of $f(x)$ from the definition, (*b*) in finding the points of inflection on the curve $y=f(x)$.

2. Differentiate the following functions:

(*a*) arc tan $\sqrt{1-x^3}$; (*b*) log $\sqrt{\frac{1}{2}(1-\cos x)}$; (*c*) $\left(\frac{1}{x}\right)^x+(x)^{\frac{1}{x}}$.

3. Given $y^2+y=x^2$, show that $\frac{d^3y}{dx^3}=-\frac{24x}{(1+2y)^5}$.

4. State the comparison test for convergence of an infinite series, and apply it to prove that the series $1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\cdots$ is convergent.

5. Integrate the following functions:

(*a*) $\frac{1-2x}{9x^2+4}$; (*b*) $\tan^2 x+\tan^4 x$; (*c*) $\frac{x^2+x-1}{x^3+x^2-6x}$.

6. Find the value of x for which $f(x)=3000$, given $f(10)=1754$, $f(15)=2648$, $f(20)=3564$.

7. (*a*) When two terms are missing from a series of n equidistant terms, show how to find the missing terms.

(*b*) When several equidistant values of a function are given, and one isolated value, show how to find other values.

8. (*a*) Prove that $\sum_0^{x-1} u_x=xu_0+\frac{x(x-1)}{2!}\Delta u_0+\frac{x(x-1)(x-2)}{3!}\Delta^2 u_0+\cdots\cdots$.

(*b*) Employ this formula to find the value of $1^3+2^3+3^3+\cdots\cdots+17^3$.

9. Find the standard deviation of the following frequency distribution:

Ages of pupils,	12	13	14	15	16	17
Frequencies,	7	45	186	114	61	8

10. (*a*) When the correlation coefficient r is given by $r=\frac{1}{n}\Sigma x_i' y_i'$ explain how the values of x_i' and y_i' are obtained from the pairs of values (X_1, Y_1), (X_2, Y_2), $\cdots\cdots$, (X_n, Y_n).

(*b*) Show that the above formula is equivalent to

$$r=\frac{\frac{1}{n}\Sigma(X_iY_i)-XY}{\sigma_x\sigma_y}$$

PART 5

(Table of $a_{\overline{n}|}$ for use with this paper is printed on the last page.)

1. (*a*) Express $(1+i)^n$ in terms of j, d, δ, and δ', where i is the effective rate of interest, j the nominal rate, d the rate of discount, δ the force of interest, δ' the force of discount.

(*b*) Reduce $\frac{1}{\bar{a}_{n|}}-\frac{1}{s_{\overline{n}|}}$ to an ordinary interest function, showing your work in detail.

2. (*a*) (1) Prove algebraically that $\delta=i-\frac{i^2}{2}+\frac{i^3}{3}\ldots\ldots$

(2) Give the formula in terms of i for the amount at the end of twenty years of \$1.00 per annum payable monthly in advance and accumulating at the effective rate i per annum.

(*b*) Life insurance proceeds of \$25,000 were left to a boy aged sixteen on condition that they were to be held by the company at interest until he reached age twenty-one and at that time applied to purchase an annuity-certain of nineteen annual payments, first due at age twenty-one. The payments for ages twenty-one to twenty-four are to be one-third as large, and for ages twenty-five to twenty-nine to be two-thirds as large, as for ages thirty to thirty-nine. The company operates on a 3 per cent basis, but allowed an extra 1 per cent—a total of 4 per cent—interest while the procedse were at interest. The boy having now reached age twenty-one, how much will his first annuity payment be?

3. A municipality issues \$100,000 in \$100 bonds at 4 per cent, maturing in sixteen years, and \$500,000 in \$500 bonds at 6 per cent, maturing in four years. Purchasers are required to take an equal number of each issue. A certain purchaser desires a yield of 6 per cent on his whole investment over the entire sixteen years, but calculates that he can reinvest the principal of the larger issue on maturity at only 5 per cent for the remaining twelve years. What maximum price should he pay for each unit of one \$100 bond and one \$500 bond?

4. A war loan of $25,000,000 bearing 5 per cent interest is to be retired in full, both principal and interest, as follows: The interest of $1,250,000 is to be paid at the end of each year for eight years, and a sinking fund earning 4 per cent accumulated during this period, first payment at the end of one year. At the end of eight years the sinking fund is to be applied to the loan, and the balance refinanced by the payment annually at the end of each year for twelve years of $2,000,000, to include both principal and interest. What sum must be set aside annually during the first eight years toward the sinking fund, and what will be the amount of the sinking fund at the end of the eight-year period?

5. A $4\frac{1}{2}$ per cent bond, par value $1,000, with semiannual coupons, matures in fourteen years and was bought for $950.11 just after payment of semiannual coupon.

(*a*) Find approximate yield convertible semiannually, using the formula $i=\dfrac{g-\dfrac{k}{n}}{1+\dfrac{n+1}{2n}k}$.

(*b*) Derive formula in (*a*).

6. On July 1, 1926, a 4 per cent bond of $1,000 par value with semiannual coupons and maturing July 1, 1929, is purchased for $972.46, yielding an effective rate of interest of 5 per cent, convertible semiannually. The purchasing corporation uses the amortization method of valuation. Assuming principal and interest paid as due:

(*a*) State the various book entries, giving the actual amounts and dates necessary until and including the maturity of the bond.

(*b*) What asset items as of December 31, 1927, will result from the purchase of this bond?

7. (*a*) Discuss briefly how a company might treat depreciation of buildings in its accounting transactions.

(*b*) Discuss briefly the nature and purpose of Controlling Accounts.

8. From the following Trial Balance make up a Profit and Loss Statement and a Balance Sheet:

TRIAL BALANCE

DECEMBER 31, 1925

Cash	$ 1,000	
Notes Receivable	2,000	
Accounts Receivable	3,500	
Merchandise Inventory	5,000	
Furniture and Fixtures	1,000	
Notes Payable		$ 1,000
Accounts Payable		1,000
Capital		10,000
Sales		19,000
Purchases	15,000	
Expenses	4,000	
Interest and Discount		500
	$31,500	$31,500

The adjustment data as of December 31, 1925, is: Merchandise Inventory, $7,000; Expenses Paid in Advance, $100; Accrued Expenses, $500; Interest Receivable, $200; Depreciation of Furniture and Fixtures, $100; Bad Debts Estimated at $200.

TABLE OF $a_{\overline{n}|}$ FOR USE IN CONNECTION WITH THIS PAPER

n	1½ Per Cent	2 Per Cent	2½ Per Cent	3 Per Cent	4 Per Cent	5 Per Cent	6 Per Cent
1	0.9852	0.9804	0.9756	0.9709	0.9615	0.9524	0.9434
2	1.9559	1.9416	1.9274	1.9135	1.8861	1.8594	1.8334
3	2.9122	2.8839	2.8560	2.8286	2.7751	2.7232	2.6730
4	3.8544	3.8077	3.7620	3.7171	3.6299	3.5460	3.4651
5	4.7826	4.7135	4.6458	4.5797	4.4518	4.3295	4.2124
6	5.6972	5.6014	5.5081	5.4172	5.2421	5.0757	4.9173
7	6.5982	6.4720	6.3494	6.2303	6.0021	5.7864	5.5824
8	7.4859	7.3255	7.1701	7.0197	6.7327	6.4632	6.2098
9	8.3605	8.1622	7.9709	7.7861	7.4353	7.1078	6.8017
10	9.2222	8.9826	8.7521	8.5302	8.1109	7.7217	7.3601
11	10.0711	9.7868	9.5142	9.2526	8.7605	8.3064	7.8869
12	10.9075	10.5753	10.2578	9.9540	9.3851	8.8633	8.3838
13	11.7315	11.3484	10.9832	10.6350	9.9856	9.3936	8.8527
14	12.5434	12.1062	11.6909	11.2961	10.5631	9.8986	9.2950
15	13.3432	12.8493	12.3814	11.9379	11.1184	10.3797	9.7122
16	14.1313	13.5777	13.0550	12.5611	11.6523	10.8378	10.1059
17	14.9076	14.2919	13.7122	13.1661	12.1657	11.2741	10.4773
18	15.6726	14.9920	14.3534	13.7535	12.6593	11.6896	10.8276
19	16.4262	15.6785	14.9789	14.3238	13.1339	12.0853	11.1581
20	17.1686	16.3514	15.5892	14.8775	13.5903	12.4622	11.4699
21	17.9001	17.0112	16.1845	15.4150	14.0292	12.8212	11.7641
22	18.6208	17.6580	16.7654	15.9369	14.4511	13.1630	12.0416
23	19.3309	18.2922	17.3321	16.4436	14.8568	13.4886	12.3034
24	20.0304	18.9139	17.8850	16.9355	15.2470	13.7986	12.5504
25	20.7196	19.5235	18.4244	17.4131	15.6221	14.0939	12.7834
26	21.3986	20.1210	18.9506	17.8768	15.9828	14.3752	13.0032
27	22.0676	20.7069	19.4640	18.3270	16.3296	14.6430	13.2105
28	22.7267	21.2813	19.9649	18.7641	16.6631	14.8981	13.4062
29	23.3761	21.8444	20.4535	19.1885	16.9837	15.1411	13.5907
30	24.0158	22.3965	20.9303	19.6004	17.2920	15.3725	13.7648

PART 6

1. (*a*) Develop by general reasoning: $P_{x\overline{n}|}=\frac{1}{a_{xn|}}-d.$

(*b*) If $m_x=\frac{dx}{\frac{1}{2}(l_x+l_{x+1})}$ prove that $p_x=1-m_x+\frac{1}{2}m_x^2-\frac{1}{4}m+$ . . . . .

2. Given the following mortality table deduce the complete expectation of life, the curtate expectation of life, the most probable after life time, and the vie probable for age ninety-one.

Age	l_x
90	847
91	462
92	216
93	79
94	21
95	3
96	0

3. (*a*) Express as definite integrals ${}_{n|}\bar{a}_x$, $\bar{A}_x$, $\bar{V}_x$.

(*b*) Find expressions for $\frac{d}{d_x}\mathring{e}_x$, $\frac{d}{d_x}\bar{A}_x$.

4. (*a*) It is desired to provide for a child aged six a temporary annuity of $1,000 for four years, first payment at age eighteen. In event of the death of the child before the age of eighteen, the net premiums will be returned without interest. Derive the formula in select commutation symbols for the net premium.

(*b*) Derive the formula for calculating the return premium factor by which a twenty-payment life premium should be multiplied to provide for the return of all premiums paid in event of the death of the insured within twenty years.

5. Derive in commutation symbols the annual premium payable for n years for an annuity of 1 on the life of x deferred n years, with no return in the event of death during the premium-paying period; but with a return in one sum at death, if that occurs after the annuity is entered upon, of the excess of the total premiums paid over the sum of the annuity payments received. What difficulties will appear in the use of the formula and how can they be met?

6. (*a*) Prove that $\mathring{a}_x^{(m)} = \bar{a}_x\left(1 - \frac{\delta}{2m}\right)$ (approximately).

(*b*) Demonstrate algebraically whether a_x is greater, equal to, or less than $a_{\overline{e_x}|}$.

7. (*a*) What would be the result of entering with a_n
(1) An annual premium conversion table?
(2) A single premium conversion table?

(*b*) Show algebraically that for a mortality table following Makeham's Law a constant increase in the force of mortality has the same effect on a_x as an increase in the rate of interest.

8. (*a*) Prove that ${}_tV_{[x]}=\dfrac{P_{[x]+t}-P_{[x]}}{P_{[x]+t}+d}$.

(*b*) Give the "equation of equilibrium," that is, the equation expressing relationship between the reserve at the beginning of a year, the interest earned thereon, and the net cost of insurance with the corresponding reserve at the end of the year in such a form that the actual interest earned on the reserve may be easily compared with the interest required to maintain the reserve, and the actual net cost of insurance may be compared with the expected cost of insurance or death-strain.

PART 7

1. (*a*) Compare and analyze the following probabilities:

$${}_t|q_{xy}; \quad {}_t|q_x \times {}_t|q_y; \quad \text{and } {}_t|q_{\overline{xy}}.$$

(*b*) Show in the case of a mortality table following Gompertz' Law that

$${}_t|q^1_{xy}=Q^1_{xy}\times {}_t|q_{xy}.$$

2. (*a*) Analyze the property of Makeham's Law that justifies a substitution of a joint life annuity on m lives of equal age w in place of a joint life annuity on m lives of unequal ages.

(*b*) Show that doubling the force of mortality is equivalent to taking two lives of equal age in calculating the expectation of life, the life annuity, and the single premium for insurance.

3. (*a*) Obtain the value in terms of simple functions of $a^{\;3}_{\overline{wxyz}}$.

(*b*) There are two expressions for the value of a reversionary annuity to (x) after (y), namely:

$$\Sigma v^n({}_np_{x-n}p_{xy}) \text{ and } \Sigma v^n\frac{l_{x+n}d_{y+n-1}}{l_xl_y}(1+a_{x+n}).$$

Give an algebraical demonstration that these are identical.

4. Express $\bar{A}_{x:y:z}$, $\bar{A}^{\overline{1}}_{\overline{xy}:z}$, and $\bar{A}^{3}_{w\overline{xyz}}$ as definite integrals.

5. (*a*) Give an approximate value of $a^{(m)}_{\overline{xyz}}$ in terms of a_{xyz} and forces of interest and mortality.

(*b*) Prove that $\frac{d}{dx}\bar{a}_{xy}=\mu_x\bar{a}_{xy}-\bar{A}^{1}_{\overline{xy}}$.

6. If the probability that (x) will die before (z) is .1996, the probability that (x) will die before both (y) and (z) is .1610, the probability that (y) will die before (z) is .2990, the probability that (y) will die before both (x) and (z) is .2602, find the values of the following probabilities:

(1) That the survivor of (x) and (y) will die before (z);

(2) That (x) will die before (z), (y) having died first.

7. Deduce a formula for the annual premium for an insurance payable if a life aged (x) dies within the next five years, or if he lives five years and dies after another life now aged (y).

8. Give the simplest symbol representative of each of the following integrals and explain fully the meaning of each function:

$$(a)\frac{1}{l_x l_y l_z}\int_0^\infty v^t l_{x+t}l_{y+t}(l_z-l_{z+t})\mu_{y+t}\bar{a}_{x+t}\,dt\,;$$

$$(b)\int_0^\infty v^t {}_tp_{wxy}(1-{}_tp_z)\mu_{w+t}dt\,;$$

$$(c)\frac{1}{l_x l_y l_z}\int_0^\infty v^t l_{x+t}\{l_{y+t}\mu_{y+t}(l_z-l_{z+t})+l_{z+t}\mu_{z+t}(l_y-l_{y+t})\}\bar{a}_{x+t}\,dt\,.$$

PART 8

1. In the construction of tables of the functions

$$(1+i)^n,\ v^n,\ s_{\overline{n}|}\text{ and }a_{\overline{n}|}\,,$$

what methods of calculation and of verification would you employ to insure the accuracy of the results?

2. Illustrate how you would compute select commutation columns based on American Men Table with $3\frac{1}{2}$ per cent interest.

3. Discuss the construction of a service table involving both the mortality and the withdrawal rate. Illustrate the commutation

columns that you would use in evaluating benefits depending on these functions.

4. Explain the commutation columns that you would construct in combining the rate of disability and the rate of mortality after disability to be derived from a new investigation with the American Men Ultimate Table to produce a new table similar to Hunter's Disability Table.

5. (*a*) Give the formula for the annual premium for the waiver of premium benefit in connection with a twenty-year endowment contract. Explain the meaning of the symbols used.

(*b*) Prove that developing the active life reserve on a premium waiver benefit on ordinary life policies with benefit and premium each limited to age sixty on the retrospective basis gives a formula equivalent to that on the prospective basis.

6. Obtain the formula for the annual premium for a waiver of premium benefit on a twenty-payment life form using Hezlett's Disability Table and explain the symbols used.

7. (*a*) Discuss how you would calculate the premium for the additional indemnity benefit in connection with a ten-payment life policy issued at age thirty where the additional indemnity provision is applicable in the event of death due to certain causes before age seventy—the extra premium to cease with the regular life premium; and how you would calculate the reserve at the end of the tth year from inception.

(*b*) Discuss briefly the history of life insurance in the United States before 1880.

8. Discuss very briefly the development of eight important advances which life insurance companies of the United States have quite generally made in the past fifteen years toward increasing service to policyholders or in extending service to new classes of policyholders.

EXAMINATION FOR ADMISSION AS FELLOW

PART 9

1. A number of companies undertake a joint investigation of their mortality experience from the anniversary in 1900 to the anniversary in 1925 under all immediate single life annuities exposed within that period.

(*a*) The investigation will be both by lives and by amounts; what precautions are necessary on this account?

(*b*) The policy year method will be used; what advantages are thereby gained?

(*c*) Describe the columnar arrangement of a perforated card for recording the data.

(*d*) Select tables are to be constructed; state exactly how you would find $E_{[x]+t}$, the exposed to risk t years after issue at age x.

2. Apply the graduation formula, $G=\frac{[3]\,[5]}{90}\,(4[3]-3\gamma_2)$, to the series $y=n^2$ from $n=1$ to $n=13$. Explain results.

3. What special problems arise in the compilation and graduation of mortality tables from census reports? Illustrate your answer by references to the U.S. Life Tables, 1910.

4. (*a*) Comment on the methods employed in the Swedish Investigation of Mortality among insured lives subject to extra hazard published in 1921, as to the fundamental assumption, the data required, and the manner of deriving mortality ratios, stating what advantages and disadvantages you see in this plan of investigation.

(*b*) State the method employed in graduating the following tables: American Men, American Annuitants', Carlisle, Combined Experience.

5. Name the eight chief factors into which a life insurance risk may be resolved and make a concise classification under these eight headings of the information usually elicited by the home office for the purpose of passing upon applications for new insurance, it being assumed that the usual complete medical examination has been required.

6. (*a*) Discuss the importance of family history in selection, stating the results shown in published investigations with your comments thereon.

(*b*) What is to be said for and against the practice of granting short-term endowment insurance at standard rates to certain applicants declined standard whole-life policies?

7. (*a*) Name five of the most frequent causes of permanent total disability in the order of importance.

(*b*) A company analyzes its disability claims to find what percentage each cause of disability represents of the total number. State in detail what company characteristics you would bear in

mind when comparing the results with the published experiences of other companies, and how you would expect the percentages to be affected thereby.

8. Define each symbol in the following expression:

$$ {}_{\frac{1}{k}}a_{\overline{xn-1}|}^{(ai)i} = \frac{{}_{\frac{1}{k}}\mathbb{N}_x^D - {}_{\frac{1}{k}}\mathbb{N}_{x+n}^{DD} + D_x^{ii}\left(\frac{\mathbb{N}_{x+n}^i}{D_x^i}\right)}{{}_{\frac{1}{k}}D_x^{aa}} . $$

PART 10

1. An established company with ample surplus will introduce a new scale of premiums and henceforth write only participating policies, reserving by the American Experience Table on the full level net premium plan. State with your reasons what general formula and basis you would recommend for the gross premiums and what interest rate for the reserves; also explain what tests you would apply to the premium scale before releasing it.

2. A company attaches to an ordinary life policy an agreement to hold the proceeds on death of the insured intact until subsequent death of the beneficiary and to pay the beneficiary interest during such time at 6 per cent per annum. Recommend the basis for a suitable non-participating extra premium and state your reasons.

3. Discuss the problem of determining suitable premiums for industrial insurance.

4. (*a*) Explain four office methods used in the United States for policy valuations, giving the advantages of each and the circumstances under which each would be suitable.

(*b*) What collateral advantages are desirable in an office policy valuation system?

5. Discuss the advantages and disadvantages of preliminary term valuation.

6. How would you determine the following liability items?

(*a*) Liability on policies cancelled but not included in "net reserve" upon which a surrender value may be demanded.

(*b*) Reserve for net death losses incurred but not reported.

7. (*a*) Define "surrender charge" and state what limit is set upon it by the laws of New York State.

(*b*) Give a retrospective and prospective reason for a surrender charge as well as your opinion of the merits of the reasons that you give.

8. It is suggested that a simple rule for change calculations would be to use the difference in reserves in all cases. (*a*) Give the arguments for and against the adoption of the rule for changes in general, and (*b*) indicate what exceptions to the rule you would consider imperative.

PART 11

N.B.—Candidates that did *not* pass Part 10 prior to 1926 are required to answer questions 1–8 and should omit 9 and 10. Candidates that passed Part 10 prior to 1926 should omit questions 1 and 2 and are required to answer 3–10.

Required only of candidates that did not pass Part 10 prior to 1926:

1. (*a*) Give four important requirements of a satisfactory method of surplus distribution.

(*b*) How would the requirements of (*a*) be met in the case of a steadily growing company with level premium reserves and ample contingency funds and with actual mortality and interest experience slightly better than the A.M. Table and $4\frac{1}{2}$ per cent, if the dividend formula is 1.045 × (gross premium—$8\frac{1}{2}$ per cent gross premium—$2.00—net premium on basis of A.M.$^{(5)}$ at $4\frac{1}{2}$ per cent)?

2. (*a*) How would you estimate the drain on surplus of a given amount of business likely to be written during the year by a company carrying full level premium reserves?

(*b*) Discuss the problem of issuing participating immediate life annuities.

Required of all candidates:

3. What objections are there to the Convention Blank Annual Statement and what can you say in favor of the form?

4. Show how the company's mortgage loan, bond and policy loan transactions appear in the items of the annual statement.

5. Indicate how the principles underlying the second part of the Gain and Loss Exhibit giving the changes in surplus according to classes or lines of business differ from the principles underlying the first part of the Exhibit.

6. What objections have been made to the amortization method of valuing bonds?

7. Discuss farm loans and railroad bonds as investments for life insurance companies. Would you recommend the purchase of Equipment Trust Securities and why?

8. (*a*) State the chief advantages and disadvantages of the sole proprietorship, the partnership and the corporation as forms of business organization.

(*b*) Give five important defects of the national banking system of the United States met by the Federal Reserve Act of 1913 and explain the corresponding remedies that the Act provided. Have any defects become apparent in the operation of the Federal Reserve System?

Required only of Candidates that passed Part 10 prior to 1926:

9. Under a certain death claim the net proceeds are $11,120.60, consisting of $10,000.00 face amount, $2,740.00 dividend additions, and $47.20 post mortem dividend minus $142.00 outstanding premium instalments and $1,500.00 principal and $24.60 accrued interest on policy loan; the proceeds are to be paid under a settlement option in ten equal annual instalments of $1,266.00, the first due immediately.

Indicate every place where this transaction will appear at the end of the year in pages 1–6 of the Convention Blank Annual Statement, except the liabilities.

10. Outline the course of the rate of interest earned by life insurance companies during the past twenty-five years and explain four important factors responsible for the trend during that time, showing to what conclusions they lead as to the future trend.

PART 12

N.B.—Candidates that did *not* pass Part 11 prior to 1926 are required to answer questions 1–8 and should omit 9 and 10. Candidates that passed Part 11 prior to 1926 should omit questions 1 and 2 and are required to answer 3–10.

Required only of candidates that did not pass Part 11 prior to 1926:

1. (*a*) What elements are necessary to make a contract valid? Discuss how these requirements are met in a life insurance contract.

(*b*) Enumerate the various federal, state and local taxes to which a life insurance company operating exclusively in the United States is subjected by statutory requirements.

2. Suppose that a well-established company incorporated under the laws of *either* Illinois *or* Canada contemplates operating in New York State. Give details of the legal requirements that you, as its actuary, would consider when studying the question.

Required of all candidates:

3. Give a brief statement of the limitations that a company should observe in connection with the special income settlements for which it will make provision in its policies.

4. (*a*) Draft a participation clause in which the usual options are included.

(*b*) What advantages and disadvantages do you see in the general practice of effecting policy changes by indorsement of the original policy as against issuance of an entirely new contract.

5. A state teachers' retirement fund is advocated. What advice should an actuary give the commissioner of education who asks his suggestions for making the plan as effective as possible.

6. Discuss any published experience under the double indemnity feature.

7. The entire capital stock of an established non-participating life insurance company is to be sold and you have been asked to set a value upon it. What information would you consider necessary, and how would you use it in determining the value of the stock?

8. Discuss the position occupied by the assessment system in the life insurance field of the United States and Canada at the present time.

Required only of candidates that passed Part II prior to 1926:

9. State the main points to be covered in a treaty under which a life insurance company cedes its excess to a reinsurance company.

10. Outline the British scheme of National Health Insurance.

RESULTS OF EXAMINATIONS

Examinations were held by the Institute on April 13 and 14, 1926, with the following results:

ASSOCIATESHIP

Part 1

One hundred forty-four candidates presented themselves for examination and the following fifty-six passed:

Abernethy, J. R.
Amerman, G. H.
Auden, M. F.
Barthel, C. E.
Batho, M. B. W.
Benson, C. H.
Bower, P. S.
Breithaupt, W. H.
Brown, E. D.
Bucknell, E. F.
Clair, J. J.
Clark, G. C.
Decker, M. F.
Delahunty, G. T.
Domingues, M. C.
Doran, A. B.
Erway, R. E.
Faith, E. L.
Feldman, I.
Fisher, G. A.
Fletcher, J. G.
Grassby, G. M.
Harper, R. J.
Higdon, J. C.
Hilliard, W. L.
Hollies, W. G.
Holmes, B. T.
Hondrop, P.
Houston, F. E.
Jones, O. H.
Keith, M. W.
Kirkpatrick, C. C.
Kuhlemeier, C. A.
Larsen, M. N.
Lehane, L. J.
Lemon, H. B.
Long, L. L.
MacLean, N. B.
Mackenzie, R. S.
Mansz, A. A.
Martin, R. J.
Mincks, W. W.
Orloff, C. A.
Pakes, E.
Pendergast, D.
Perkins, F. P.
Pierce, R. W.
Porter, R. R.
Schirm, O. G.
Stocking, L. A.
Thompson, E. W.
Tibbs, E. O.
Turnbull, D. McI.
Twiss, J. F.
Unsworth, N. R.
Yates, S. D.

Part 2

One hundred thirty-one candidates presented themselves for examination and the following fifty-nine passed:

Abernethy, J. R.
Abrahamson, T. A.
Ambler, A. B.
Amerman, G. H.
Andrews, L. R.
Armantrout, E. D.
Auden, M. F.
Batho, M. B. W.
Benson, C. H.
Bower, P. S.
Brown, E. D.
Burchell, J. R.
Chen, S. T.
Clair, J. J.
Clark, G. C.
Craig, D. S.
Decker, M. F.
Domingues, M. C.
Doran, A. B.
Elander, A. E.
Estes, E. F.
Faith, E. L.
Feldman, I.
Fletcher, J. G.
Fletcher, R. V.
Frederick, R. A.
Grassby, G. M.
Higdon, J. C.
Higginson, G. W.
Hilliard, W. L.
Hollies, W. G.
Holmes, B. T.
Huston, F. E.
Jennings, W. C.
Johnson, H. E.
Jones, O. H.
Keene, E. L.
Keith, M. W.
Kuhlemeier, C. A.
Lehane, L. J.
Lemon, H. B.
Long, L. L.
MacLean, N. B.
Mackenzie, R. S.
Mincks, W. W.
Pendergast, D.
Perkins, F. P.
Pullen, J. G.
Reilly, G. E.
Roberts, D. A. H.
Schirm, O. G.
Smith, L. G.
Sullivan, C. O.
Turner, P. A.
Twiss, J. F.
Ungerleider, A. L.
Vernon, G. H.
Vetter, L.
Whitbread, R. B. T.

Part 3

Seventy-eight candidates presented themselves for examination and the following thirty-four passed:

Abernethy, J. R.
Ambler, A. B.
Amberman, G. H.
Auden, M. F.
Brown, E. D.
Clair, J. J.
Danzinger, L. J.
Decker, M. F.
Doran, A. B.
Fletcher, J. G.
Higdon, J. C.
Hilliard, W. L.
Hogan, F. J.
Hollies, W. G.
Holmes, B. T.
Jennings, W. C.
Kirkpatrick, C. C.
Kuhlemeier, C. A.
Larsen, M. N.
Lehane, L. J.
Long, L. L.
MacLean, N. B.
Mincks, W. W.
Nicholson, E. H.
Pakes, E.
Perkins, F. P.
Reilly, G. E.
Tuck, E.
Turner, P. A.
Twiss, J. F.
Ungerleider, A. L.
Unsworth, N. R.
Wood, D.
Yates, S. D.

PART 4

Sixty-seven candidates presented themselves for examination and the following thirty-eight passed:

Amerman, G. H.
Angerer, J. W.
Armantrout, E. D.
Atwater, R. B.
Auden, M. F.
Bell, W. M.
Brown, E. D.
Carson, C. E.
Chen, S. T.
Clair J. J.
Craig, D. S.
Decker, M. F.
Ede, C. F.
Ferguson, A. A.
Graeser, A. O.
Hilliard, W. L.
Holmes, B. T.
Jennings, W. C.
Keene, E. L.
Kuhlemeier, C. A.
Lemon, H. B.
Long, L. L.
MacLean, N. B.
Martin, R. J.
Mincks, W. W.
Myers, G. W.
Myers, O. J.
Pakes, E.
Perkins, F. P.
Reilly, G. E.
Sarson, H. M.
Skelding, G. A.
Tuck, E.
Turner, P. A.
Twiss, J. F.
Ungerleider, A. L.
Wadleigh, W. L.
Wood, D.

PART 5

Forty-three candidates presented themselves for examination and the following twenty-two passed:

Armantrout, E. D.
Atrubin, H. M.
Batho, E. R.
Beveridge, J. R.
Buckner, D. E.
Conrod, S. F.
Danzinger, L. J.
Estes, E. F.
Green, E. C.
Holtzman, E.
Lyle, A. M.
Menge, W. O.
Michener, S.
Mims, J. L.
Motts, T. M.
Nelson, C. E.
Pall, G.
Shannon, S.
Shepherd, P.
Wallace, W.
Wellman, A. C.
Woolery, J. M.

PART 6

Thirty-seven candidates presented themselves for examination and the following eighteen passed:

Atrubin, H. M.
Batho, E. R.
Beveridge, J. R.
Buckner, D. E.
Conrod, S. F.
Cox, L. C.
Longnecker, J. V.
Lyle, A. M.
Menge, W. O.
Michener, S.
Nelson, C. E.
Pall, G.
Preston, R. F.
Schillig, E. M.
Shannon, S.
Shepherd, P.
Wellman, A. C.
Woolery, J. M.

Part 7

Twenty-eight candidates presented themselves for examination and the following nineteen passed:

Anderson, T. L.
Atrubin, H. M.
Batho, E. R.
Beveridge, J. R.
Buckner, D. E.
Coates, C. S.
Conrod, S. F.
Evans, J. H.
Huehl, W. H.
Irwin, P. C.
Linder, J.
Little, J. E.
Menge, W. O.
Michener, S.
Pall, G.
Shannon, S.
Shepherd, P.
Smith, L. G.
Wellman, A. C.

Part 8

Twenty-four candidates presented themselves for examination and the following fifteen passed:

Atrubin, H. M.
Batho, E. R.
Beveridge, J. R.
Buckner, D. E.
Conrod, S. F.
Evans, J. H.
Irwin, P. C.
Longnecker, J. V.
Menge, W. O.
Nelson, C. E.
Richter, O. C.
Shannon, S.
Wellman, A. C.
Woolery, J. M.
Young, F. E.

FELLOWSHIP

Part 9

Twenty-one candidates presented themselves for examination and the following fourteen passed:

Bjorn, W.
Clark, H. L.
Cross, C. F.
Dandy, J. P.
Davis, M. E.
Kineke, F. D.
MacKinnon, W. D.
Martin, L. R.
Moyer, R. E.
Prentice, G. T.
Shepherd, B. E.
Shudde, L. O.
Stowe, H. J.
Warren, L. A. H.

Part 10

Twenty candidates presented themselves for examination and the following seventeen passed:

Budinger, J. A.
Clark, H. L.
Cross, C. F.
Dandy, J. P.
Davis, M. E.
Eastlack, A. C.
Kalmbach, L. J.
Kineke, F. D.
MacKinnon, W. D.
McClurg, D. R.
Martin, L. R.
Moyer, R. E.
Prentice, G. T.
Shepherd, B. E.
Shudde, L. O.
Stowe, H. J.
Young, J. R.

PART 11

Fifteen candidates presented themselves for examination and the following eleven passed:

Budinger, J. A.	Kineke, F. D.	Moyer, R. E.
Coler, W. P.	Lehman, A. T.	Oates, M.
Davis, M. E.	McAndless, A. J.	Stowe, H. J.
Good, A. C.	Martin, L. R.	

PART 12

Eighteen candidates presented themselves for examination and the following twelve passed:

Budinger, J. A.	Goodrich, C. R.	McAndless, A. J.
Coler, W. P.	Kineke, F. D.	McClurg, D. R.
Davis, M. E.	Koch, E. B.	Martin, L. R.
Good, A. C.	Lehman, A. T.	Moyer, R. E.

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THE RECORD

AMERICAN INSTITUTE *of* ACTUARIES

November, 1926

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THE RECORD
of the
AMERICAN INSTITUTE *of* ACTUARIES
November, 1926

ADDRESS BY THE PRESIDENT, J. G. PARKER

In acting as your presiding officer for the first time I would like publicly to express my very keen appreciation of the honor you have done me in electing me to this high office. I would also take this occasion to express to you my gratitude for giving me the opportunity of serving with you for the good of this Institute, first as one of your members, then as a member of your Board, and now as your President.

Before making any remarks I would review briefly the work of your Board of Governors since the last meeting.

A suggestion was made in June in regard to a change in the order of business so that it might be unnecessary every year to put through a motion changing the hour of the election of the officers and the Board. An amendment to the by-laws has been drafted, submitted to the Fellows in accordance with the constitution, and will be considered by you at this meeting.

A committee of the Board has for some years had under consideration an amendment to the constitution with reference to the status of the past presidents on the Board of Governors. The work of this committee has been embodied in a suggested change in the constitution, which has also been submitted to the Fellows and will be open for your consideration at this meeting.

With the notices which you received concerning our Convention here today there was forwarded to you an outline of a plan for the securing of bound copies of our *Record* at the small additional cost of one dollar per volume. Your Board, through its Editor and Secretary, has made this arrangement which, I am sure you will agree

with me, is an admirable one for every member. May I draw attention to the fact that it is possible for you to make arrangements at this time with our printer for the binding of your old copies at an especially favorable rate, due to the fact that many will be availing themselves of the privilege. A memorandum was prepared by the Secretary showing in detail all the matter contained in a complete set of *The Record* and giving complete instructions in regard to the binding of the various volumes.

Your Board hopes that during the year a start may be made upon the work of preparing an Index of the first fifteen volumes of *The Record*.

In making this, my first presidential address, to the members of the American Institute I have thought it might be timely to speak of the rather recent interest acquired by Canadian companies and Canadian actuaries in the work of our Institute, touching upon the reasons for this interest, commenting upon the value which the work of the Institute had been to its Canadian members, and, therefore, necessarily the value which they on their part have been to the Institute. I also purpose to sketch somewhat briefly the reciprocal value of membership accruing both to the Institute and to its new members in the work which lies before us in the coming years.

A search through the numbers of *The Record* will reveal the fact that up to a few years ago only one Canadian actuary was a member of the American Institute and no company was sufficiently interested in its existence to be enrolled as a Contributing Member. The *Year Book for 1926* shows in the list of Contributing Members 12 Canadian companies, and I have the hope that before many meetings will have been held most of our Canadian companies will be here represented. The *Year Book*, moreover, shows that there are 7 actuaries practicing in Canada who are Fellows of the Institute, while one who secured his Fellowship through examination has since died. In addition, 14 Associates are engaged in actuarial work in various capacities while 30 Students from our offices and colleges are enrolled as such in the records of the Institute and have taken one or more of the examinations prescribed for Associateship.

It seems but natural that the question should be raised as to the reason for this growth in interest within the body of Canadian actuaries, Canadian companies, and Canadian students.

Undoubtedly the primary reason lies in the similarity of the problems affecting the Canadian companies and the companies of the middle western states. We live in a country of nine or ten million people scattered from the Atlantic to the Pacific. Our basic industry is agriculture. We have, in the process of development, tremendous stores of natural resources similar in their character to those in the United States. Large sections of our population are engaged in lumbering, in the manufacture of pulp and paper, in mining. We have before us, as you have had, the problem of the assimilation of many thousands of foreign-born settlers who will be responsible in the days to come for the development of our country and its citizenship.

Our agency problems, then, must be those of companies interested in developing to the fullest extent a business among a rural population—a population prone to frequent changes of habitat, an ever increasing population. Our expense problems, which are vitally linked up with our agency management, present to us the same difficulties as are presented to you who are engaged in developing a similar business in the United States, particularly in those states of the Union which may be termed the "agricultural states."

Similarly, the situations forced upon us in attempting to solve the problems of the conservation of our business are the same as confront the companies in these states. Our old policyholders are scattered, as is our population, over wide areas, making the cost of personal contact for conservation purposes almost prohibitive.

In our struggle for new business we have had the same urge for volume which has been felt by you. There has been an apparent willingness on our part to make almost any sacrifice to further the amount of new business which we can secure and pay for year by year. This has created within our companies, just as it has created within your companies, the problems of forced business, the switching and changing of policies, brokerage business, the handling of the high-pressure agent, and the conserving of business written under high-pressure methods. Such problems, you may say, are not necessarily actuarial problems but they have been forced upon the actuaries of many of our companies in their executive capacity. Particularly is this the case in companies of more moderate size where all of the burdens from every department rest upon the shoulders of a few official heads.

In the underwriting departments we have been experimenting

along with you in an endeavor to find the proper rating for standard and substandard lives and for risks applying for disability benefits or Double Indemnity Accident benefits.

In our actuarial departments we have been confronted with the necessity of approximating rates for Disability insurance; with the adaptation of statistical machinery not only for the valuation of policy liabilities but for accounting purposes, agency records, and many other general-office purposes. We have had to endeavor to foretell the effect of a possible change in the basis of valuation. We have had to face the necessity of meeting the competition of companies much larger in size, much older in business and in experience, the bulk of whose business is obtained from urban territories and the more thickly populated sections of the country.

The problems here enumerated are, you will realize, only a few of those confronting all of us but they are vital and are of the same especial interest to companies operating in Canada and in the middle western states.

It seems to me that the interest of our Canadian members was aroused by an attempt to find a solution to our many problems in common with those companies who are operating under similar conditions and in somewhat similar territories.

There has been in the Institute a very pronounced freedom of discussion, particularly in the open discussion of subjects of current interest which have been a part of the program of this Institute at each of its meetings. It, therefore, seems to me that the credit for the interest which the Canadian companies have shown in the American Institute must be given to the character of our informal discussions. That this interest was active rather than passive may be judged by the fact that company after company has sought membership in the Institute, and today we have members in attendance representing the majority of the progressive Canadian companies.

In thinking over the reason why our informal discussions have been of such value to our members it has seemed to me that there has been one predominant factor in the success which has always attended this part of the program of the Institute. Under the constitution of the Institute duly credited representatives of contributing companies are entitled to all the privileges of the floor in regard to the presentation and discussion of papers and in the discussion of all subjects which are informally presented. The contributions which

these representatives have made to the discussions have been of great moment in determining the success which has always attended the meetings of this Institute since it has been my good fortune to be included among its members.

But I believe you will agree with me that all of the benefit from a meeting is not derived from attendance at the various convocations. The good-fellowship created and the friendships formed with the representatives of the companies attending the meetings have been a very prominent factor of our connection with the Institute; and while a great deal of assistance has been gained by the free and frank statements which have been made on the floor, yet the opportunity of a heart-to-heart talk at the luncheon table, in the corridors before or after the meetings, at our informal dinners, in the rooms of the members, has made the meetings of the Institute of great personal value to me and to all of my friends with whom I have discussed the matter. Not only have we been able to form new friendships among the actuaries of the United States but we also now have the privilege of numbering among our friends many of the executives of the various companies whom it had not been our good fortune to meet in other years.

Naturally the benefit accruing to our Canadian members has not been entirely one sided, and the records of the Institute show many contributions from the Canadian actuaries to the discussions at all of our conventions. In the matters of underwriting practice, of non-medical insurance, of attempted solutions of many other of our common problems, we have given freely all the information at our disposal.

But we are not interested merely in a review of the recent progress of the Institute. Our future activities must be of great moment to us, and it is in contemplation of the work which lies before us that I can see the great advantage which our Canadian members will derive from their association with this body. Many matters will come before us in the near future which will necessitate the close co-operation of all of our members in order that the Institute may accomplish its highest purpose. Many things suggest themselves to me at this time. I shall dwell on only two or three.

A task which may well be undertaken by the Institute during the next few years will be the development of an experience on non-medical business. I think it has been found that the rural agen-

cies of our companies have transacted a greater proportion of their business on the non-medical plan than have the urban agencies. The data which might be furnished by the various member companies of this Institute, particularly of those companies doing largely a rural business, would be possibly more homogeneous in character than data derived from any other source or sources. A joint experience, the results of a collaboration between the United States and Canadian companies, would have a practical value warranting the best efforts of the members of this Institute.

A more difficult task, but one which should not be shirked on that account, is the preparation for a joint experience in connection with Disability insurance. The lack of homogeneous material has been a decided handicap in any such undertaking in the past and will undoubtedly be one of the great difficulties to be faced in any future investigation. It seems to me, however, that an experience derived from a larger number of companies of somewhat equal volume of business, companies doing business with similar classes of people, might offer problems less difficult of solution than an experience derived from a smaller number of larger companies. In any event I believe we would all have a feeling of greater security and satisfaction if we had at our command some disability statistics obtained from the records of companies doing business under somewhat similar conditions and selecting their risks from similar classes of people.

If it were deemed practicable an investigation could be undertaken to differentiate the degree of hazard from a disability standpoint in certain general occupational groups, or in groups showing certain physical impairments. The more companies who can share in such an experience, as long as there has been some measure of uniformity in underwriting methods and some homogeneity in risks undertaken, the better results will undoubtedly be obtained.

Another task which could be assumed with advantage by the Institute would be an investigation into the mortality experience in various groups of impaired lives and in various occupations. I am thinking of an investigation along the lines of the *Medico-Actuarial Investigation*, which was published over fourteen years ago and which dealt with policies issued up to and including the year 1908. The companies associated together in this Institute, both of the United States and of Canada, would be peculiarly well fitted for the

furnishing of data along occupational and physical lines which would be homogeneous in character due to the transaction of business among somewhat similar classes of people, due to certain groups having had a similar set of underwriting rules, both for standard and substandard business, and due to the fact that there would be no preponderance of material from any one company.

These are only some of the problems with which the actuaries must be actively concerned during the next few years. In their solution the Institute must do its full share, and I am of the opinion that constituted as it is, having the active assistance of many contributing companies, along with the trained assistance of its Fellows and Associates, it is peculiarly well fitted to undertake the tasks which are before us.

May I add a further word in regard to the Institute itself. We have from year to year been making some advancement in the accumulation of funds hoping that at some future time we shall have a certain revenue from these funds which will enable us to maintain permanent headquarters. The accomplishment of this aim should ever be before our members because as our Institute grows in size the work which is thrust upon the Secretary becomes a burden too great for an executive of any company to bear for any length of time. With, however, the aid of permanent headquarters and the assistant who would be employed in such an office, a large part of the detail work would be taken from the Secretary's shoulders and the burden would thus be relieved. This will tend to a greater stability in the position of the Secretary of the Institute, which is an end greatly to be desired by any body constituted as we are.

One of the surest ways to increase the revenue of this Institute is to increase the number of the Contributing Members. I know from experience with our own company, which experience I am sure is borne out by the representatives of those contributing companies here today, that the advantage of belonging to the Institute compensates the company many times over for the small annual fee which it is called upon to contribute. In going over the lists of both United States and Canadian companies it has seemed to me that there are many in both countries who would appreciate the advantage of membership in the Institute and whose membership reciprocally would be of real advantage to the Institute.

In conclusion I would like to put before you the same thought

which has been suggested by many of the presidents who have preceded me.

This Institute can be a success only if the members will devote time and attention to its progress and to its affairs. One of the most certain ways to increase the influence is by the presentation of formal papers. I need not disguise from you the fact that this involves work, but this work will redound not only to the advantage of the Institute but also to yourselves in no uncertain way. Your officers and Board would, therefore, express the hope that during the next six months the members of our Institute will give the matter of the presentation of suitable papers their most earnest consideration.

The next and also an important service which our members can render is a regular attendance at the meetings, having prepared themselves in some measure before the meeting for an active participation in the informal discussion on the subjects which are included in the program.

With the earnest and sincere co-operation of the Fellows, Associates, and Contributing Members your officers and Board look forward to the continued prosperity and an ever increasing usefulness for this Institute.

OSCULATORY INTERPOLATION: NEW DERIVATION AND FORMULAS

SECOND PAPER

By W. A. JENKINS

In the author's first paper on this subject[1] osculatory interpolation formulas were developed which were of minimum degrees except in certain cases. The object of this paper is to present a method of correcting the formulas there developed, in which the degrees of one or more of the unmodified coefficients exceed the degrees of the final modified coefficients, so that the resulting formulas are of minimum degrees in all cases; those degrees being actually less than those of corresponding formulas derived by any other method except when the order of contact is 1.

The method consists of the substitution of polynomials, of the same degree as the final modified coefficients, in place of the offending unmodified coefficients.

THE METHOD: A PARTICULAR CASE

For the seventh-difference formula of contact order 1, we obtained the following:

$$u_x = xu_1 + {}_{x+1}C_3\Delta^2 u_0 + {}_{x+2}C_5\Delta^4 u_{-1} + \frac{x^2(x-1)}{60}\Delta^6 u_{-2}$$

$$+ \xi u_0 + {}_{\xi+1}C_3\Delta^2 u_{-1} + {}_{\xi+2}C_5\Delta^4 u_{-2} + \frac{\xi^2(\xi-1)}{60}\Delta^6 u_{-3}\ .$$

u_x, considered as a function of x, is obviously of third degree except for the coefficients ${}_{x+2}C_5$ and ${}_{\xi+2}C_5$. Let us therefore assume a polynomial $\phi(x)$ of third degree or lower such that, if we replace ${}_{x+2}C_5$ by $\phi(x)$ and ${}_{\xi+2}C_5$ by $\phi(\xi)$, the formula will remain osculatory. If we demand that $\phi(x)$ and its derivative take on the same values as

[1] *Record of the American Institute of Actuaries*, XV (1926), 87–98.

${}_{x+2}C_5$ and its derivative when $x=0$ and $x=1$, our assumption will be satisfied. Thus

$$\phi(0)=0 \qquad \phi'(0)=\tfrac{1}{30}$$
$$\phi(1)=0 \qquad \phi'(1)=-\tfrac{1}{20}$$

can be satisfied if

$$\phi(x)=a_0+a_1x+a_2x^2+a_3x^3 .$$

These equations determine uniquely that $a_0=0$, $a_1=\frac{1}{30}$, $a_2=a_3=-\frac{1}{60}$, and that

$$\phi(x)=-\frac{x(x-1)(x+2)}{60} .$$

The final formula of third degree and having contact of order 1 is

$$u_x=xu_1+{}_{x+1}C_3\Delta^2u_0-\frac{x(x-1)(x+2)}{60}\Delta^4u_{-1}+\frac{x^2(x-1)}{60}\Delta^6u_{-2}$$
$$+\xi u_0+{}_{\xi+1}C_3\Delta^2u_{-1}-\frac{\xi(\xi-1)(\xi+2)}{60}\Delta^4u_{-2}+\frac{\xi^2(\xi-1)}{60}\Delta^6u_{-3} .$$

THE METHOD: "IN GENERAL"

Let $\phi(x)$ be the polynomial to be substituted in place of ${}_{x+i}C_{2i+1}=C_i(x)$, $(i=1, 2, 3, \ldots)$.

Considering first the formulas of order of contact $p=2k$, we found the degree of the final coefficients to be $3k+1$. If ${}_{x+i}C_{2i+1}$ needs to be corrected, that is, to be removed to make way for $\phi(x)$, $3k+1<2i+1$. Let

$$\phi(x)=a_0+a_1x+\ \ldots\ +a_{3k+1}x^{3k+1} .$$

Demanding (1) that $\phi(x)$ and its first k odd derivatives take on the same values as do $C_i(x)$ and its first k odd derivatives when $x=0$ and $x=1$, and (2) that the first k even derivatives of $\phi(x)$ and $C_i(x)$ take on the same values when $x=0$—demanding that these conditions be satisfied insures osculation of order $p=2k$.

These demands are coextensive with the following $3k+2$ equations which determine the $3k+2$ coefficients of $\phi(x)$.

$$\begin{array}{ll}
\phi\ (0)=C_i\ (0)=0 & \phi'\ (0)=C_i'\ (0)\\
\phi\ (1)=C_i\ (1)=0 & \phi'\ (1)=C_i'\ (1)\\
\phi''(0)=C_i''(0)=0 & \phi'''(0)=C_i'''(0)\\
\text{———} & \phi'''(1)=C_i'''(1)\\
\cdot & \cdot\\
\cdot & \cdot\\
\cdot & \cdot\\
\cdot & \cdot\\
\cdot & \cdot\\
\phi^{(2k-2)}(0)=C_i^{(2k-2)}(0)=0 & \phi^{(2k-1)}(0)=C_i^{(2k-1)}(0)\\
\text{———} & \phi^{(2k-1)}(1)=C_i^{(2k-1)}(1)\\
\phi^{(2k)}(0)=C_i^{(2k)}(0)=0\ . &
\end{array}$$

These equations reduce to equations (a) as follows:

$$\left.\begin{array}{l}
a_0+a_1+\ \ldots\ .+a_{3k+1}=0\\
a_{2s}=a_{2k}=0\\
a_{2s+1}=\dfrac{C_i^{(2s+1)}(0)}{(2s+1)!}\\
(2s+1)^{(2s+1)}a_{2s+1}+(2s+2)^{(2s+1)}a_{2s+2}+\ \ldots\ .+\\
\qquad\qquad (3k+1)^{(2s+1)}a_{3k+1}=C_i^{(2s+1)}(1)\\
s=0,\ 1,\ 2,\ \ldots\ .,\ k-1\ .
\end{array}\right\}\ (a)$$

When formulas of order of contact $p=2k-1$ are considered,

$$\phi(x)=a_0+a_1x+\ \ldots\ .+a_{3k}x^{3k}$$

and equations (b) determine the $3k+1$ coefficients.

$$\left.\begin{array}{l}
a_0+a_1+\ \ldots\ .+a_{3k}=0\\
a_{2s}=0\\
a_{2s+1}=\dfrac{C_i^{(2s+1)}(0)}{(2s+1)!}\\
(2s+1)^{(2s+1)}a_{2s+1}+(2s+2)^{(2s+1)}a_{2s+2}+\ \ldots\ .\\
\qquad\qquad +(3k)^{(2s+1)}a_{3k}=C_i^{(2s+1)}(1)\\
s=0,\ 1,\ 2,\ \ldots\ .,\ k-1\ .
\end{array}\right\}\ (b)$$

Values of $C_i^{(2s+1)}(0)$ are tabulated with a recursion formula in the author's first paper.[1] For $C_i^{(2s+1)}(1)$ we have Table I.

[1] *Op. cit.*, pp. 94–95.

TABLE I

VALUES OF $C_i^{(h)}(1)$

i \ h	1	2	3	4	5	6
0	1	0	0	0	0	0
1	$\frac{1}{3}$	1	1	0	0	0
2	$-\frac{1}{20}$	$-\frac{1}{12}$	$\frac{1}{4}$	1	1	0
3	$\frac{1}{105}$	$\frac{1}{90}$	$-\frac{1}{15}$	$-\frac{1}{6}$	$\frac{1}{6}$	1

$$C_i^{(h)}(1)=\frac{1}{2i(2i+1)}\{h(h-1)C_{i-1}^{(h-2)}(1)+2hC_{i-1}^{(h-1)}(1)+(1-i^2)C_{i-1}^{(h)}(1)\}.$$

It was not necessary to demand that the derivatives of $\phi(x)$ reproduce those of $C_i(x)$ as certain other arbitrary values would serve. However, these values were deemed best.

RESULTS

By the method presented we obtain osculatory formulas whose degrees depend only upon the order of contact, as follows:

Order of contact	1	2	3	4	5	6	. . . . .
Degree	3	4	6	7	9	10	. . . . .

By Dr. Reilly's generalization of Sprague's method of development,[1] the degrees are the following:

Order of contact	1	2	3	4	5	6	. . . . .
Degree	3	5	7	9	11	13	. . . . .

By Lidstone's method[2] the degrees are identical with those obtained by Sprague's method, or higher. It is seen that the proposed method produces formulas of minimum degrees in all cases, those degrees being equaled by other methods of development only when the order of contact is 1.

In Table II are given a few of the polynomials to be substituted in the formulas under consideration, as developed by the author.[3]

[1] John F. Reilly, *Record of the American Institute of Actuaries*, XIII (1924), 4–22.

[2] *Ibid.*, XIV (1925), 12–20.

[3] *Op. cit.*, p. 96.

${}_p\phi_i(x)$ is used to replace the unmodified coefficient ${}_{x+i}C_{2i+1}=C_i(x)$, when contact of order p is desired, provided that the degree of ${}_p\phi_i(x)$ is less than that of $C_i(x)$, i.e., $2i+1$.

TABLE II

VALUES OF ${}_p\phi_i(x)$

Order of Contact	$i=2$	$i=3$
$p=1$	$\frac{-x(x-1)(x+2)}{60}$	$\frac{x(x-1)(x+3)}{420}$
$p=2$	$\frac{x(x-1)(x^2-2x-2)}{60}$	$\frac{-x(x-1)(2x^2-3x-3)}{420}$
$p=3$		$\frac{x(x-1)(13x^4-57x^3-52x^2+144x+144)}{20,160}$

A few of the corrected formulas, whose degrees are now a minimum, are as follows. The first terms omitted are those of Everett's interpolation formula. The order of contact is p: the final terms involve $\Delta^{2n}u$.

$$\left.\begin{matrix} p=1 \\ n=3 \end{matrix}\right\} \left\{\begin{matrix} \ldots\ldots +{}_1\phi_2(x)\Delta^4u_{-1}+\frac{x^2(x-1)}{60}\Delta^6u_{-2} \\ \ldots\ldots +{}_1\phi_2(\xi)\Delta^4u_{-2}+\frac{\xi^2(\xi-1)}{60}\Delta^6u_{-3} \end{matrix}\right.$$

$$\left.\begin{matrix} p=2 \\ n=3 \end{matrix}\right\} \left\{\begin{matrix} \ldots\ldots +{}_2\phi_2(x)\Delta^4u_{-1}+\frac{x^3(x-1)}{60}\Delta^6u_{-2} \\ \ldots\ldots +{}_2\phi_2(\xi)\Delta^4u_{-2}+\frac{\xi^3(\xi-1)}{60}\Delta^6u_{-3} \end{matrix}\right.$$

$$\left.\begin{matrix} p=1 \\ n=4 \end{matrix}\right\} \left\{\begin{matrix} \ldots\ldots +{}_1\phi_2(x)\Delta^4u_{-1}+{}_1\phi_3(x)\Delta^6u_{-2}-\frac{x^2(x-1)}{280}\Delta^8u_{-3} \\ \ldots\ldots +{}_1\phi_2(\xi)\Delta^4u_{-2}+{}_1\phi_3(\xi)\Delta^6u_{-3}-\frac{\xi^2(\xi-1)}{280}\Delta^8u_{-4} \end{matrix}\right.$$

$$\left.\begin{matrix} p=2 \\ n=4 \end{matrix}\right\} \left\{\begin{matrix} \ldots\ldots +{}_2\phi_2(x)\Delta^4u_{-1}+{}_2\phi_3(x)\Delta^6u_{-2}-\frac{x^3(x-1)}{280}\Delta^8u_{-3} \\ \ldots\ldots +{}_2\phi_2(\xi)\Delta^4u_{-2}+{}_2\phi_3(\xi)\Delta^6u_{-3}-\frac{\xi^3(\xi-1)}{280}\Delta^8u_{-4} \end{matrix}\right.$$

$$\left.\begin{matrix} p=3 \\ n=4 \end{matrix}\right\} \left\{ \begin{matrix} \cdots\cdots +{}_3\phi_3(x)\Delta^6 u_{-2}+\dfrac{x^4(x-1)(265x-409)}{40{,}320}\Delta^8 u_{-3} \\ \cdots\cdots +{}_3\phi_3(\xi)\Delta^6 u_{-3}+\dfrac{\xi^4(\xi-1)(265\xi-409)}{40{,}320}\Delta^8 u_{-4} \end{matrix}\right.$$

These are the only formulas requiring correction which do not involve tenth differences.

AN ALTERNATE METHOD

The problem of developing such formulas as those above can also be solved by starting afresh and assuming: (1) that

$$u_x=\sum_{i=0}^{m-1} {}_{x+i}C_{2i+i}\Delta^{2i}u_{1-i}+\sum_{i=m}^{n}\phi_i(x)\Delta^{2i}u_{1-i} + \sum_{i=0}^{m-1} {}_{\xi+i}C_{2i+1}\Delta^{2i}u_{-i}+\sum_{i=m}^{n}\phi_i(\xi)\Delta^{2i}u_{-i}\,,$$

where $m=p-\left(\text{the lower integer of }\dfrac{p-2}{4}\right)$ and p is the order of contact; (2) the continuity of the first p derivatives at $x=0$ and $x=1$ when the formula is applied to successive unit intervals; and (3) the satisfying of one extra restriction on each $\phi_i(x)$, except the last, for each odd derivative made continuous.

If these extra restrictions are such that the odd derivatives of $\phi_i(x)$ reproduce the odd derivatives of $C_i(x)$ at the ends of each interval, the formulas obtained are identical with those developed by the first method presented. This second method is, however, far more cumbersome than the first.

DISCUSSION OF PAPERS READ AT THE PREVIOUS MEETING

PREMIUM RATES AND SURRENDER VALUES: THEIR RELATION TO COMPANY POLICY

By C. O. SHEPHERD

MR. JAMES S. ELSTON:

Mr. Shepherd's paper is very valuable. It gives a lot of new material that has not previously been available in published form, and throws much further light on old methods.

I think that the subject of the calculation of premiums might be clarified somewhat if we consider that the use of formulas such as Mr. Shepherd gives leads us to values involving, to the best of our ability, the influences of mortality, interest, and expense. These values may be considered as first approximations to the desired premiums. Then with these as a basis we work history assumptions to obtain the asset shares or "natural reserves," as Mr. Shepherd very aptly calls them. This process introduces the effects of lapsation and of surrender values and possibly further refinements. A study of the results probably leads to slight modifications of the premiums we first obtained. The new results are closer approximations to the desired results and may be again used to obtain still better results if desired. This is the mathematical process of using a first approximation to obtain a more accurate second approximate value.

It may be surprising to some but it is probably true, as Mr. Shepherd notes, that in many cases surrender values are greater than the natural reserves.

What he says about deficiency reserves and the indirect regulation of premium rates through them certainly furnishes food for thought.

Our methods in general agree with those Mr. Shepherd outlines, but of course we use different factors, in fact, I think in every case. The only one that I feel inclined to question is the possible use of

an ultimate rate of mortality of 90 per cent of the American Men Ultimate Table. Of course I understand that Mr. Shepherd uses this only for the purpose of illustration, but some may imply that this is a proper rate. Undoubtedly, the ultimate mortality is well under the American Men Ultimate, in most companies at least, but it is my belief that much of the difference is due to the mortality in the sixth, seventh, and eighth policy years, which is very much under the table. In many companies the exposure in these years is sufficient to have considerable influence on the total ultimate mortality. I am not convinced that there will be any considerable saving on which it is safe to count in the real ultimate mortality, as, for instance, after the tenth year. I doubt whether it is advisable to assume any savings after attained age seventy.

Mr. Shepherd makes note of the possible modification in premium calculations that might be introduced on consideration of the average size of policies, but apparently does not make much use of the differences as between forms of policies, calling attention rather to the possible incongruities in the rates between different forms of insurance at the same ages. Of course, it is necessary to eliminate all the incongruities we can, but it does seem to me that the average size of policy should be allowed for in the calculation, if for no other reason than to indicate to the company, itself, which plans are the most profitable, and which the least, and therefore which should be most encouraged. Some discussions of this subject seem to discountenance consideration of the average size, because it would be discrimination to charge comparatively higher rates on a $1,000 Ordinary Life policy than on a $10,000 Ordinary Life policy at the same age. As I see it, the question is not that, but whether allowance shall be made for the fact that all the Ordinary Life policies issued at age twenty average perhaps $2,000, although those issued at age fifty average nearly $5,000, while many of the expenses of issuing them are as much per policy on the age-twenty policies as on the age-fifty ones. The same problem enters in studying the comparative expenses per policy on Ordinary Life, Ten-Year Endowment, and Ten-Year-Term policies issued at the same age.

MR. ROBERTSON G. HUNTER:

Mr. Shepherd's paper may be discussed under three general headings: (1) the basis of mortality, interest, and expense for pre-

miums; (2) the effect on the premium of a variation in these factors; (3) the addition to the premium due to legal reserves or surrender values being in excess of the asset share or "natural reserve," as he calls the accumulation of effective premium. I shall confine myself to a discussion of the first and third headings.

In the discussion following Mr. Cammack's and Mr. Arthur Hunter's papers appearing in the *Transactions of the Actuarial Society* and *The Record* of the Institute, objection was made to the use of the American Men Table as being an average table compiled from the data of a large number of companies, of which some would naturally show a lower mortality and some a higher than the average.

Since that time there has been a marked improvement in mortality as shown by the experience of individual companies, which leads me to believe that the American Men Table is no longer an average but a maximum table and as such would now be acceptable to those actuaries who previously liked it not.

The author assumes a mortality of 90 per cent of the American Men Table. Such an assumption seems to be justified by recent mortality experience. The experience of the Equitable of Iowa for the years 1920-24, inclusive, on all business in force more than five years was 67 per cent of the American Men Ultimate.

However, because of the objections that have been made to the use of the American Men Table and the desirability of agreeing upon a table other than the American Experience, I would prefer the use of the American Men Table unmodified as the basis for computing premiums.

The interest trend will always remain difficult of prophecy. The reduction of the rate of interest within the last three or four years has undoubtedly made us more conservative than we would have been in 1920 or 1921. Nevertheless, the use of $4\frac{1}{2}$ per cent interest appears to give a reasonable margin of safety. Possibly I might not be so optimistic if it were not for the fact that in the mortality basis there is an ample margin of safety, and, as Mr. Shepherd well points out, "the test of boldness or cautiousness does not lie in the standard of mortality or rate of interest assumed, but in the premium rates actually adopted."

The assumption made by the author of $4\frac{1}{2}$ per cent for the first twenty years and $3\frac{1}{2}$ per cent thereafter seems unduly conservative

and unnecessarily complicated. There is also the question of whether such an assumption is not discriminatory in its effect upon the premiums at the young and old ages.

In discussing expense assumptions the author is of the opinion that effect should be given to the variation in the size of the policy according to age and plan of insurance. In a previous part of his paper he discards the more logical assessment of the general expenses as a constant per policy rather than a constant per thousand as being against the principle firmly held in this country that the small policyholder should pay proportionately the same rate as the large policyholder.

Here, it would appear, are two contradictory opinions. If premiums are loaded according to the size of the policy, then we are in fact charging a larger rate per thousand for the small policyholder than we are for the larger policyholder, while at the same time penalizing the larger policyholder who happens to fall in the small-sized group. From a competitive standpoint the non-participating company cannot afford to take cognizance of the added expense at the younger ages due to the smaller average size of policy since the participating companies do not, so far as I know, make any allowance for this factor in their dividend formula.

In order to make this discussion as concrete as possible a table is appended (Table I) showing the premiums computed recently for the Equitable of Iowa compared with the author's gross premiums without adjustment. The bases of our premiums are as follows:

1. Mortality: American Men Select
2. Interest rate: $4\frac{1}{2}$ per cent
3. Expense: Medical inspection fee, $3.00 per thousand
   First-year overhead, $8.00 per thousand
   Renewal overhead, $1.25 per thousand
   Premium tax, $2\frac{1}{2}$ per cent of the premium
   First-year commission, 50 per cent graded
   Second-year commission, 15 per cent graded
   Third to tenth year, 7 per cent graded
   Tenth year and thereafter, 2 per cent

You will notice that we are converts to the opinion that the overhead expenses are due very largely to new business and should be so assessed, but such distribution makes little difference in the

premium from a distribution that throws more of the expenses against renewal premiums.

The premiums computed on the bases in Table I are shown in the exhibit without any modification or without any addition for contingency. The premiums actually charged by us were equated to the net premiums on the American $3\frac{1}{2}$ per cent basis with or without a loading. Below age forty the computed premiums were less than the net American $3\frac{1}{2}$ per cent premiums, being as much as 11 per cent less at age twenty.

TABLE I

Age	Ordinary Life		20-Payment Life		20-Year Endowment	
	Shepherd's Gross Premiums Unadjusted	Equitable of Iowa Gross Premiums	Shepherd's Gross Premiums Unadjusted	Equitable of Iowa Gross Premiums	Shepherd's Gross Premiums Unadjusted	Equitable of Iowa Gross Premiums
20.....	13.30	12.00	19.88	17.24	39.87	39.73
30.....	17.40	16.20	24.62	22.12	40.42	40.32
40.....	24.62	23.87	32.07	30.14	42.76	42.80
50.....	37.47	37.70	44.07	43.22	49.44	49.96
60.....	60.54	62.40	64.72	65.49	65.57	67.33
65.....	78.41	80.35	81.05	81.93	80.08	82.25

Mr. Shepherd has in the last part of his paper placed before us a rather baffling problem, one I think that we have treated in the past with slight consideration.

We have always recognized that the natural reserves have been less than the full level premium reserves during the early years of a policy's history and have tried to correct this deficiency by devising the modified preliminary-term plan of valuation. Now Mr. Shepherd comes forward to show that the natural reserves accumulated from his standard premiums are less to a considerable degree and for a longer period of time than the legal reserves on the Illinois standard.

Knowing that few companies, if any, could stand such a drain on their surplus by reason of this considerable deficiency for a long period of time, he computes the extra premium that would be required to bring his hypothetical company to technical solvency at the end of ten years. Is it not a strange train of circumstances that compels an addition to the premium rates adequate to meet policy

obligations for no other purpose than to comply with an artificial standard of solvency? Surely the solution of this problem is not in increasing the premiums but in taking steps to amend the valuation laws to conform more nearly to the assumptions made in the computation of the premiums.

The addition of a loading to a premium to cover the payment of surrender values in excess of the natural reserves is not a wholesome practice. It is generally agreed that an insured who discontinues his policy should be charged with the expenses that he creates in the procuration and administration of the business and should receive no more than the natural reserves accumulated from the effective premiums he has paid.

The author of the paper subscribes to this principle, but because of stress of competition, and to a much less degree statutory laws, he indicates that we cannot hold to this principle but must give surrender values in excess of the natural reserve. Therefore, in order to compensate for this loss he distributes it over the insured as a body by loading the premium accordingly.

I am unable to see the necessity or wisdom of such a practice. There are many companies that are not paying surrender values in excess of the natural reserves. Mr. Morris, in his discussion of Mr. Cammack's paper, showed that if the lapse rate and surrender values applicable to his company were taken into consideration, the natural reserves would be greater than those brought out by Mr. Cammack.

The addition, therefore, of an extra premium to meet competitive surrender values makes it impossible to meet competition on premium rates. Of the two horns of the dilemma, I would choose the competitive premium rate and make surrender values fall within the natural reserves so far as the laws permitted.

MR. H. W. BUTTOLPH:

I rather regret that Mr. Shepherd in a paper of so much interest and of such value should have introduced a controversial question in what I believe he terms the worthlessness of the American Experience Table. I am not going to take the bait and indulge in any argument on that. I will simply content myself with saying that those of us who still cherish some kindly thoughts in regard to our old friend may get a little satisfaction out of the statement later in the

paper by Mr. Shepherd to the effect that all mortality tables are worthless and always will be because they are out of date before they can be published.

Neither am I going to argue the other point which he so temptingly holds out to me—the question of reduction in premium rates. There, again, I will content myself with one statement to the effect that many of us still believe that the chief business of life insurance companies is to pay claims and not to pay dividends or reduce premium rates.

The one point that I really want to emphasize is brought out by Mr. Shepherd in the statement: "I have been particularly anxious not to detract from the value of my investigation by overconservatism."

There is, again, on page 10 another welcome caution against too much optimism regarding some of the theory on which his tables are based. This, I am glad to notice, appears again on page 21, where he cautions the casual reader against underestimating the unfavorable effect of a higher mortality or a higher first commission.

There is one thing that seems to me a little confusing. Mr. Shepherd asks the question: "May not the mortality at the younger ages be less than 80 per cent and the mortality at the higher ages more than 80 per cent?" The question was asked in a form suggesting the answer "Yes"; and yet he goes on in the same paragraph to state that "the more recent improvement appears to have been more evenly spread [referring to the improvement in mortality] and it is safe to assume that the older ages have benefited materially."

I, for one, should like to have had some definite reference to the experience on which he bases that statement of what will be a safe assumption. I fully realize that our company has not enough experience at the older ages to warrant me in expressing any opinion on the subject at all, based on our own experience; but I have never seen any reference to an experience which would make me willing to echo that statement of the author's.

One point touched on by Mr. Hunter is the statement in the paper to the effect that the deficiency reserve is in effect a regulation of premium rates. That, of course, is perfectly true; but it is also the maintenance of the principle of a prospective valuation. Whatever table of mortality or whatever interest rate is adopted for the determination of the tabular reserves which this particular part of

the paper seems to advocate, it seems to me that the abolition of deficiency reserves emphasizes a present inconsistency. We now say in the matter of valuations there need be no attention paid to expense; there is no deficiency reserve charged if the gross premium equals the net. If we adopt Mr. Shepherd's suggestion we go a step farther and say: "Not only need we pay no attention to expenses in the determination of reserves but we need pay no attention to so-called 'net premiums.'"

On page 26, Mr. Shepherd, in referring to surrender values, speaks of the minimum permitted by law, and he shows that in some instances the minimum permitted by law is more than what he terms the "true reserve." It does not seem to me that those instances are of sufficient importance to cause us much worry; and certainly we do not follow the theory to which he calls attention when he says: "The terms of the option to withdraw should be such that its exercise will not be unfavorable to the company. . . . ." All of us, I think, admit the truth of that from a theoretical point of view; but an investigation of practice of companies generally on the question of surrender values shows that they fall far, far short of complying with that theory, even to the limited extent permitted by law.

There is so much of value in this paper that I rather hesitate to call attention to what seems to be an omission. The reference in the paper to lapses and the effect of lapsation is, in my judgment, briefer than the subject warrants. Mr. Hunter's suggestion, in which I fully concur, as to the propriety of assessing a larger amount of the general expense against the first year than Mr. Shepherd's formulas contemplate increases the importance of this question of lapsation. If a reasonable rate of lapse be taken into consideration, it will materially affect the so-called "true reserves" which Mr. Shepherd calls attention to; and the assessment against first-year business of a larger proportion of the general expense of the company will emphasize that condition very materially.

MR. W. A. P. WOOD:

There are two or three points in connection with Mr. Shepherd's paper which have occurred to me to which I should like to refer.

I think that Mr. Shepherd is to be commended for putting before the Institute a paper which in some ways is of such practical

importance. He has not taken for granted, or rather he has not claimed, that what is assumed in regard to his premiums should be taken as an index of what companies should employ in the calculation of non-participating premiums.

These standard assumptions, as he calls them, evidently are not strictly correct, because if you will refer to page 13 of *The Record*, you will find that under the "Gross Premiums 'Standard Assumptions'" for age sixty-five, the Life Twenty-Payment premium is higher than the Twenty-Year Endowment premium—the Life Twenty-Payment premium being $81.05 and the Twenty-Year Endowment premium being $80.08, being about $1.00 less for the Twenty-Year Endowment. This arises, of course, from the fact that he has assumed 70 per cent first-year commission in the case of the Life Twenty-Payment premium and 50 per cent in the case of the Twenty-Year Endowment. This should not be taken as detracting from the value of the paper at all. It merely shows that the particular assumptions which he has made here should be modified to some extent.

Mr. Shepherd has referred in the paper to the variation in mortality and has said that probably the betterment in mortality is now somewhat approaching uniformity as between different ages, or something to this effect. He infers that companies now experiencing better mortality than the American Men Select would probably show this improvement for the different ages at entry, not necessarily in exactly the same proportion but without any very great variation.

I am afraid, judging by the experience of our own company, that this proportionate improvement is hardly correct. I have had occasion to make some investigation of our mortality in different sections —the United States, Great Britain, and a couple of the provinces in Canada—and I have found that our mortality experience has been considerably better than the American Men Select. In fact, on the whole it is better than the Canadian Men Select, a table obtained at the same time. But that improvement is more noticeable for the younger ages at entry. In the case of the United States for ages forty-five and over at entry, the experience has been slightly unfavorable, but for ages under forty-five it has been quite favorable. In Canada for ages forty-five and over at entry it has been as favorable as for ages forty-five and under; and in Great Britain for ages

forty-five and over at entry it has been relatively more favorable than for ages forty-five and under. But to repeat, as far as the United States goes, it has been less favorable for ages forty-five and over. For our combined experience of all sections for ages sixty and over, it has been quite unfavorable.

So I think, then, if you are going to make certain departures from the American Experience Table in your assumptions in regard to mortality, it would be advisable to make provision for mortality which is relatively heavier for the older ages at entry.

Mr. Shepherd has, I think, done a good service for the Institute in calling our attention to the high cash values in the policies compared with the true reserves or the amount to which the net premiums will accumulate. He has shown us, I think, the folly of having values which are greater than the funds which would be accumulated. Most of us should profit from this in getting up new premiums and guaranteed values.

There is another point to which I might refer which has not been mentioned in connection with this matter, and that is that if a company is transacting both participating and non-participating insurance, I think that some recognition should be made of the relative net cost between these two branches of the business. It does not seem to me to be fair to the policyholders who do not always know the different plans, nor fair to the agents who are selling the business, to have any great discrepancy between the net cost in the participating and non-participating sections, especially if the plan in which the net cost to the policyholder is the lower is the one which yields the agent less renumeration and which he will not probably place before the applicant unless he is forced to do so.

I just want to sum up the points which have occurred to me in connection with Mr. Shepherd's paper. I think that these variations in assumptions to which he has called our attention are very valuable to us in getting us to think of the relative importance of expenses, interest, and mortality. In placing the matter before us in the way that he has, without saying definitely what the premiums should be, but just making different suggestions and laying the whole thing open, he has rendered a service to the Institute.

MR. JAMES F. LITTLE:

I am not rising to discuss Mr. Shepherd's paper; I wanted to mention one rather small point that came out in Mr. Robertson

Hunter's remarks, where he spoke of assessing a part of the expenses per policy rather than per amount of insurance.

That point came up with me some years ago, and I reasoned somewhat after this fashion: The smaller policies are very much more numerous than the larger policies. The expense rate of a company is kept within practical limits only by having a large number of policies on its books, and the smaller policies as an aggregate might contribute toward bringing down expenses as much as or more than do the larger policies.

I tried to check this up. I could not find companies suitable in the United States but I was aware of the fact that there were some English companies that do business among the people in England taking smaller policies; and there are one or two that largely confine themselves to where the larger policies come from.

I took one company whose average policy was about $1,200, and another company that had an average of about $6,000, so that one company's average was about five times that of the other. Naturally the company with the larger average had a much smaller number of policies on the books. I found that the expense rate of the company with the larger policies was greater than that of the company with the smaller policies. This result seems to afford justification for the theory that the smaller policies in the aggregate do contribute toward a feasible expense rate as much at least as the larger policies.

The conclusion seems to follow that a uniform rate of premium for large and small policies is not only very convenient in practice but justified in theory, and, of course, with a uniform premium would go, in the case of a participating company, a uniform rate of dividend.

So far as the extremely large policies are concerned, which might be in a class by themselves, we have this fact to remember: That up to the present, in the United States at least, these policies have always shown a noticeably higher rate of mortality than the smaller contracts. So that at the present time there seems to be every justification, theoretical as well as practical, for using a uniform rate of premium for policies irrespective of size.

MR. FRANKLIN B. MEAD:

This is a noteworthy, important, and instructive paper. I only wish I had had the opportunity to prepare a written discussion so that I could have said what I should like to say.

We have been in the midst of confusion for about twenty years by the use of a fictitious table of mortality. I think it has confused the issue and retarded the development of actuarial science and of actuarial students, young and old practitioners alike, more than any other feature. We have been so progressive in America that it is an anomaly that we should be the least progressive of all nations when it comes to a table of mortality.

I would have given anything if I had had the advantage twenty years ago of the principles and the advantages of the use of such a paper as we have before us today. We struggled hopelessly to determine premium rates on a basis of fictitious net premiums. Preliminary Term companies were even attempting to load Preliminary Term premiums and give consistent premium rates, not to speak of the inconsistency of loading by a constant percentage throughout the American Table on a net level basis

I might say for myself, personally, that I have been in favor of a new table for the last twelve years. As early as 1915 in the Institute of Actuaries I called attention to the dilemma of the Preliminary Term companies as regards the recent reductions of rates that were taking place at that time, when one of the larger non-participating companies for the first time adopted the premium rates which were below the net premiums of the Preliminary Term companies. Then I allowed some of my friends to prevail upon me that a new table might lead to competition and reduce premium rates. And then the influenza epidemic came along, and we were glad indeed to allow things to drift. The situation came up again a year ago. Then I allowed myself to have the issue clouded, so far as I was concerned, by the terrible competition that took place in connection with group insurance, which grew worse, when finally the group companies, their actuaries and executives and agency men, had to call in one of the commissioners to save them from themselves.

That condition, however, is not parallel. Group insurance has its temptations because rates are assessed in individual cases; one group man told me a number of years ago that he purposely made rates lower than he had ever seen before to see what would happen; they were always met by the other companies.

I allowed that to confuse the issue further.

I believe that when actuaries come to compute premium rates

which are fixed, that take care of the whole spread of business and which are to last for a long time in the future, they are going to use other methods, and be sane and conservative.

As far as the American Experience Table is concerned, we have an erroneous affection for that table. In fact, it could not very well be worse but it was probably the best we had. You could draw with a free hand and produce offhand twenty or thirty tables within thirty minutes which would give better results and give the same average rate of mortality as the American Experience Table and give approximately the same reserves. The question, of course, is to secure equity between the different ages; and the American Experience Table is about the most inequitable table that exists, especially at the present time.

In this connection I might say a few words with reference to our company's experience based on the American Men Table. Our own mortality from organization up through the year 1923, including the influenza epidemic and excluding reinsurance, was 85 per cent of the American Men Select Table. We had a heavy incidence of mortality in the early years due to large policies, upon which claims were incurred after the financial *débâcle* of 1921. Our mortality the first year was 104 per cent; the second year, 93 per cent; the third year, 91 per cent. The ultimate percentage was 72 only, making a general average of 85 per cent. However, our experience by age groups paralleled very nicely and satisfactorily the American Men Table. I might say that we had always suspected a heavy mortality above age sixty but we found it was entirely satisfactory.

Our mortality in recent years has been more favorable. In 1925 it was only 64 per cent of the American Men Select on our own direct business. It was even less in 1924 and even less this year, slightly. I might say that the mortality varied very little from 64 per cent according to age groups and duration. On our business more than five years old it was 65 per cent; it averaged the first three years about 65 per cent too. The other durations, in the fourth and fifth years, were more favorable. From fifteen to thirty-nine it was 67 per cent as compared with 64 per cent for all ages; forty to forty-nine, 67 per cent; fifty to fifty-nine, 59 per cent; sixty and over, 61 per cent—62 per cent nearly. And this low mortality has resulted from a very free selection of risks.

At one time I was rather favorably inclined toward taking

cognizance of small policies and assessing premiums on a higher basis of computation because of the small policies at younger ages but I am not now so much inclined to do that, because we get a much more favorable mortality on smaller policies than we do on the large one at the higher ages.

We have in our own company access to quite an experience on large policies. I might say that whereas our own mortality on our own direct standard business was 64 per cent in 1925, on our standard reinsurance business, large policies, it was 110 per cent. It has been running consistently that from year to year until this year, averaging 106 per cent through 1923. The incidence is satisfactory as regards age groups but we have had a heavy first-year mortality, 164 per cent, showing the adverse selection incident to large policies. This year the mortality is much heavier as the mortality for all durations is about 175 per cent; that is, on standard reinsurance, due to the incidence of large claims.

I am practically in accord throughout with Mr. Shepherd's technique and will talk for the most part upon the significance of his paper. I will say, however, that first-year home-office overhead is very heavy and should, therefore, be taken care of as a first-year proposition as is done with first-year commissions.

Now a word with reference to the false position in which we find ourselves at the present time in reference to deficiency reserves, due to the fact that we have a fictitious net premium in the American Experience Table. Many companies are charging at the younger ages premium rates that are down to that net premium. Naturally, then, it is not a true net premium; it is a fictitious net premium which includes premium loading, and that premium loading should be taken into consideration in determining the position of a life insurance company.

Right is right, and it is only a question of time when we will face the facts and the idiosyncracies will be removed from our laws. As a matter of fact, in most states we can use the American Men Table at once, if we desire to do so as a basis of valuation. Take the Indiana and the Michigan statutes, for instance. The Michigan statute (and the Indiana statute is practically equivalent) states that the minimum standard of valuation is the American Experience with 4 per cent interest or that some other standard giving higher reserves may be used. Now, higher reserves cannot mean

higher reserves in every individual instance. It must be reserves in the aggregate, because there is no existing table of mortality that can be found or none that can be constructed which will give higher reserves in every instance than any other mortality table that exists or that can be constructed.

The standard provision acts require that you must state in your policy contract the basis of premium rates, the basis of mortality, and the interest rate, and that is all that is said on that subject.

Many tests have been made with the American Men Table for the average distribution of business; any distribution you can find in practice will give higher reserves in the aggregate than the American Men Table, and it is therefore a "higher standard," which is the term used in the Indiana law, the minimum standard in that state being the Actuaries 4 per cent or the American Experience 4 per cent.

I think no attorney-general or commissioner will stand in the way when a company presents the matter to them in that light. We are too inclined to look at reserves narrowly because of cash values. The main thing we are considering is solvency, which has to do with measuring aggregate liabilities with aggregate assets. Cash values will take care of themselves.

There is possibly some objection to the question of extended insurance with reference to the new table. However, if you take a maximum surrender charge there will not be an undue term; and most of the companies, besides, are using automatic premium loans. When you grant automatic premium loans it will be found that there is very little extended insurance granted.

I want to call attention to a remarkably apt statement made by Mr. Shepherd. He has said with true inspiration: "I have been particularly anxious not to detract from the value of my investigation by overconservatism. After all, the test of boldness or cautiousness does not lie in the standard of mortality or the rate of interest assumed but in the premium rates actually adopted." After all, that is the true test of conservatism; we should always keep that in mind.

New mortality tables are bound to come, and we might as well cast aside our old machinery. I know at least two companies that are about to adopt the New Table and there are more to follow. Most companies have been using the New Table in the computation of their premium for many years.

I am reminded of a dialogue that I heard in vaudeville a number of years ago. One of the men was an Irishman named Kelly, who was very deaf, and the interlocutor was having a hard time making him understand. They happened to be in the presence of Niagara Falls. The interlocutor said to Kelly: "Aren't those Falls magnificent? Aren't they grand?" And Kelly would say, "Eh?" He would repeat it and would make a few more expressions as to the remarkable quality of those Falls and their impressiveness. Finally Kelly looked up and said: "But what's to prevent it?"

MR. R. A. HOHAUS:

Mr. Mead's remarks compel me to read an excerpt from a very interesting discussion by George King in 1906, which I found in some old magazine files, of the proposed New York expense margins which were then being recommended for adoption.

> Therefore, if there were a legal enactment restricting expenses in the way set forth in these two questions, there would be a strong inducement offered to Insurance Companies to retain faulty tables, and this would be detrimental to life assurance.

MR. C. O. SHEPHERD:

Anyone who has seriously studied a problem of this magnitude can realize how welcome and helpful criticism is. I have been very much interested in these discussions.

My principal object was to show the effect of *variation* in premium factors, and while the opportunities for several interesting digressions were too great to be resisted, my viewpoint was quite different from what it would have been if I had merely been trying to arrive at current office premiums. In the latter case, for example, I feel safe in saying I should not have mentioned an 80 per cent A.M. table of mortality. For the purpose of studying mortality variation, however, assuming that 80 per cent A.M. mortality at the younger ages is not entirely in the realm of fancy, it did not seem necessary to introduce complications in order to avoid the use of that same percentage at the higher ages. I did not intend to convey the impression that I believed there was any experience available to support the assumption of 80 per cent A.M. mortality at higher ages. In the recent blood-pressure investigation of the Association of Life Insurance Medical Directors and the Actuarial

Society, the mortality on all business submitted was 72 per cent of the A.M. Select and Ultimate for all ages combined and 81 per cent for ages at issue fifty and over. Obviously, the margin on a percentage basis at older ages was not as great as at younger ages, but I did feel that if we had experienced as low a rate as 72 per cent for all ages and 81 per cent for ages over fifty, we might expect something less than 100 per cent for ages considerably above fifty —even ultimate mortality.

Mr. Elston believes that to a considerable extent the improvement in mortality, which we have noted, is an improvement for the sixth, seventh, and eighth policy years; in other words, that we are enjoying the effects of our selection for more than five years. I recall that Mr. Hutcheson drew the same conclusion when he presented the mortality experience of his company. A very large part of the experience contributed to the blood-pressure investigation was confined to the first eight policy years, and if Mr. Elston is correct, we should be rather careful in assuming that the results indicate an improvement in ultimate mortality. Mr. Wood mentioned an experience which shows no improvement in the mortality in the United States above age forty-five. If this experience is typical, it is hard to reconcile with the results shown by the material in the blood-pressure investigation.

I am much encouraged to find that Mr. Buttolph and Mr. Hunter feel that a large proportion of general expenses should be apportioned to the first policy year. I thoroughly agree with them. I hesitated to base my computations on such an assumption because, in the absence of facts, I should have had to assume an arbitrary rate which might have provoked useless controversy. Furthermore, my tables lend themselves very well to a study of *variation* in expenses according to either formula. Personally, I feel that a ratio of 6:1, such as Mr. Hunter has used, is much nearer the truth than a 1:1 ratio, such as I have used. As someone has pointed out, the value assigned to this ratio does not have any striking effect on premium rates although it does have a decided effect on natural reserves.

Several of the speakers have commented on the fact that the tables which I have presented demonstrate that most of us are using surrender values in excess of natural reserves. Unfortunately, the demonstration is unnecessarily feeble. If I had thrown a larger

proportion of general expenses into first year, where I believe it belongs, natural reserves in the earlier policy years would have been considerably smaller and the excess mentioned above correspondingly greater. If our theory of apportionment of general expenses is correct, I doubt if there is a company in the United States that is not paying surrender values in excess of natural reserves. I grant that it will be difficult to bring about a change in the face of the obstacles which other speakers have pointed out. I do not believe, however, that we should resign ourselves to the situation. There seems to be a general agreement among our profession as to the fact, but there is surprisingly little evidence of it in print, and outside of our profession it is practically unrecognized. I believe we should see that the facts become better known.

Mr. Hunter slightly misunderstands my attitude on this question. I heartily agree with him that the correct solution is to modify surrender values, and I am disposed to go just as far in that direction as possible. But if surrender values cannot for legal or practical reasons be brought within natural reserves, then I think it is in order to increase our premiums. The solution of the problem lies in a general recognition of the facts and modification of legislative requirements. If it is widely known that such a condition exists, I feel that time and the free play of existing forces will bring about the change.

Mr. Hunter feels that the assumption of a lower rate of interest after twenty years is unduly conservative. It seems to me that this is the logical solution of the interest-rate question. The future course of interest rates will be affected by so many forces that it defies prophecy. I advocate the use of a lower rate, not because I am convinced that interest rates will drop to such a level, but because I do not know *what* the level will be. In the matter of mortality rates we have a factor of safety in the improvement which may reasonably be anticipated. We should introduce a similar factor of safety in our interest rate. The effect of such an assumption on natural reserves is of more importance than the effect on premiums. I feel strongly that legal reserves today are unnecessarily high during say the first ten years of the life of our policies, but I feel quite different about reserves at the longer durations.

Notwithstanding the arguments advanced, my own view is that it is quite in order to introduce the average size of our policies at

different ages and on different plans as a factor in determining premium rates. It cannot be denied that to some of us it is a question of considerable practical importance. No doubt, policies for $50,000 and $100,000 should not be weighed in the scale at face value, but I think we can distinguish between a class of $5,000 and $10,000 policies and a class of $1,000 policies.

Theoretically, at least, I am not convinced of the necessity of introducing lapse rates in order to determine our surrender values. If surrender values in all cases were exactly equal to natural reserves, the lapse rate would not have a particle of effect. Surrender values can only be increased above natural reserves by a redistribution of margins between natural reserves and surrender values exacted from other policyholders.

Mr. Wood points out the discrepancy between my Ordinary Life and Twenty-Year Endowment rates at age sixty-five and correctly attributes it to inconsistent commission assumptions. I find in looking back over my papers that this discrepancy was recognized when the tables were prepared. I am rather glad it occurred because it illustrates a practical point. Unless we take special precaution to avoid just such a situation, we are apt to find that our commission rates are inconsistent with our premium rates as between different plans and as between certain ages. Roughly speaking, theory would suggest that commission rates should be decreased as the premium increases whether the increase is due to the plan of insurance or increase in age. It is very generally recognized that there should be a material difference in the rate of commission paid on a Twenty-Year Endowment and an Ordinary Life at the younger ages. There is no such good reason for a similar difference at age sixty or sixty-five because the two plans are much more nearly alike.

OSCULATORY INTERPOLATION WITH UNEQUAL INTERVALS

By J. F. REILLY

MR. W. A. JENKINS:

As osculatory interpolation with unequal intervals has always seemed to be rather an awkward problem, Dr. Reilly's methods of development are very ingenious and surprisingly simple. The fact

that he completed the developments through his previous solutions of the cases where the intervals are equal suggested to me that a new development for unequal intervals might be worked out along the lines of my solution of the equal-interval case as presented to the Institute in June.

I found that such an approach does not lead far. The crux of the method then presented is in the first assumption of the form of the formula particularly in the choice of the ϕ function and the variable there involved. The most logical selection that I could see led to the solution of three of the simpler cases and no farther. Perhaps another choice of the function would complete the general development.

Dr. Reilly's methods seem to stand alone for osculatory interpolation with unequal intervals.

While working on this problem, however, an ordinary, non-osculatory interpolation formula came to my attention which I do not recall having seen before. It is a central-difference, divided-difference formula of which Everett's formula is a special case. It is as follows:

$$u_x = \frac{x-a_0}{a_1-a_0}\,u_{a_1} + \frac{(x-a_1)(x-a_0)(x-a_{-1})}{a_2-a_{-1}}\,\Delta'^2 u_{a_0}$$
$$+ \frac{a_1-x}{a_1-a_0}\,u_{a_0} + \frac{(a_2-x)(a_1-x)(a_0-x)}{a_2-a_{-1}}\,\Delta'^2 u_{a_{-1}}$$
$$+ \frac{(x-a_2)(x-a_1)(x-a_0)(x-a_{-1})(x-a_{-2})}{a_3-a_{-2}}\,\Delta'^4 u_{a_{-1}} + \ldots.$$
$$+ \frac{(a_3-x)(a_2-x)(a_1-x)(a_0-x)(a_{-1}-x)}{a_3-a_{-2}}\,\Delta'^4 u_{a_{-2}} + \ldots.,$$

where Δ' indicates the divided difference.

MR. H. L. RIETZ:

A clearly written mathematical paper frequently calls for relatively little discussion, and the paper of Professor Reilly seems to me to be of this character.

The contribution in the paper under discussion involves two main points. The first of these points consists in showing both by the Sprague method and by the Lidstone method that the change

from equal to unequal intervals of the independent variable does not materially complicate the problem of developing osculatory interpolation formulas by the use of the fifth-degree polynomial so determined as to satisfy the well-known six prescribed conditions. The second of the main points consists in pointing out that the formulas for unequal intervals lend themselves to the generalizations of osculatory interpolation with which Professor Reilly has dealt in previous papers published in *The Record*. These results give us another important generalization of the theory of osculatory interpolation.

VALUATION CONSTANTS FOR AN ATTAINED-AGE VALUATION, ILLINOIS STANDARD

By WILBUR M. JOHNSON

MR. EDWARD W. MARSHALL:

Mr. Johnson derives a general formula for attained-age valuation, applicable to both the Preliminary Term and Modified Preliminary Term methods of valuation, which will be of help to students in their study of the subject. He also furnishes tables of valuation constants for the Illinois Standard method of valuation which will be of assistance to companies wishing to make use of the attained-age method of valuation on the Illinois Standard basis.

The attained-age method of valuation seems to be gaining ground year by year. Since looking into the matter two years ago I have learned of several other companies which have decided to adopt this method of valuation. Where it can be adapted to existing office conditions it has been found exceedingly valuable, not only from the point of view of economy, but of facility of operation and collateral advantages. The company with which I am connected continues to find the attained-age method of valuation by far the most satisfactory one it has employed.

The tendency in actuarial departments using perforated card systems is to throw more and more burden upon such systems wherever possible. The use of the perforated cards, not only for attained-age valuation and collateral data obtained from the system but for many other statistical and accounting purposes as well, is being developed to a high degree in the modern office. Improvements in

the sorting and tabulating machines have opened up new avenues of development, and opportunities for further economies and increased efficiency are undoubtedly being presented. It is well worth while to keep in touch with any improvements in such machines and to instal the latest equipment wherever justifiable.

Our experience has shown that when new improved types of machines are installed, we begin to think of our work in terms of what the machines will accomplish, and that before long better methods are discovered and new uses are developed, which often result in great economies, or at least in greater effectiveness in the organization.

MR. E. A. PORTER:

In the paper Mr. Johnson has derived the formula for attained-age valuation by algebraic transformation of a retrospective expression for the Illinois Standard terminal reserve. A corresponding formula may be obtained prospectively with more direct reasoning which is entirely general and usually superior in application.

Assume that ${}_tV_x$ is to be obtained for a policy of $1,000 volume, which volume remains constant for the next succeeding $n-t$ years, during which period $n-t$ constant net premiums are payable, each π in amount, and that the terminal reserve at the end of $n-t$ years is ${}_nV_x$. Then we have:

$$
\begin{aligned}
{}_tV_x &= 1{,}000A^{\,1}_{\overline{x+t:n-t}} + {}_nV_x\, A^{\,1}_{\overline{x+t:n-t|}} - \pi\, a_{\overline{x+t:n-t|}} \\
&= 1{,}000\,\frac{M_{x+t}-M_{x+n}}{D_{x+t}} + {}_nV_x\,\frac{D_{x+n}}{D_{x+t}} - \pi\,\frac{N_{x+t}-N_{x+n}}{D_{x+t}} \\
&= 1{,}000\,\frac{M_{x+t}}{D_{x+t}} - \pi\,\frac{N_{x+t}}{D_{x+t}} + \frac{1}{D_{x+t}}\left\{{}_nV_x\,D_{x+n} - 1{,}000M_{x+n} + \pi N_{x+n}\right\} \\
&= 1{,}000A_{x+t} - \pi\, a_{x+t} + \frac{10^5}{D_{x+t}}\left\{\frac{{}_nV_x\,D_{x+n} - 1{,}000M_{x+n} + \pi N_{x+n}}{10^5}\right\}.
\end{aligned}
$$

Developing the $t+1$st mean reserve, we have:

$$
\begin{aligned}
{}_{t+\frac{1}{2}}M_x &= 1{,}000\,\frac{A_{x+t}+A_{x+t+1}}{2} - \pi\,\frac{a_{x+t}+a_{x+t+1}-1}{2} \\
&\quad + \tfrac{1}{2}\left\{\frac{10^5}{D_{x+t}} + \frac{10^5}{D_{x+t+1}}\right\}\left\{\frac{{}_nV_x\,D_{x+n} - 1{,}000M_{x+n} + \pi N_{x+n}}{10^5}\right\}.
\end{aligned}
$$

Comparing with the formulas of the paper, we note that we have identical results, except that where Mr. Johnson uses

$$\Theta = \frac{(\pi - P)N_x + rN_{x+1}}{10^5}$$

we now have

$$\Theta = \frac{{}_nV_x D_{x+n} - 1{,}000M_{x+n} + \pi N_{x+n}}{10^5} .$$

At first sight, the last expression appears somewhat formidable. However, by choosing n properly, considerable simplification is usually effected. For Illinois Standard constants, n is chosen as the premium period if the net renewal premium remains uniform, otherwise it is chosen as a period terminating when the change in net premium takes place. A change in Θ occurs, of course, whenever the net premium changes, and whenever the policy becomes paid up.

It will be noticed that the new expression for Θ does not involve the difference of two net premiums. On account of this, the propagation of any error due to discarded decimal parts of a cent is much reduced, and reliable constants may be obtained from regular tables of net premiums and reserves. Certain tables computed in this way have been compared with Mr. Johnson's, which latter were computed from net premiums to one-hundredth of a cent, and the slight differences found would be of negligible practical importance.

For term policies, the value of Θ is negative. This has proved a source of some difficulty in application, and several ways of handling the negative constants have been suggested: making separate totals of positive and negative Θ's; using the complement of negative Θ's, carrying the 9's to the left to the limit of the machine; and using the complement, but so wiring the machine that a single 9 totals on all the adding wheels to the left. The following is a further suggestion: Increase each Θ by the volume. That means add 1,000 to each Θ except that for pure endowment, which has no volume. Call the result Θ'. Use in the regular way. The final working formula then is amended:

$$\sum_{n=1}^{n=y} {}_{n-\frac{1}{2}}M_{y-n} = (\Sigma S)A + (\Sigma^s\Theta' - \Sigma S)B - (\Sigma^s P)C .$$

The tables published are of valuable character, and will be much referred to. An extremely high standard of accuracy appears to have been reached. I do note, however, a misleading heading to Tables I, II, and III, to wit: "Value of Θ for Renewal Years." The values of Θ given apply to the first policy year as well as to renewal years.

We are all indebted to Mr. Johnson for his valuable paper.

MR. S. C. KATTELL:

(Written discussion presented by Miss Margaret Walker)

The chief object of Mr. Johnson's paper was apparently to put in print tables of constants to be used in an Illinois Standard valuation by Dr. Karup's attained-age method. A discussion of the paper in line with its object would seem to call for a check as to the correctness of these constants. Values of Θ were compared with those obtained from another insurance company and a few differences noted. These slight discrepancies seem to be due to the number of places of decimals to which the insurance functions, used in the computations, were carried. For practical purposes the constants and mean insurance functions of Mr. Johnson's tables appear to be sufficiently accurate.

Dr. Karup's method of attained-age valuation seems to be an extremely simple one and to all appearances a very practical one. Any paper that focuses attention upon the method, or makes its use easier for a company without a large actuarial staff, is of genuine service to the insurance world.

My experience with Dr. Karup's method is only theoretical, and therefore further remarks will probably not be of much value. However, there are a few considerations which might be mentioned with profit at this time.

Mr. Johnson has given in his tables values of Θ for renewal years. This would seem to imply that these values cannot be used for the first year.

If

P_{x_1} = First-year net premium

P_{x_r} = Renewal net premium

$\pi^{1}_{x\overline{1}|}$ = One-Year-Term net premium

π_x = Ordinary Life net premium

then the reserve at the end of n years is given by the formula

$$A_{x+n}-P_{x_r}\cdot a_{x+n}+\frac{(P_{x_i}-\pi_x)D_x+(P_{x_r}-\pi_x)N_{x+1}}{D_{x+n}}$$

or

$$A_{x+n}-P_{x_r}\cdot a_{x+n}+\frac{(P_{x_i}-\pi_{x\overline{1}|})D_x+(P_{x_r}-\pi_{x+1})N_{x+1}}{D_{x+n}}.$$

If n in either of the foregoing formulas is equal to zero, the formula produces a result of $P_{x_i}-P_{x_r}$. This negative correction at the beginning is just sufficient to allow the use of the regular formula containing the renewal premium, P_{x_r}, and thus bring out the correct reserve at the end of the first year.

During the last couple of years, the Modified Life policy (i.e., one with premiums at half-rate during the first five years) has become quite popular. The reserve during the first five years on the Illinois Standard basis is

$$A_{x+n}-P_{x_r}\cdot a_{x+n}+\frac{(P_{x_i}-\pi^{\mathrm{I}}_{x\overline{1}|})D_x+(P_{x_r}-\pi_{x+1})N_{x+1}}{D_{x+n}}$$

for $n\leqq 5$, and the reserve for years subsequent to the fifth is

$$A_{x+n}-2P_{x_r}\cdot a_{x+n}+\frac{(P_{x_i}-\pi^{\mathrm{I}}_{x\overline{1}|})D_x+(P_{x_r}-\pi_{x+1})N_{x+1}}{D_{x+n}}+\frac{P_{x_r}N_{x+5}}{D_{x+n}}$$

for $n\geqq 5$, where the net premium to be multiplied by the life-annuity is doubled and an extra term appears in the Θ constant factor.

If the net premium punched on the card is P_{x_r} and space is provided for two Θ factors, it would be possible to use the same card for valuation both during and after the first five years. Of course, it would be necessary that the cards representing business more than five years old be kept separate so that the net premium could be doubled and the correct Θ factor used. However, it would probably be more economical to repunch the cards at the end of five years doubling the net premium and adjusting the Θ factor, and thereby be able to keep the cards all in one file.

In previous discussions of Dr. Karup's method, it has been

stated that both premium-paying and paid-up term policies must be valued separately, or the Θ factor punched in complement with sufficient 9's to carry beyond the adding device of the machine, because the Θ factor is negative. Mr. E. A. Porter has used the simple device of adding the amount of insurance to the Θ factor, thereby making it positive, and of course subtracting it out again when figuring the reserve. If this is done with all policies, the term business may be included and valued with the other business. The formula then becomes

$$(\Sigma S)A_{x+n}-(\Sigma^{s}P)a_{x+n}+\frac{(\Sigma^{s}\Theta+\Sigma S)}{D_{x+n}}-\frac{\Sigma S}{D_{x+n}}.$$

In case of the valuation of a pure endowment, the amount to be added to the Θ factor would be zero. However, as pure-endowment policies are themselves a rarity but usually exist only in connection with policies running under extended-term insurance, there is no reason why a Θ factor could not be used consisting of the sum of the Θ's under the pure endowment and extended-term insurance plus the amount of extended-term insurance. Hence it would seem that all business, regardless of plan, could be grouped together by attained age for the valuation.

MR. J. A. BUDINGER:

Mr. Johnson develops his formulas retrospectively and then resolves them into attained-age form but the formulas for gross-premium valuation on pages 29 and 30 of Mr. Bell's paper, *J.I.A.*, Volume XXXIX, are directly applicable to any form of Preliminary Term valuation in which the extra allowance or loading for acquisition expense is limited to the first year because all such forms of valuation are obviously special cases of gross-premium valuation as dealt with in that paper.

Viewed in this way the ϕ_x of Mr. Bell's formulas represents the difference between the Preliminary Term renewal net premium and the corresponding level net premium, $\phi_x \cdot a_x$ being the extra first-year loading or margin on Ordinary Life policies and always $\leqq P_x+\phi_x-P^{I}_{x1}$, and $\phi_x \cdot a_{xn|}$ being the extra first-year loading or margin on n Payment Life or Endowment policies and on those Ordinary Life policies on which the full reserve is attained in n

years, $\phi_x \cdot a_{x\overline{n}|}$ being always $\leqq {}^{L}P_x + \phi_x - P^{1}_{x\overline{1}|}$ where ${}^{L}P_x$ is the level net annual premium for the policy in question.

For the convenience of those not having access to Bell's paper the following formulas are given:

Ordinary Life attaining full reserve at end of n years, $t<n$,

$$ {}_{t}V'_x = A_{x+t} - (P_x + \phi_x)a_{x+t} + \phi_x \frac{N_{x+n}}{D_{x+t}} \qquad (1) $$

n Payment Life, $t<n$,

$$ {}_{t:n}V'_x = A_{x+t} - ({}_{n}P_x + \phi_x)a_{x+t} + \frac{({}_{n}P_x - P_x) + \phi_x N_{x+n}}{D_{x+t}}, \qquad (2) $$

which is identical with formulas (12), (13), and (14) in Mr. Johnson's paper when $n=20$ and $\phi_x = {}_{19}P_{x+1} - {}_{20}P_x$. When the full reserve is attained in m years, then $t<m<n$, it is only necessary to substitute N_{x+m} for N_{x+n} in the last term in the formula.

n Year Endowment, $t<n$,

$$ {}_{t}V'_{x:\overline{n}|} = A_{x+t} - (P_{x\overline{n}|} + \phi_x)a_{x+t} + \frac{(P_{x\overline{n}|} - P_x) + \phi_x N_{x+n}}{D_{x+t}}, \qquad (3) $$

which is modified in the same way as the preceding formula when the full reserve is attained in m years. The substitution of ${}_{n}P_{x\overline{r}|}$ for $P_{x\overline{n}|}$ gives the corresponding formula for an n-Payment r-Year Endowment.

An alternative method of developing these formulas is to view the policy as a combination of the corresponding level net premium contract with a special endowment policy whose benefit equals the extra first-year margin under the Preliminary Term policy and is payable at the inception of the contract and whose premiums are payable concurrently with those of the level net premium policy or for that period at the end of which the full level net premium reserve is attained. The reserve on this special endowment element must, of course, always be taken retrospectively, is always negative, and when combined with the level net premium reserve produces the Preliminary Term policy value.

These formulas give correct reserves for all durations including those less than 1, except 0, so that no separation of new issues from others is necessary for valuation and cost calculations.

As Mr. Gerhard points out on page 153, *T.A.S.A.*, the reserve

for duration o is negative and equal to the extra first-year margin. Consequently it is not necessary to have the first-year net premium on the punch card because if the first-year net premiums on say the new issues are required they can be obtained by taking the sum of the renewal net premiums and these negative reserves. The addition of these quantities may be done automatically in combination with taking off the reserve for duration o by decreasing by unity the annuity factor in the second term in the formula.

This relation may be used to check the accuracy of the valuation factors on the punch cards as they are made, because then the first-year net premiums are readily available and may be easily compared with the result of the calculation described in the preceding paragraph.

It seems that the attained-age method of valuation may be applied with as much facility to Preliminary Term reserves as to level net premium reserves.

I should like to add informally that Mr. Marshall said the student would find it a very interesting exercise to apply the attained-age valuation formulas to Preliminary Term reserves. He will not only find it an interesting exercise, but I believe it will give him some unconventional and unusual views of Preliminary Term valuation.

MR. J. L. McLACHLIN:

Many of these remarks have reference to the general problem of valuing by the attained-age method rather than to Mr. Johnson's paper on "The Valuation Constants for the Illinois Standard."

1. I should like to discuss the problem of office routine in connection with this system of valuation. In Mr. Marshall's paper, referred to by Mr. Johnson, the method outlined for obtaining the reserve at the close of the year is similar to the general practice with a group valuation, i.e., a tabulation of the business in force at the beginning of the year, which is then valued as at the end of the year, plus a valuation of all new business and reinstatements, and less a valuation of all withdrawals. This method produces the reserve for the balance sheet in a very short space of time after the close of the books for the year, and if no subdivisions of the total reserve are necessary, the additions and subtractions are required for each attained age only, which is not a serious consideration when done as described in the paper mentioned above.

The problem of our company is more complicated than this. We have three tables of mortality and two rates of interest. In twelve foreign countries in which the company does business, reserves are required for mortality statistics or expense accounts. The company carries on business in Great Britain, and the Board of Trade return requires a subdivision of the business into three dividend groups, and eight plans of insurance. Also the company issues joint life policies at equivalent equal ages, requiring a separate grouping for them. As a result of these features of our problem, it did not seem feasible to value as at the beginning of the year, and carry the valuation through the year in so much detail. Since 1920, when we first installed this system, we have, during the year, merely filed in new cards and pulled withdrawals, and at the close of the year tabulated the whole file. In practice, the tabulation has been done twice: first, in as contracted a form as possible to obtain the balance-sheet figure as soon as possible; and, second, in the more detailed form required to complete the various schedules.

A point to be guarded against in this method is a peak load on the tabulator. On a Hollerith tabulator printer we have added 100,000 cards in three days, taking care of a few other routine tabulations at the same time. The second tabulation is done at more leisure during the month of January. We should like to hear how other companies using this attained-age method have dealt with a similar problem.

2. Mr. Johnson has set out his values of Θ to five significant digits at age twenty. When we made our installation in 1920, the question of space on our card had to be reckoned with, and as a result, we decided to use only four significant figures. Our tables of factors contain five digits, as do Mr. Johnson's, but we only use the final digit for obtaining a more correct value of Θ for sums assured greater than 1,000. The results, when compared with a group valuation, have been quite satisfactory in the aggregate. Also we cut our multipliers to five significant digits, as against seven and six in the case of those given by Mr. Johnson, and we believe that the results are quite accurate enough.

In making a comparison of the units of work involved in our company as between the group method and the attained-age method, the saving effected has not been so great as that noted by Mr. Johnson or Mr. Marshall. Our experience has shown about 40 units

of work as against 100 by the group method, but this does not take into consideration the fact that loadings, and mortality costs, are obtainable from an attained-age valuation with very little additional work.

3. I should like to raise one more point in connection with Mr. Marshall's paper. He outlines his method of checking his reserves by a calculation based on the previous year's result. We have not done so in the past, and should like to hear if many of the companies follow that practice, and what results are obtained.

MR. WILBUR M. JOHNSON:

When Mr. Marshall presented his excellent paper on "Attained-Age Valuation" two years ago he contributed much to the information of many who were studying the questions of improving valuation and kindred problems.

Mr. Porter has given in an excellent way a formula derived prospectively while Mr. Kattell has given a mathematical demonstration for the first year.

I desire to thank all the men who have taken part in the discussion and thus contributed to the available information concerning this method of valuation.

If the valuation constants as given prove to be of any convenience or service to others investigating this method, I shall feel repaid for the pains taken in their preparation.

REINSTATEMENT OF POLICIES IN FORCE AS EXTENDED TERM INSURANCE

By E. G. FASSEL

MR. JAMES F. LITTLE:

Mr. Fassel has done the spade work for arriving at a plan, which I doubt any company has in existence at the present time, of scientifically handling applications for reinstatement where there is a greater or lesser period of extended insurance to run.

In one respect, I think, we will have to go beyond Mr. Fassel's plan. Mr. Fassel assumed that the increased mortality would be a constant percentage of the normal for the remainder of life. That, of course, we know is not typical of the extra mortality due to any

form of impairment whatever. The normal curve of extra mortality would show in the early years as a rule a very considerable extra mortality; but after not very many years the additional mortality, not merely as a percentage of the normal, but often as an absolute excess mortality, diminishes. It is very often the case that within anywhere from ten to twenty or twenty-five years the extra mortality that would be found in the survivors of a group subject to some impairment at the commencement of the period would practically have disappeared. There is a very strong tendency for a group of impaired lives and a group of newly selected lives to approximate each other as to the mortality of the survivors after a number of years.

The net effect of assuming that the mortality in the case where there is a considerable period of extended insurance would be very high among impaired lives during that period, and very much lower thereafter, would be to reduce greatly the period of extended insurance necessary to enable a policy to be reinstated without inquiry at all as to the health of the insured.

With the view of establishing a rule to be used prior to inquiry as to present condition, Mr. Fassel has assumed an average degree of impairment, his idea, I take it, being to fix certain rules as to periods of time so that if a policy comes up for reinstatement with not less than the prescribed period of extended insurance still to run, the policy can be reinstated without further inquiry. But in other cases where the medical examination has already been made and an impairment is revealed, I think we cannot assume that we will ignore the type of impairment or the degree of impairment and use a uniform rule in deciding whether to reinstate or not. In such cases, I think we must take the facts into consideration and base our decision on the individual situation.

There is one additional item that Mr. Fassel's tables do not appear to take directly into account. If I am wrong he will no doubt tell us in what way it has been done. Account does not seem to have been taken of the number of back premiums to be paid up in order to reinstate the policy. That, of course, must have its effect. If already five years had run of an extended term and the policyholder in order to reinstate had to pay those five back premiums with interest, the situation would be different from that where, with the same remaining unexpired term, the policy had lapsed just a few

months earlier. In the first case, with the larger sum to come in, the company can afford to be more liberal in allowing reinstatement.

There is, however, one point not covered in the paper that is of very great importance today, particularly in such a company as the Prudential. We have, as most of you know, in all policies, or practically all of them, a disability provision. Now, it may happen that on the reinstatement of a policy, the disability provision would immediately become operative, and where a disability provision is included in the policy some difference in the treatment might very easily be required. There might be a case where, we will say, with five or ten years to run, we would feel we could safely reinstate life insurance because the prospect of either death or recovery within that period was very great. But, if, on the other hand, he is likely promptly to become a disability claim even though recovering, it might be unwise from a financial standpoint to reinstate the policy.

The problem, therefore, is complicated by the disability provisions which may be included in the policy; and that is one of the reasons that led me to say at the outset that I have not yet found a satisfactory solution for this problem.

Mr. Fassel, however, has started something, and I hope some other members of the Institute will carry on the good work and give us some sort of an entirely workable plan for handling these reinstatements.

MR. OLIVER W. PERRIN:

The subject of Mr. Fassel's paper has been given consideration on several occasions, directly or indirectly, both by the Institute of Actuaries and the Actuarial Society, but no doubt sufficient time has elapsed since the last discussion of the subject to make the time ripe for another. The interval has witnessed a growing tendency toward more liberal surrender values, including extended term insurance granted automatically upon lapse, and a more or less concurrent tendency toward more liberal reinstatement conditions, particularly as to policies which have lapsed and are in force under the extended term insurance provision.

Any theoretical consideration of the subject must, of course, be based upon certain assumptions which may or may not accord with the facts, and the best that we as actuaries can do is now and then

to check up these assumptions with actual experience and make such modifications in practice as may seem to be justified thereby.

Mr. Fassel has given us a thorough analysis of the problem of reinstatement on the broad principle, which I do not dispute, that reinstatement may be granted provided the strain on the company is not increased. In general, his calculations confirm the statement that the more impaired a risk is the shorter should be the period of unexpired extension required for reinstatement. He deals only with policies in force as extended term insurance, and the practical question arises in my mind as to what bearing his conclusions may have on the rule followed by some companies of reinstating, without medical examination, all policies having at the time of reinstatement a relatively small number of years of unexpired extension, such as three or five; it is undeniably difficult to find support for such a rule solely on theoretical grounds. The problem, as I see it, is inseparably linked up with the causes and effects of lapse and with other practical circumstances, such as the interval between the time of lapse and the time of reinstatement. As a matter of fact, is the mortality of a cross-section of policies, so to speak, having say five years of unexpired extension at the time of reinstatement, actually worse than that of the class of policies to which they may be considered as belonging? Do they actually represent lives which are impaired to the degree assumed in Mr. Fassel's calculations?

The company with which I am connected is maintaining a Department of Conservation which undertakes to reinstate policies, or prevent their surrender, by direct correspondence with the policyholder after the effort of the agent fails; and as a guide to this service, the agent is requested to give the company a statement of the reason for the policyholder's decision to discontinue payment of premiums. To throw some practical light upon this discussion, two thousand cases were investigated as to the stated causes of lapse or surrender, with the following results:

The lapse in approximately 24 per cent of the cases was stated as due to financial difficulties or lack of ready money; in 19 per cent, to inability to obtain any reply from the policyholder; in 14 per cent, to indifference; in 13 per cent, to failure to locate the policyholder; in 8 per cent, to uninsurability as evidenced by the policyholder's failure to meet the company's requirements, or by the agent's information as to the undesirability of the risk; in 3 per

cent, to domestic difficulties; in 3 per cent, to the passing of the need for the protection; in 2 per cent, to replacement by insurance in another company; in 2 per cent, to an expressed preference for extension or paid-up insurance; and in the remaining 12 per cent, to no reason whatever, or else to one of a miscellaneous nature which could not be included in any of the foregoing classes.

The two thousand cases were chosen at random, and the stated reasons indicate that a large proportion, perhaps one-third of the total, decided to discontinue premiums on account of overpersuasion or indifference or the passing of the specific need for which the insurance was taken out. Only about 8 per cent are clearly impaired risks, and an additional group of about one-fourth of the total, where the reasons are financial or domestic trouble, may be regarded as including a larger proportion than the other groups of risks below the average.

Reasoning from these results and bearing in mind that policyholders in poor health, except through force of circumstances, are unlikely to let their policies lapse, there appear to be good grounds for the opinion that the mortality of a group of policies reinstated under a rule of unexpired extension would compare satisfactorily with a similar group of continuing policies.

Such a rule, it may be admitted, is at present based largely upon the individual judgment of the actuary and, especially when measured by the results of Mr. Fassel's calculations, must depend for its justification upon experience. Mr. Hunter in 1915 reported the experience of the New York Life under reinstated policies in which were included policies with five or more years of unexpired extension reinstated without evidence of insurability, and this was generally favorable. A similar reinstatement rule is followed by the company with which I am connected, and in the effort to ascertain what additional light might be thrown upon the problem by the Penn Mutual's experience, I used the only records available for the purpose which were the reinstatements for the five years of issue—1917, 1918, 1919, 1920, and 1921. These reinstated policies were traced to their anniversaries in 1926, except for the anniversaries falling after October 1, 1926, and although the experience is small and of short duration, it covers the early years immediately following reinstatement during which any selection against the company would be most apparent.

The results are as follows:

1	2	3	4	5
Year of Issue	Total No. of Policies Reinstated	Total No. of Deaths under Column 2	No. of Policies Reinstated on Unexpired Extension	Total No. of Deaths under Column 4
1917.....	134	0	98	0
1918.....	236	0	167	0
1919.....	489	3	319	·3
1920.....	391	2	280	1
1921.....	223	0	189	0
	1,473	5	1,053	4

Owing to the small number of actual deaths and the fact that they occurred in only two out of the five years of issue, the calculation of the expected deaths was not attempted. These figures, which are given because they are the only ones available, are put forward not as a justification of one company's liberal rule from an empirical point of view, but rather as showing that the practice, so far as it may be judged from these years of issue, has not developed any unfavorable features. The theoretical assumptions upon which Mr. Fassel's calculations are based require among reinstatements a large proportion of lives below the average and, so far as it appears from the experiences already available, they do not apply to policies reinstated under an unexpired extension rule. For confirmation of these assumptions, we must await the results of some other experience.

It may be added in concluding these remarks that the increased efforts of companies along lines of conservation of policies strengthens the impression that reinstatements under such conditions will contain a larger proportion of policies which might otherwise have remained lapsed through indifference, overpersuasion, or reasons of a similar nature—with a correspondingly favorable effect upon the mortality under reinstatements.

MR. J. CHARLES SEITZ:

Is it not a wise provision that, in a case of reinstatement such as under discussion, the policyholder be advised that the company's action shall not be a waiver of the conditions of the policy in con-

nection with any future reinstatement? If after paying premiums for a number of years, he then defaults and desires to reinstate, there being a loan against the policy, the company may not wish to give such liberal treatment, but may desire to exact the terms of the policy and require evidence of insurability. The company might be prevented from doing so unless protected against waiver.

MR. G. F. HOLLENBERG:

I might say that it has been the practice of the Union Central, if there be Disability or Double Indemnity attached to a contract, to be less liberal. Ordinarily, we will reinstate a policy without any evidence of insurability if the extension is more than five years from the date of application; but if it has Disability or Double Indemnity, we require some form of new evidence of insurability.

We also recognize the question of Disability benefits and Double Indemnity in this way: Our agents are authorized to accept settlements on the basis of an informal application, if that informal application is satisfactorily completed and settlement is received within ninety days. However, if we extend the use of this informal health certificate beyond ninety days because of a certain length of extended insurance, he is not permitted to accept that settlement automatically but must first present the informal health certificate to our Medical Department; and then if the medical director feels that we should require a regular form of health certificate completed by an examiner, it is requested at the applicant's expense.

MR. C. O. SHEPHERD: I should like to ask a question that may fall a little outside of the scope of the paper, as to the practice of some of the other companies. In case the application for reinstatement and the accompanying papers show that the risk is impaired to a point where the company would not feel justified in reinstating except on a rated basis, is it customary to offer to reinstate on a rated basis?

I raise that question because I think the legal point has been raised as to whether the company has a right to do that. Unquestionably it has the right to decline the application for reinstatement or to reinstate the policy as it formerly stood; but has it the right or is it a safe practice to impose the extra rating and offer to reinstate on that basis?

MR. HOLLENBERG: I might answer that the Union Central has

done it without any difficulty. And if a policy had been issued after the Union Central had begun writing substandard business, we would revive it on a substandard basis, otherwise it would be reinstated on the Endowment plan or some other higher-priced plan, subject to an amendment being signed by the applicant in which he consents to the change.

MR. LITTLE: I might mention in connection with that, that a legal opinion has been given that in such case: If the policy is reinstated subject to the extra premium the insured may refuse to pay the extra, and the company must continue the policy at the original rate of premium.

So far as I know that opinion has not been contested in the courts, but I know that opinion has been given by an insurance lawyer.

MR. HENRY MOIR:

(Written discussion presented by the Secretary)

This paper by Mr. Fassel is an interesting one, and develops in a theoretical way a question in which all life insurance executives take practical interest. In discussing the subject in 1910 (*T.A.S.A.*, XI, 296), I took particular pains to point out that the practical question is an important one, and that the theories and mathematical solutions are subordinate, and cannot be applied in any hard-and-fast way, but must be subjected to an intelligent judgment. My reason for this position is that these mathematical solutions never take into consideration all of the factors governing the problem. We have to make fundamental assumptions which are never correct, and which are rarely even roughly correct.

It has always been my practice to study difficult problems by considering what would happen in cases which go to the limit. One of these limit cases, applied to this question, would be that a policyholder, having carried a policy as extended term insurance for several years, and still having x years to run, might apply for reinstatement while on his deathbed. The legal reserve for the extended term insurance may be some \$50 per \$1,000. The unpaid premiums over a period of ten years with interest are likely to be as much as \$250—perhaps a great deal more. It is in the interest of the company to collect the back premiums and reduce the risk; all the more so if the insured is likely to die almost at once. We may therefore think of two limiting cases: the first, where the man is thoroughly

healthy and should be reinstated because of his good health; the second, where a man is completely unhealthy, and should be granted reinstatement because it is profitable for the company. I formerly pointed out rather carefully the distinction which should be drawn between average legal reserves and true reserves for impaired risks. The distinction is fundamental in discussing this problem especially since cases of reinstatement of impaired risks can scarcely be reduced to average conditions.

My practical experience of substandard mortality, and of mortality curves relating to impaired lives, is that none of them follow any rule laid down by Mr. Fassel. It is therefore necessary that this question be approached from a broad, practical standpoint rather than from a narrow and technical mathematical viewpoint. So long as the practical considerations are recognized as being the chief factors in the case, then the mathematical study is of interest and of value. My reason for sending this note is that students may be carried away by the mathematical niceties, and may forget that the fundamental assumptions on which the mathematics are based are far from being in accordance with the actual conditions affecting impaired lives.

MR. E. G. FASSEL:

The discussion has brought out a number of valuable points, and our thanks are due to those who have participated. I am especially glad for the note struck by Mr. Moir. The results of mathematical analyses should not be accepted blindly. The problem treated in this paper is of such a nature that the deduction of an office rule from the calculated results calls for the free exercise of practical judgment.

Mr. Little's question as to the allowance that the paper makes for the payment of back premiums is answered by referring to the second paragraph of page 55. The analysis and the results are of a general nature and apply whether there are many back premiums or only one.

I am glad for the emphasis given in the discussion to the type of extra mortality that decreases and may be considered as eventually disappearing. To have introduced such a class in the paper would have made the work prohibitive as involving the select-table principle. I have, however, worked out two examples. Mortality was assumed to be 320 per cent A.M.(5) on reinstatement decreasing 12

per cent for each age to the uniform level of 80 per cent after twenty years. It will be remembered that 80 per cent is the assumed basis of general mortality in the paper. Inasmuch as $320 \div 80 = 400$ per cent, this basis assumes extra mortality of three times the normal at reinstatement, disappearing after twenty years, as compared, for example, with C, which is seen near the bottom of page 59 to have extra mortality of one and one-half times the normal throughout. The results are compared in the "Special" column of Table A with an excerpt from Table III of the paper.

TABLE A

CRITICAL VALUE OF t, IN YEARS

Duration	B	C	D	E	F	Special
	Ordinary Life—Age 25 at Issue					
25	14	12	11	11	10	8
	Ordinary Life—Age 45 at Issue					
15	10	9	7	9	8	6

Even with the severity of the "Special" assumption, t is two-thirds the C values, which form the basis of the later tables in the paper. This seems to bear out the conclusion reached in the paper that average values of t can be used as doing substantial justice over a wide range of mortality variation. In other words, it is not unreasonable to apply fixed values of t for reinstatement applicants without reference to the degree of impairment. The tendency of the "Special" calculations is to reduce the values of Table VII. Giving the "Special" calculations their full weight, the values of t appearing in Table VII will be reduced by one-third.

As regards an office rule, it seems clear that a uniform value, say 5, for all plans, ages, and durations is too rough; that the high values appearing at some points in Table VII are rather severe; and that when adjusting Table VII by reducing the values one-third to give effect to decreasing extra mortality disappearing after a period, Ordinary and Limited Payment Life plans should be compromised into a single column, separate columns being a refinement no longer justified. Two-thirds of the value of t in columns 2 and 3

of Table VII may thus be compromised at 60 per cent of the values in column 2. In Table B, t runs at about 60 per cent of the values in Table VII subject to a maximum of 10 and minimum of 2.

TABLE B

UNEXPIRED TERM

Number of Years Required for Reinstatement of Policy on Impaired Risk if in Force as Extended Term Insurance

Age Attained	Required Number of Years	Years to Maturity	
Under 45	10	Over 30	
45	10	30	
46	9	29	
47	9	28	
48	8	27	For Life plans and for Endowment plans maturing after age 75 enter table according to "Age Attained"
49	8	26	
50	7	25	
51	7	24	
52	6	23	
53	6	22	
54	6	21	
55	5	20	For Endowment plans, except those maturing after age 75, enter table according to "Years to Maturity"
56	5	19	
57	5	18	
58	4	17	
59	4	16	
60	4	15	
61	4	14	
62	3	13	This table does not apply to policies with Disability benefits
63	3	12	
64	3	11	
65	3	10	
66	3	9	
67	2	8	
68	2	7	
69	2	6	
70 and over	2	5 and under	

When using Table B in the reinstatement of policies in force as extended term insurance, it is recommended that the presence of indebtedness or paid-up additions be ignored.

The paper has been developed by assuming what we may call reinstatement practice (1), requiring medical examination of all

cases, with approval first of the normal risks and then approval of those found to be impaired but to have sufficient unexpired extension. In the discussion reference has been made to what we may call reinstatement practice (2), approving all applications with sufficient extension without regard to health and requiring examinations for only the residue. It seems clear that the length of required extension is to be taken the same under the two practices and that the length must be determined by considering the mortality of those understandard reinstatement applicants that cannot be included as normal. In other words, the method of the paper gives results applicable to both practice (1) and practice (2).

In a given group of applicants for reinstatement there is no difference in result between (1) and (2). Assume 100 applicants, being 35 normal risks with the required extension and 35 without, also 15 impaired risks with the required extension and 15 without. Practice (1) approves the 70 normal risks without reference to the extension, approves 15 of the impaired, and rejects the 15 impaired risks that do not have the required extension. Practice (2) approves the 50 with required extension without reference to health, approves the remaining 35 normal risks upon examination, and rejects the same 15 as before. Results by the two practices therefore differ only as they involve differences in the groups of reinstatement applicants.

The group applying for reinstatement under practice (2) may be expected to contain those under (1), and it has been argued that many additional normal risks will be brought in, being persons deterred by the expense and trouble of being examined. There is also every reason to suppose that the group will include additional understandard risks, being persons deterred from examination by the knowledge that they would not pass. If those approved under (2) could be classified into normal and impaired risks, it is a fair assumption that the latter group would show much the same mortality as the corresponding group under (1).

The word "normal" is being used to imply mortality on the average as good as that of continuing policies, as referred to on page 52, first paragraph. If all lapsed policies were reinstated in a body they could all be regarded as normal because the company would be in the same position as if no default had occurred. The under-average lives in the group would merge with all the others in produc-

ing the normal mortality, and there would be no question of a specified unexpired extension to be considered. Many policyholders, however, do not apply to reinstate their lapsed insurance, and the incentive is strongest for those in bad health. Only a part of the underaverage lives, therefore, can be merged in the normal group, and the remainder must be treated on their merits. This is recognized in the paper by omitting column A of Table III from consideration and by assuming the rather heavy mortality of C in Tables IV and subsequent. It is in treating the foregoing remaining underaverage lives on their merits that the question of a specified unexpired extension arises, and it is immaterial whether reinstatements are according to practice (1) or practice (2).

THE INCONTESTABLE CLAUSE

By H. W. BUTTOLPH

MR. PERCY H. EVANS:

I must confess to having had some difficulty in grasping exactly what Mr. Buttolph is alarmed about and what advantage he thinks we have overlooked. The voluntary insertion by life insurance companies of a clause designed to divest them of ancient common-law rights after a specified period might have been expected to result in some confusion in the decisions, and especially when such clause is made to turn on the meaning of a single word. There is, for example, a line of decisions to the effect that a provision excluding from risk death by suicide at any time is limited by the clause to the term of the incontestable period in the case of a simple statement of incontestability although that result was not intended. But the later cases cited by Mr. Buttolph appear to reflect a more satisfactory definition of the word "incontestable."

It is fundamental in contract law that fraud vitiates the contract. It is also fundamental in contracts of insurance where the insurer must rely upon the statements of the applicant, that misstatements as to facts material to the risk similarly affect the contract even where made without fraudulent intent. Originally, therefore, the basic law gave insurance companies a complete and un-

limited defense against misstatement of essential facts, and also, under the old practice of warranties, against misstatements which had nothing whatever to do with the risk.

Now came the life insurance companies saying in effect that for business reasons they did not desire the protection of the law in defense of misstatements after a certain length of time. In fact, a few companies proposed to dispense entirely with this defense. This extreme liberality was, however, held to be contrary to public policy. Most of the companies proposed to waive their legal defense against misstatements only after a stipulated period—variously one, two, or three years—and this proposal was held to be within the privilege of contract. As the Supreme Court of Georgia said (*M.B.L.A.* vs. *Robinson*, 30 S.E. 918): "This does not violate in any sense the well-settled principle that fraud is to be abhorred, vitiates everything it touches, and the person guilty of it is not to be countenanced in any way by the courts."

The rule is thus stated in 14 Ruling Case Law, section 380, to which Mr. Buttolph refers:

> A provision in a contract of insurance limiting the time in which the insurer may take advantage of certain facts that might otherwise constitute a good defense to its liability on such contract, precludes every defense to the policy other than the defenses excepted in the provision itself, including false answers to the application and even fraud where the time fixed by the contract is not unreasonably short.

Mr. Buttolph's thesis is in effect a criticism of this statement as a definition of incontestability and particularly the idea that all defenses retained by the insurance company must be specifically excepted.

When the insurance companies concluded to limit their legal defenses against misstatements to a stipulated period they and not the courts or statute writers constructed the policy provision to accomplish that end. Instead of making a definite and concise statement that after two years the liability of the company shall not be disputed on account of any misstatement made in the application, they said: "This policy shall be incontestable after two years." This made the meaning depend on the definition of one word. There was no more intention of enlarging the coverage than of waiving the premium consideration. If suicide was excluded from the risk during the life of the policy, the incontestability clause was not

intended to bring it into the coverage at the end of the period. Nevertheless, some of the courts so held.

I am not competent to judge as to what extent there may have been judicial confusion in defining "incontestability," but there has certainly been confusion in the lay-mind as between contesting the policy and contesting a claim. Mr. Buttolph appears to think insurance officers, including counsel, have been negligent in their acceptance of legislation and rulings on the subject. Yet he cites leading cases in which the distinction between coverage and avoidance is clearly set forth. It seems to me that the fault, if any, lies in the form of language originally adopted by the companies themselves for the purpose of waiving their defense after the stipulated time. It seems to me that in adopting the word "incontestable" they used an ambiguous word, ambiguous because the word "contest" alone applies to any dispute whatsoever, whereas it was not intended to apply to claims for benefits not promised in the policy.

Why was the word "incontestable" adopted instead of some more definite formula such as "after two years the liability of the company under this policy shall not be disputed on account of any misstatement made in the application"? I suspect that a preference for rhetorical euphemy is the answer, and euphemisms are from their nature peculiarly liable to ambiguity.

If the word "incontestable" is truly somewhat ambiguous the fact will account for the advisability of reciting the exceptions as a part of the clause, however numerous such exceptions may be. Thus the definition quoted above (14 Ruling Case Law) says the provision "precludes every defense to the policy *other than the defenses excepted* in the provision itself."

Mr. Buttolph says that in addition to confusion relative to the distinction between a contest of the policy and contest of the claim there is also a misconception that the incontestability clause is binding on the company but not on the insured as related to alleged misrepresentation of the conditions of the policy which may not be brought forward against the company until after many years. It seems to me the failure of companies to plead the incontestability clause in defense, say, of what the insured claims the agent told him twenty years before about the results of tontine accumulation, can be explained on the ground that when the companies wrote the word "incontestable" into the contract they meant to limit their

own defense but did not mean to limit the protection of the insuring public against misrepresentation of the benefits promised, and that they have never raised the point because they have at all times acted in good faith in their contract relations and have generally been able to show good faith as a sufficient defense, or at least as all the defense they cared to make. It is difficult to conceive of any reputable company entering a plea which would tacitly admit that a misrepresentation had been made by a responsible agent authorized to bind the company but that such misrepresentation was outlawed by the incontestability clause.

MR. W. H. McBRIDE:

It is a privilege to participate in Institute discussions and an especial one when the subject is a paper of such conspicuous merit as Mr. Buttolph's. He makes two points which I will consider in inverted order. His second is that the clause under discussion is binding with equal force on both parties. To this I subscribe my hearty approval and later will make but a passing allusion to it. In his first he says, in substance, that this clause is frequently construed by the courts, in some way not clearly explained, as negativing other clearly stated provisions. It is the reason for such judicial errors that I have to supply. In his remarks on another subject Mr. Mead used the words I have been seeking as a statement of the difficulty. The courts are merely "unable to see the forest for the trees."

I hope no one will mistake the turn my discussion takes as meaning that I indulge a hope that we can purge our laws at once of anything as firmly established by long use as the things I am about to denounce. That wrong things have been so for a long time does not preclude discussion of how they should be nor warrant the omission of any effort to make them better.

The note of optimism, with regard to abatement of judicial confusion, in Mr. Buttolph's paper is echoed by two lawyers in the *Proceedings of the Legal Section, A.L.C.*, Detroit, 1926, and one, reviewing seven cases, including but two cited by Mr. Buttolph, said: "These recent cases seem to indicate a growing tendency on the part of the courts to give the expression 'this policy shall be incontestable' its true meaning." The last thirteen words just quoted from Mr. William Ross King are likewise a verbatim quotation from Mr. Buttolph's paper, so I am not digressing from the latter.

I am denying that we want to give the expression "this policy shall be incontestable" "its true meaning," for I shall prove that doing so reduces the law to an absurdity if, indeed, it does not make it unconstitutional. In this, I am not at all out of harmony with the paper under discussion, in which the stated purpose is to show how judicial confusion arose from adding exceptions in conflict with the intent of the principal clause or, at best, confusingly redundant. I am going back of these addenda in the belief that there is a deeper-seated cause of our trouble. In law, the true meaning of an act is its intended meaning, but to arrive at the intent, the language must be interpreted. In this case, it seems the courts have failed to distinguish the "true meaning" of "the expression" from the intended meaning of the act in which it is used, and here I must forestall any apprehension that I am about to quibble over words.

Not having access to any South Carolina reports later than 1924, and lacking the time to carry my study back of 1906, my conclusion concerning the state of the law in that jurisdiction is based on the nineteen years indicated, which should be sufficient for present purposes. It may be that South Carolina claimants are unlike those of other states or that companies doing business there have refrained from appeals, but, if these hypotheses fail, unless that state has a monopoly on judicial wisdom, I am forced to conclude that some virtue dwells in its statute by which even trial courts have felt constrained to dispense justice on this issue, for in all these nineteen years the Supreme Court of South Carolina has never reviewed a case in which it was sought, under the statute I will quote, to extend coverage to contingencies not insured against by the policy, or to modify in any way its clearly stated provisions. It avoids the word "incontestable," with no attempt at substitution, by saying:

> All life insurance companies that shall receive the premium on any policy for the space of two years shall be deemed and taken to have waived any right they may have had to dispute the truth of the application for insurance, or to claim that the assured person had made false representations, and the said application and representations shall be deemed and taken to be true.

This act is, of course, unsatisfactory in that it leaves uncertain just when the defense of misrepresentation ceases to be available, but it does possess the virtue, lacking in the language of other legis-

latures, of giving no ground for the perversions of which Mr. Buttolph voices our complaint. That virtue is apparent upon reading the act, but what is more convincing, it works! Surely, when two acts have the same purpose, distinguishing the language of the one that fails from that of the one that succeeds is more than a quibble, for the virtue found in the latter, by the difference in practical results, implies a vice in the former.

The statute quoted might be condensed without sacrificing the intent, but if fewer words are used they should all be such as express the meaning intended instead of obscuring it, as the word "incontestable" has done in other acts having the same purpose.

We know why the fraud exception was omitted, and will pass that, but (as far as I know) we have never been told why the word "incontestable" was chosen when it describes a quality no one ever intended a life insurance policy to possess.

Webster defines "contest," first: "To call to witness; to introduce a lawsuit by calling witnesses; to bring an action." With special reference to law, he defines: "*to make the subject of litigation;* to dispute or resist by course of law; to defend, as a suit; to controvert." If Webster knows what words mean, then, after the expiration of the specified time, the courts are closed to all life insurance litigation where the word "incontestable" is used. That might not be so bad if Mr. Buttolph is successful in his contention that the clause is double acting and binds the insured as firmly as the insurer. If we cannot be sued under an "incontestable" policy we shall have no occasion to defend.

Several years ago I thought a Daniel had come to judgment when the Supreme Court of New Mexico was the first to distinguish the contest of a policy from the enforcement of its terms, and now we all approve Mr. Buttolph's distinction between the contest of a policy and the contest of a claim. Neither of these distinctions goes deep enough and, but for the miswording of "the expression," it would not be necessary to make them. If the courts do not perceive that, if a contract is really "incontestable," it cannot be made "the subject of litigation" at all, and, recognizing the unfitness of the word, discern the intent of the act, how can we expect them to distinguish the contest of a policy from its enforcement or the contest of a claim from the contest of the contract under which it arises? These distinctions are too sound and substantial to be dismissed as

subtleties, but they are not obvious enough to restrain a loose-thinking or hostile court from erroneous decisions.

The above-quoted authority defines "rescind," specifically; to vacate or make void, etc., and says: "In the law of contracts the technical rescinding of the contract avoids the contract *ab initio* and requires that both parties be put as far as possible in the same position as before making the contract." If the company's defense at law or plea in equity be misrepresentation (and that is all we are concerned with), there is no contract in existence and consequently no claim. The relation of company and applicant has never developed into that of insurer and insured. The company has prepared a document, called a "policy," which, in the absence of misrepresentation is a contract or, with misrepresentation, (even fraudulent), may at the end of a certain time ripen into a contract of life insurance. If founded on misrepresentation, until the taint thereof be cured by time, the policy is still a mere document representing a void (or voidable) attempt to contract. The dictum quoted would be clearer in these words: "In the law of contracts technical rescission voids *ab initio* the attempt to contract, etc." If the phrase *ab initio* has any meaning at all, it is the policy which may be rescinded, for no contract ever existed.

A contest (making the matter a subject of litigation) may be necessary for the rescission of an attempt to contract, but it may also be necessary to enforce the terms of a document when its subject matter has, or acquires, the status of a contract. An incontestable policy is therefore an absurdity, and a law requiring a policy to be incontestable (as defined) would deprive persons of property (or rights) without due process of law.

If a policy is void *ab initio* because of the suppression of material facts in the applicant's statements, made as inducements to its issue, and the insurer be not sued at law within the time allowed by law for defending on grounds of misrepresentation, he must take some affirmative action (in some jurisdictions at least) or lose his defense. The safest step (and in some jurisdictions the only safe one) is to file a bill in equity (or chancery) for the rescission of the contract. If the insurer's case has sufficient merit in the eyes of the chancellor the (attempted) contract is rescinded by decree, provided the action has begun in time. If an attempted contract can be "rescinded" on certain grounds within a certain time, what could

be more natural than to say that after a certain time it shall be unrescindable?

The expression, "this policy shall be incontestable," does not mean, even approximately, what was intended by those who secured its adoption, and all the added exceptions only confound the confusion which naturally arose from the use of a wrong word in the beginning. I therefore submit that a policy that is rescindable within a certain time, if founded on misrepresentation, and represents a contract that is thereafter unrescindable, is what the law intends and should prescribe.

I have agreed with every thought in Mr. Buttolph's paper, with praise for his able presentation, but I have gone a step farther and have challenged the aptness of the fundamental word in the clause, showing why it is dangerously misleading, and that in its absence confusion is likewise absent, even in trial courts. Why stop there? The next step may be futile so far as influencing legislation or company practice is concerned, but the fact remains, to my mind at least, that an "unrescindable clause" (miscalled from the beginning "incontestable") does not belong in a policy at all, except in such as intend to reduce the rescindable period to one shorter than the law allows. With the exception named, its only proper place is in the statutes. Correctly drawn, it should be there and not elsewhere. In the statutes it does all the good it can do, and I further submit that a provision dealing with the rescindable period is as out of place in a life insurance policy as a statement of a statute of limitations (full or condensed) would be in a promissory note or commercial contract. Left in the statute where it belongs, such a regulation cannot be dragged under the well-known doctrine of adverse construction which can be invoked, on the slightest pretext, or none, when it becomes the insurer's language by appearing in the policy. When a court resorts to this doctrine, we usually find the word "adverse" an outstanding example of accurate expression.

MR. WENDELL M. STRONG:

(Written discussion presented by the Secretary)

Mr. Buttolph's paper deals with two important matters relative to the incontestable clause. The first of these has to do with the application of the incontestability clause to claims where the contingency insured against has not occurred. We find the incontest-

able clause invoked in cases of suicide within the first year although the contract specifically states that suicide is a risk not assumed. We also find the clause invoked in cases where claim is made for Double Indemnity and where the cause of death was specifically excluded from the risks assumed under the Double Indemnity provisions.

Such a situation could hardly occur if there had not grown up ideas in regard to life insurance which are at variance with the general law. One of these is the general idea held by beneficiaries, and corroborated by decisions of courts of certain states, that if the insured dies the insurance is payable and any refusal to pay is a contest of the policy.

Another is the tendency to incorporate exceptions in the incontestable clause. A typical example of this is the incontestable clause set forth in the section of the Illinois statute requiring standard policy provisions, one of which is that the policy

> shall be incontestable after it shall have been in force, during the lifetime of the insured, for two years from its date except for non-payment of premiums and except for violation of the conditions of the policy relating to naval or military service in time of war and except as to provisions and conditions relating to benefits in event of total and permanent disability and those granting additional insurance specifically against death by accident.

The exceptions in this statute are extraneous and tend to befog the issue. In the first place, the exception as to non-payment of premiums is not necessary. The policy provides what happens in case premiums are not paid, and it is not a contest of the policy within the meaning of the incontestable clause for the company to decline payment because of non-payment of premiums.

The same considerations apply with even greater force to the exception of the provisions for Double Indemnity and Disability benefits. These provisions define the risk assumed by the company and have no relation to the question of incontestability. Certainly it would not be considered a contest of a fire insurance policy if a company refused to pay a claim for the destruction of a house by flood when the policy insured only against destruction by fire. The company determines its liability entirely by the contract, and the exceptions incorporated in the incontestable clause can only mislead as to what constitutes a contest of the policy.

Recent decisions on this point would indicate that the courts are beginning to take the proper view of the question. Mr. Buttolph has referred to the decision of the United States Circuit Court of Appeals, Fifth Circuit, in the case of *Sanders* vs. *Jefferson Standard Life Insurance Company* (10 Fed. Rep. 2d 143). Since that decision was rendered there has been another important decision in the Eighth Circuit in the case of *Mack* vs. *Connecticut General Life Insurance Company* (12 Fed. Rep. 2d 416). The policy was an Illinois contract, and the incontestable provision of the Illinois law, which I have quoted, was involved. The Circuit Court of Appeals made the following distinction between the incontestable and suicide clauses:

> A contest made within two years is not to be confused with a defense of death by suicide committed within two years. In the incontestable clause the two year period is a period of limitation. The contest by suit or answer must be instituted within two years from the issuance of the policy. But in the suicide clause the two year period is a period of exclusion of risk on account of suicide. That clause does not undertake to limit the time within which the defense of suicide may be made nor does the statute of Illinois above quoted undertake to do so.

The United States courts have generally adopted the proper viewpoint as to the distinction between what constitutes a contest of the policy and what constitutes the contest of a claim where the cause of death was not one of the risks insured by the policy. Earlier decisions in the state courts tended toward what seems to me the erroneous interpretation by applying the incontestable clause to the case of suicide. Mr. Buttolph, however, cites two or three recent cases from the state courts which indicate a changing attitude on the question. It is to be hoped that we shall soon have a sufficient number of such decisions so that the difficulties which might otherwise come up will be avoided.

The second question raised by Mr. Buttolph is whether the incontestable clause cannot be invoked by the company as well as by the claimant; in other words, whether if the company is barred from contesting for fraud after two years the claimant should not be equally barred from contesting the policy after two years on the grounds of fraud or misrepresentation.

The question, however, is whether the insured or one claiming under him ever does wish to contest the policy. He may, and not

infrequently does, desire some change in the contract, but after all his claim is made under the contract and it is rather a question of reformation than of contesting the contract.

The cases that arise most frequently, it seems to me, are where an attempt is made to redate the contract from the date that it bears to the date when it was delivered, in order either to make the premium payment come before the end of the days of grace or extend the insurance to a time that would cover the insured's death. The trend of one line of decisions has been to consider the premium-due date as the anniversary of the date on which the first premium was paid rather than the date provided for in the policy, thus effecting by judicial action a change in the contract between the insured and the company. These decisions to some extent have been based on a decision rendered many years ago in the United States Supreme Court in the case of *McMaster* vs. *The New York Life Insurance Company* (183 U.S. 25). The circumstances of this case differ from those of many of the cases of more recent date.

The policy was dated the date of the application instead of the date when it was written, on request made by the agent, of which the insured did not know. Also the insured did not have actual notice of this through reading the policy, since there was evidence that when he received the policy he put it away without reading it. He died on what would have been the last day of grace after the end of the first year had the policy been dated the date it was issued instead of being dated the date of the application.

In addition to the foregoing there was evidence that the insured was told if he took the policy he would receive thirteen months' insurance. Also, the lapse occurred at the end of the first year, so that the insured had not acquiesced in and affirmed the dating back of the policy by repeated premium payments as of the anniversaries of that date.

The decision, however, has been used as a precedent by both state and federal courts in cases in which the underlying facts were entirely different and to which it does not properly apply. An essential thing in the McMaster case was that the insured had never, so far as the evidence showed, accepted or ratified the dating of the policy back to the date of the application; on the contrary, there was evidence to show that he believed the date to be the later date. Such a situation could not occur if the dating back had been by the

insured's request, as it sometimes is, or if the date had been accepted and affirmed by the insured many times by his paying premiums as of the anniversary of that date, without protest.

Thus the courts have arbitrarily redated contracts in force for some time and have applied to life insurance contracts a different law from that applied to other contracts. In no other contract would the courts allow a change in the written provisions years afterward when the contract has been continually in the hands of the party under whose interest the claim was made. Certainly, if the company is to be prohibited from contesting a policy after two years the beneficiary should be estopped in such a case from obtaining a change.

While some of the line of cases I mention have been very extreme in disregarding the ordinary doctrine of estoppel and also the principle upon which a contract can be corrected when it did not express the contract intended to be entered into by the parties, nevertheless they are questions of reformation, rather than contest, of the policy.

The policy imposes no duty on the insured, or those claiming under him, enforceable by the company and when there is a contest on any point brought by the insured or beneficiary it is not (with an almost negligible number of exceptions) under a claim that the policy is void, but rather a contest to enforce the policy reformed to meet their claims.

MR. R. A. HOHAUS:

Mr. Buttolph discusses two apparent misconceptions of the meaning and intent of the incontestability provision required by law and emphasizes the desirability of a clean-cut distinction between a question of coverage as opposed to the question of forfeiture. I agree with his attitude that the amendments permitting the specific exception of total and permanent disability, accidental death, etc., are not very advantageous and that they do not add any rights not previously had by the insurer. While I do not feel that these amendments offer any particular disadvantage, I think the author's warning as to the possible effects of continual amendment of the clause is well given.

The second misconception stated is that the incontestability clause is binding on the company and not on the insured. In refer-

ence to the statement in the *American and English Encyclopedia of Law* that "the practical and intended effect is to create a short period of limitation within which the insurer must test, if ever, the validity of the contract," the author asks, "Why insurer and not also insured?"

Probably most of us, in reading this paper, were unusually struck by this question because it gave a viewpoint entirely new to us. While at first reading I could not satisfy myself that it was not a very important point, the proverbial attitude of doubt or contrariness which many impute to actuaries seemed to operate immediately, especially since the author surmised that "defendant companies have never brought it strongly to the attention of courts," and therefore I tried to discover the reason for absence of this defense.

The contentions of the plaintiff or the defendant in actions in court concerning life insurance policies may be classified in four groups according to their purpose. They are:

1. To prove the contract *void*
2. To prove the contract *voidable*
3. To obtain the correct interpretation of the contract
4. To enforce fulfilment of the obligations of the contract

Which of the foregoing are affected by the incontestability provision?

The first group includes contentions, which, if sustained, declare the contract void *ab initio* because of fundamental misunderstanding or because an essential condition was never fulfilled or the contract never completed. There having been no meeting of minds, there was no contract at any time and hence none to contest. An extreme example would be the issue of a Group Life policy to an employer who thought he was obtaining Compensation Insurance. Another example might be a case where the applicant obtained possession of a policy without making payment of the necessary premium.

The second group includes contentions attempting to establish misrepresentations, misleading statements, etc., which make the contract voidable. In such cases the contract is contested because the actual consideration differed from that alleged. However, unlike the first class, the contract would be in effect if the action were not taken, and hence the expression "voidable" as contrasted with

"void." An example of this class is an action based on a misstatement by the insured in his application.

The third group—correct interpretation—includes the contentions based on alleged ambiguity or efforts to overcome provisions alleged to be contrary to state statutes. An example of the first type is *Johnson* vs. *American Central Life Insurance Company*, cited by the author, and in which the question was "the meaning of and construction to be placed on the contract," and the court stated "there is a conflict and an ambiguity" and "the construction should be applied which is favorable to insured, rather than to the insurer." An example of the second type is *Lillard* vs. *Metropolitan Life Insurance Company*, in which the plaintiff maintained that the provision liquidating an indebtedness in event of extended term insurance, by reducing the face amount in such proportion as the indebtedness bears to the cash surrender value instead of by shortening the length of term, was not in accord with the state statutes and that the policy provision fell before the statutory requirement. In both types no contest of the validity of the contract was made but the actions were instituted to determine just what the obligations and the coverage of the contract were. I do not see how the incontestable clause can be invoked by the insurer to prevent an action questioning its interpretation of the contract as written.

The fourth group is self-explanatory and obviously includes no contests of the contract but efforts to enforce it. Hence the incontestability provision does not affect this group.

Of the four groups, then, only the second embraces contentions which contest the contract and therefore it is the only one affected by the incontestability provision.

Let us grant that the incontestable clause is binding on the insured as well as the company. It can bind the insured or beneficiary, but what is there to bind? Does he have occasion to present contentions of the second group—to prove the contract voidable? Does the insured contract to do anything or does he assume any obligation? Why should any insured wish to contest the validity of any policy when he has no obligations of which he may wish to be relieved? Practically every court action instituted by an insured or beneficiary is begun as a "question of coverage" and not "a question of forfeiture," the latter question, if it enters, being raised by the defense of the insurer. Hence if the first misconception stated

by the author be removed and the application of the incontestability apply only to "a question of forfeiture," it is of little avail to the insurer to enforce the incontestability clause against the insured since the insured's contention is not on the latter question.

Mr. Buttolph's paper is very interesting and stimulating but I cannot agree as yet that his second point offers "a very important defense against a certain class of unjust and fraudulent claims." However, the insurance companies can well use any additional arguments for defense, and I shall not be unhappy if the foregoing reasoning be proved incorrect.

MR. JAMES F. LITTLE:

The first life insurance company that I worked for more years ago than I like to think about was established in 1869 and incorporated by act of Parliament, and this act included a provision that all policies should be incontestable from date of issue. How much the meaning of the word "incontestable" has been changed you will gather when I say that we contested fraudulent claims with equal facility and with equal success as compared with other companies. The only idea back of the incontestable clause was that we could not put restrictions on the insured's actions into the policy. If he wanted to travel to any part of the world he was free to do so; and that was all that "incontestability" really meant at that time.

We have traveled a long way from that, and in recent years — not so very recent but some time back—the legislatures of the various states were moved to put life insurance policies in somewhat the same position as other property. Where a thief has stolen some movable property and sells it in the open market to an innocent purchaser, the purchaser is protected. What the legislatures probably intended was to do something similar. But see what they did do.

What they did do was to say, in effect, if the thief retained the property for a certain length of time and was then caught with the goods, as we say, he was to have absolute possession of it notwithstanding. That is actually what has been done in the case of this incontestable clause. That was intended no doubt to protect originally a purchaser for value. There was another meaning to it, namely, protection in the case of innocent misstatements of the original applicant, but it was primarily intended to protect a purchaser for

value. If merely that provision had been inserted, probably not more than one out of ten of the claims that we now pay under this incontestable clause would have been covered. The cases of innocent misstatements are few and rarely of any importance.

So we have the striking and unsatisfactory result that where fraud has been committed, in order that 10 per cent of honest purchasers might be protected, the life insurance companies have to pay ten times the amount necessary to secure such protection.

This indicates on the part of the legislatures an enactment of laws without a clear understanding of what they were actually doing. Unfortunately, the trouble did not stop there. When the laws came to be interpreted by the judiciary they made a most extraordinary extension even of these laws in the application thereof.

In the old days the policy would be considered void in case of fraud. That was quite inconvenient for both parties; and later on we began to understand that the policy was voidable. In other words, if the life insurance company, after it had discovered the fraud, accepted the subsequent premiums without informing the insured of the situation, it waived its right to contest the policy, and made the policy good. But, remember, it was not until the company had committed some such act that the policy actually came into force.

We all know that if a policyholder dies during the first year, where a year is the period selected for the incontestable feature, and claim is not made until after the end of that year, or if the life insurance company has not commenced suit to cancel the policy within that year, notwithstanding the death of the insured, they are not in a position then to commence suit. This is so well established by numerous decisions that we must take it today as the actual situation from the legal point of view.

Yet, the situation was this, that at the time of the death of the insured within the first year, this change in the law, which, remember, was intended to operate after the expiration of a certain period, did not change the fact that at that time the insurance had not gone into effect. It needed some action by the company to put it into effect.

Now, at the end of the year the policy of insurance was to go into effect under such wording as "after one year from date this policy shall be incontestable"; but this policy is a policy of insurance

against some event to happen. You cannot put a policy of insurance into effect on something that has already happened.

We had a decision to that effect from our general counsel, the late well-known and respected Mr. Lindabury, who told us when we were putting the disability concessions into our policies that the Board of Directors had no power to make that retroactive so as to affect people already disabled. He said: "This is insurance. You cannot put insurance into effect against an event that has already occurred."

At the end of the first policy year, therefore, clear logic and the law would have required that only a policy of insurance against an event happening thereafter could go into effect. In the event of death within the first year, the policy not otherwise in force never could go into effect to cover that death merely because of the one-year provision, yet the courts have decided otherwise and have decided it so unanimously, or almost unanimously, that we must accept this ruling as the actual situation.

We have, therefore, an initial unfortunate mistake of the legislatures greatly exaggerated by the action of the judiciary. While the legislatures intended merely that after the first year the situation should be changed and the policyholder could feel that he was then secure, the courts, by their action, have totally changed the face of affairs as existing during the first year, and in a large percentage of the cases of fraud have prevented the companies from securing a remedy that they were entitled to. Legislation in some states has relieved this situation, which, however, apparently could never have arisen on proper interpretation of the earlier laws.

As Mr. McBride said, there is probably not very much hope of immediate relief; but I agree very much with Mr. Buttolph when he points out that right from the start the situation was confused by putting into the clause the absurd provision, "incontestable except for non-payment of premiums," in other words, confusing altogether the contest of a claim with the contest of a policy. That has, of course, I think, led the judges on to their, what appears to me, confused understanding of the intent of the incontestability clause.

There is one other point that occurred to me while the matter was being discussed. In the case of a policy obtained by innocent misstatement, there is reason for the legislatures to set a period

after which such innocent misstatement would not avoid the policy. That is never of much significance or consequence. But if a policy has been obtained by fraudulent misstatement (I wish some of the lawyers here would answer this question), under the present interpretation of the law after one year the beneficiary is fully protected, but as the company has suffered a loss through the fraud of the applicant, could not the company start suit against the insured's estate to indemnify it for the loss incurred by the fraud of the insured?

In other words, the company must pay the policy; but one would suppose that ordinary law and reason would suggest it had a right of claim against the man whose fraud had involved it in the loss. The law does not say anything directly to prevent the company from recovering, where the insured leaves an estate, for any loss to which they have been subjected by his fraudulent act. The point is one for the lawyers to decide.

There is one other point I should like to draw attention to in this connection. It has long, and very properly, been the rule of the courts of law when reading a policy to say that as the policy was drafted by the life insurance company, any ambiguity should be interpreted favorably to the insured; but where that ambiguity is compulsory under statute and under rulings of insurance departments, it is wrong for the courts to continue to use that rule. They should then look to the fairness of the intent, and altogether get rid of the old idea that the company was responsible for the phraseology. If a question arises as to any part of the contract where the company really has a choice of phraseology, then it would be perfectly right for the courts to decide as heretofore; but, at the present time, we have no choice, but have to find something that complies with certain specific regulations, particularly in the case of this incontestable clause, whence equity, as well as law, would suggest that at this stage the old rule of law that the interpretation unfavorable to the insurance company should be the one adopted should not be invoked. I think nowadays it should be recognized that with the aid of the law and the insurance departments the insured has been adequately represented in preparing the contract and that the insurer is only one of the two parties drawing up the agreement which must therefore be given the most reasonable interpretation.

MR. LAWRENCE M. CATHLES:

I think it is quite plain that we have established a grievance on the part of the companies against the courts and against the statutes, but I am just wondering if we have gone quite deep enough into this question of incontestability. Perhaps we should have our minds a little clearer on it if we went a little farther back.

I agree with Mr. Evans that the wording was probably used for euphemistic purposes. You don't like to use any wording that indicates the possibility of dispute. It is much nicer to say that it can be incontestable, but why did that clause ever come into life insurance policies? I don't think it answers the question entirely to say that it came in from agency pressure.

My mind goes back to the first insurance lawsuit of which I have any recollection—I don't mean personal recollection, but recollection of reading; and that was in Great Britain in 1587 where policies used to be issued for only a limited term. This particular policy was issued to insure a man for twelve months for a premium of, I think, about $80 a thousand. I don't remember the dates but I do remember that the insured died within 365 days of the issuance of the policy and the company contested the claim because they said the policy was not for one year but for twelve months, and a month consisted of something less than one-twelfth of 365 days. I think it was 28 days that they claimed the month consisted of. The insurers deservedly lost their contention and were compelled to pay the claim.

But thinking again of these old cases, we can all remember of reading cases where a policy was contested because a man misstated the age at which his grandmother died, etc. There have been a number of claims contested on trivial grounds.

I just wonder whether that is not the reason why in the development of the business the companies agreed that such a condition was absurd, and that they should try to settle without contest and get rid of the stigma which these old cases had cast upon the business.

MR. J. CHARLES SEITZ:

If it were a necessity to save time at this meeting I could begin and finish the discussion with the words, "Well done, thou good and faithful servant," thereby indicating my full approval of the author's remarks.

"Way Back When," as the cartoonist expresses it, or to be explicit, in the November, 1917, *Record* there appears about a page and one-half of discussion between myself and others in an effort on my part to put across the meaning of "incontestability"—my contention being that one cannot contest, or refuse to carry out, any terms or conditions of the life insurance contract after the expiration of such limited period as may be expressed in the contract.

The topic which called forth the discussion referred to dealt with the separation of policy provisions which decreased or restricted the benefits and the incontestability provision per se. It was specifically stated in this discussion that there was a conflict between the terms of the policy if the pure Incontestable Clause were in one part of the contract and the Military or Naval Service provision were in another part of the contract. It is no infrequent occurrence to find a person falling into the error of interpreting a contract by part instead of as a whole. It is a fundamental legal principle that the contract must be read as a whole. Please understand I am not referring to the matter of ambiguity in the contract, which can be defined as an expression of one idea in such form as to be subject to more than one interpretation.

Mr. Buttolph has initiated an excellent thought which might be used effectively in future lawsuits if we call the attention of our legal departments to it. I refer to the second misconception, viz., that the Incontestable Clause is binding on the company but not on the insured. Let us lay stress on this thought, which may mean a victory for the company in a lawsuit. Apparently little things such as this turn the tide when assumed larger ones have no effect.

It appears futile to me to suggest an effort at reconstruction of the Incontestable Clause, that is, a division of its parts by a change in the law, and therefore I shall not proceed farther on this point.

There is a practical way to increase the rapidity of the trend toward correct understanding to which Mr. Buttolph referred. Is it not possible that Mr. Buttolph's article be printed in pamphlet form and distributed to the bar associations or that a member of the Legal Department of a life insurance company submit a brief article on the subject before the Bar Association meetings of the locality wherein the company is domiciled? The correct information imparted in this manner far outweighs the effect when argued specifically before the court. In the first instance the thought will have

been crystallized, and in the latter instance it is only in process of crystallization.

In noting the trend of the discussion, I think Mr. Buttolph has drawn attention not so much to the matter of the discussion of the incontestability feature but to the addition of the exceptions to the incontestability feature. I have interpreted his paper to mean that he is laying stress on including exceptions to the incontestable clause rather than making the discussion primarily on the incontestable feature per se.

MR. W. A. P. WOOD: In view of the discussion about incontestability, I would like to take a particular example. Supposing a person engaged in aviation in the state of Illinois should make application for insurance and be unwilling to pay an extra premium; is it possible to insert in the policy a clause to the effect that in the event of death when engaging in aviation the amount payable shall be limited to the reserve on the policy without any restriction to the one-year or the two-year incontestable period? That is, is it possible to have a clause which will put restrictions after the incontestable period expires? Would there be any likelihood of danger later on in connection with the settlement of a case of that kind if the assured should meet with an accidental death and the company should tender the reserve on the policy?

MR. F. R. JORDAN: I should like to mention one thing. In the Illinois law there is a provision that is probably not found in the laws of other states which specifically states that you cannot pay less than the face amount, depending on the cause of death after the incontestable period. So that would cover that particular feature in this state.

MR. WOOD: That would apparently shut it right out.

MR. JORDAN: Yes, it would shut it right out on that, but that particular feature is not in the laws of other states.

MR. CATHLES: It is in the laws of certain other states.

MR. JORDAN: Probably in the laws of certain other states, but it is not common to the laws of all states.

MR. A. C. GOOD: I just want to point out that in the state of Missouri a company would have to pay a suicide claim in full even though the death occurred within the incontestable period and even though the usual suicide clause was contained in the policy.

MR. LITTLE: I might tell Mr. Wood that we had one case of a

man in Pennsylvania who wanted such a policy. We could not give it to him in Pennsylvania, but we could in New Jersey. The clause used was simply the thing that Mr. Buttolph properly objects to. We tacked on to the incontestable clause some provision about the event of death from aviation.

There seems to be some confusion because they tell us that we cannot put in a life insurance policy a limitation to pay less than the face of the policy on account of death from any cause; yet we can put in the policy of insurance a provision that we will pay more than the face of the policy if death is due to one cause, namely, accident. I don't know whether, if a policy carries the Double Indemnity, we may not later on be asked to pay the double amount for death from any cause under such a clause, the logical inference would be that if we cannot except one specified cause of death, we cannot except all causes but one—as we do in the Double Indemnity provision.

MR. JORDAN: In reference to Missouri, does not the law governing the suicide clause there require that you must prove the suicide was contemplated when the policy was taken out? If you can prove that, then you can limit your liability; isn't that correct?

MR. GOOD: You are right there, but I think that would be a pretty hard thing to do.

MR. JORDAN: I know, but if you can prove that suicide was contemplated when the policy was taken out, can you limit your liability?

MR. GOOD: Yes, if you can prove it to the satisfaction of the jury.

MR. CATHLES: In answer to Mr. Wood's inquiry, I have understood that in the state of Washington an exemption from liability due to death from aviation would be allowed.

MR. C. O. SHEPHERD: We have been advised by our own legal department in that connection that we would be on dangerous ground in most of the states, but there are a few where it can be done. New Jersey is one, Florida is another. No doubt there are some other states where it is possible.

MR. H. W. BUTTOLPH:

Mr. Evans began his discussion with a confession of difficulty in understanding just what I was talking about, and demonstrates his failure to understand me by quoting from section 380 of Ruling

Case Law and following the quotation by the statement that my thesis is a criticism of the definition of incontestability therein contained.

It seems to me that this shows very clearly one of the points that I tried to bring out—namely, that there is not a proper differentiation between a contest of a policy and a contest of a claim. If I understand the meaning of the words quoted by Mr. Evans from Ruling Case Law, I agree entirely with the rule there stated that such a provision "precludes every defense to the policy other than the defenses excepted in the provision itself," but this does not mean that such a provision enlarges upon the scope of coverage or precludes every defense to every claim that may be brought under the policy.

I am, of course, glad to agree with Mr. Evans and others who have brought out the same point that the use of the word "incontestable" was unfortunate and that the desired result could have been brought about by a provision very much more clearly worded.

In one other particular Mr. Evans seems to have misunderstood me. I did not question the good faith of life insurance companies nor did I suggest that any company should enter a plea, admitting that misrepresentation of the contract had been made by one authorized to bind the company, but that such misrepresentation was outlawed by the Incontestable Clause. The point which I did try to make and which seems to have been thus misconstrued was one which will appear in the answer to Mr. Hohaus' question, which I will now attempt to make.

In his classification in four groups of the contentions which form the basis of actions in court concerning life insurance policies, it seems to me that Mr. Hohaus has not covered all of such contentions as may arise unless his third group be made broader than would appear to be his intention. Certainly, his reference to the case of *Johnson* vs. *American Central Life Insurance Company* as a type of case coming within this third group does not completely substantiate his point.

In that case, the court did hold that there was conflict and ambiguity as to when the insurance first became effective and that this conflict and ambiguity should be resolved in favor of the insured, but it then went on to apply the doctrine of the Halsey case, which was based not on an interpretation of the contract but on an

enforcement of what the court held to be justice and equity—namely, that the first premium paid by the insured, having been equal to that for subsequent years, entitled the policyholder to a full year's insurance, and that the contract should be not interpreted, but reformed, with this in mind. It is such a reformation of a contract, accepted and complied with by the insured for years, that I believe the Incontestable Clause can properly be regarded as prohibiting.

There are many valuable points brought out in the other discussions of my paper with which I fully agree and which have been so well expressed that no emphasis on my part seems proper, so I close the discussion with an expression of appreciation of the attention which has been shown by the men who have so ably discussed the paper.

OSCULATORY INTERPOLATION: NEW DERIVATION AND FORMULAS

By W. A. JENKINS

MR. JAMES W. GLOVER:

(Written discussion presented by Mr. H. C. Carver)

It is strange that the method adopted by Mr. Jenkins for deriving the formula for osculatory interpolation has not been developed before. A great deal of work has been done upon this subject as illustrated by the long list of references at the beginning of this paper. The method of attack, however, seems always to have been the same, that is, of selecting auxiliary partial curves to determine the values of the derivatives at the ends of the intervals under consideration. It has always seemed to me that this was a peculiar line of approach and that it would be more straightforward to determine directly the values of the derivatives of the successive interpolation curves. Mr. Jenkins is the first one to carry this idea through to the point of developing the formulas, and it is all covered under two very simple conditions, namely: (1) when the formula is applied to any interval, the end ordinates of the interval will be reproduced; and (2) when the formula is applied to any two adjacent intervals, the two curves determined will have contact of order p at their junction.

It is interesting to note that the author assumes Everett's interpolation formula up to and including terms involving differences of order $(2n-2)$ of the third and fourth ordinates from the left end, where $-n$ is the subscript of the ordinate at the left end. The differences of order $2n$ of the first and second ordinates from the left end are multiplied by functions $\phi(1-x)$ and $\phi(x)$, respectively, whose degree and coefficients are determined by the two fundamental conditions mentioned above. Illustrating the procedure by determining the fifth-difference formula, the one used most in practice, four terms agree with Everett's formula and the remaining two terms involving fourth differences of the function at the points -1 and -2 are multiplied by $\phi(x)$ and $\phi(\xi)$, respectively, where $\xi=1-x$. The application of the second condition which means that the first and second derivatives of the adjacent interval interpolation curves must be equal, respectively, leads directly to the fifth-difference formula with contact of order 2. This formula is of degree 4 in x while the developments of Sprague and Lidstone lead to a formula of degree 5, indicating that the latter are redundant to the extent of 1 degree in the abscissa variable x. Tables II and III of the paper show that with higher degrees of contact the redundancy in the Sprague and Lidstone developments is still greater with regard to the degree of the function representing the osculatory curve. There is, however, a range for which the Sprague formulas show a lower degree for the final osculatory interpolation curve than the Jenkins formulas, although the latter are always less in degree than the Lidstone formulas except when n and p are unity. Mr. Jenkins suggests that a minimum degree curve might be obtained for the whole range of values of n and p, superseding the method proposed in this paper, when n is greater than or equal to $p+1$ (nearly). I understand that he has now carried through this suggestion and is presenting a paper today in which the minimum osculatory interpolation curve has been derived for the whole range of values of n and p.

It is to be regretted that the author has not illustrated his method by a numerical application to a mortality table, giving the comparative results by his method and that of Sprague and Lidstone. The Table IV giving the minor in terms of the major differences is simplified by the fact that the fifth minor differences are constant. An example of this kind might bring forth some facts which may be

favorable or unfavorable to the general theory, which have not been noted by the author nor the reviewer. From the point of view of theory, the author is to be commended upon greatly simplifying the theory of osculatory interpolation and putting this subject upon the basis of a common contact of adjacent interval curves without introducing the idea of partial curves.

MR. JOHN F. REILLY:

(Written discussion presented by Mr. H. L. Rietz)

The first osculatory interpolation formula was developed by Sprague in 1880 with the aid of "partial interpolation curves." This method was the only one employed until 1921 when Mr. Robert Henderson proposed to develop such formulas by an entirely different method, making no use whatever of "partial interpolation curves." He succeeded in producing a formula which is osculatory if differences of the sixth and higher orders vanish. Instead of making the two slopes at the point of juncture each equal to a third quantity, and likewise the two curvatures, as Sprague did, he made one slope equal to the other slope, and one curvature equal to the other curvature, without any reference to other quantities. This resulted in lowering the degree of the interpolation formula. Again in 1924 Mr. Henderson showed how to solve the difference equation that he had formerly approximated to by resolving it into two factors, thereby obtaining a method for the practical application of his type of osculatory interpolation formula without assuming that differences of the sixth and higher orders vanish.

In the paper under discussion Mr. Jenkins also has discarded the "partial interpolation curves" process, and, by imposing the same conditions as Mr. Henderson did, derived an interpolation formula that is truly osculatory.

Everett's formula is assumed as a basis, and the osculation is acquired by modifying the coefficients of the two terms containing fourth differences. In this respect the plan resembles that of Lidstone who modified only the final term of Newton's formula. It is worthy of note that while Mr. Jenkins modified the coefficients of two terms, one of these two coefficients being originally a function of x, the other the same function of ξ, he has modified them so that one becomes a certain function of x and the other the same function of ξ, and that we cannot say that there are two independent modifica-

tions. On the other hand, Sprague modified the requisite number of coefficients to make the resulting formula of minimum degree with the use of "partial interpolation curves." It therefore seems clear that it would be possible to lower the degree of Mr. Jenkins' formula still further in some cases by modifying the coefficients of more than two terms, and that a curve of minimum degree would be obtained by modifying the coefficients of a suitable number of terms. The author of the paper has set this as his further problem, and it is greatly to be desired that he may succeed in solving it. Mr. Henderson's fifth-difference formula with contact of order 2 is of the third degree, while Mr. Jenkins' is of the fourth degree, but Mr. Henderson's is only approximate, while Mr. Jenkins' is exact.

It will probably be clear to most readers of the paper that in Table III n is not the order of differences employed; the order is really $2n+1$, since the use of two differences of order $2n$ is equivlaent to one difference of order $2n+1$. From this table it is evident that Mr. Jenkins' method produces formulas of lower degree than either Sprague's or Lidstone's when the order of contact is high and the order of differences is low, and that the reverse is true when the order of contact is low and the order of differences is high.

The case of greatest interest to actuaries, probably, is that in which $n=p=2$, that is, second-order contact with fifth-order differences, and here Mr. Jenkins' formula is superior to Sprague's and Lidstone's, inasmuch as his is of degree 4 while theirs is of degree 5. The interest of the mathematician, however, is likely to be centered on the "generalized derivation," for he is naturally attracted by a general theory, realizing that from it he can immediately obtain any special case.

In conclusion, I should like to state that in my judgment this paper was well thought out and carefully written, and is an important contribution to the subject of interpolation.

MR. W. A. JENKINS:

My interest was just aroused in this matter by a question raised by Mr. Fassel. He said that the curves obtained under the ordinary formulas undulated a great deal, and raised the question of whether or not we could get a sort of average slope at each junction which would make the interpolations smoother. My answer to that question is that it cannot be done without going to higher differences. I

think, though, that this formula does reduce the undulations somewhat.

Mr. Carver: I have done a little work in trying to obtain smoother results. I find that if we impose conditions dealing with the equality of moments rather than the equality of derivatives of order higher than the first there is a great tendency to reduce the undulations. If you are going to impose five conditions for a function of the fourth degree or six for a fifth-degree function, etc., it is well to equate first derivatives and then make up the remainder of the conditions by imposing the equality of moments.

Mr. Jenkins: I wish to thank Professor Reilly for his clear, accurate survey of work done in this portion of the field of interpolation; Professor Glover for his careful discussion; and Professor Carver for his suggestive idea concerning moments.

Mr. Glover was quite right when he said that "a numerical application to a mortality table . . . . might bring forth some facts which may be favorable or unfavorable to the general theory. . . . ."

It seems probable that, as Mr. Carver suggested, the strict adherence to the conditions of continuity of derivatives only is not most advantageous for the purposes of mortality table or other graduations. It might even be desirable to admit a formula which does not reproduce the given values but does maintain their general trend, such curve having a continuous first derivative and being subjected to various other restrictions, perhaps involving moments.

INFORMAL DISCUSSION

DISABILITY BENEFITS

The considerations necessary when underwriting disability benefits in case of:

a) Occupational hazards
   1. Agricultural
   2. Railroad
   3. Metal trades
   4. Mining
   5. General industries

b) Physical impairments
   1. Build and family history
   2. Nervous system
   3. Circulatory system
   4. Respiratory system
   5. Digestive system
   6. Urinary system
   7. General impairments

MR. R. G. HUNTER:

I presume that most of us in the selection of risks for disability have been forced to be guided very largely by a study of the frequency of the causes of disability; and we have also considered that there is a certain relationship between the disability rate and the mortality rate. In my own company we have tried to adapt the numerical mortality rating to the selection of risks for disability.

Recognizing the important part that tuberculosis plays in the disability claims, we have refused the disability benefits where the numerical mortality rating is +15, imposed by reason of light weight or tubercular family history or a personal history of tuberculosis, even though we accept risks up to +25 for standard insurance.

On the other hand, we will accept overweights if we have to impose a rating up to +25. In fact, if we accept as standard a case of overweight, we will also accept it for the disability benefit.

There is a recent paper that has appeared in the *Transactions*

of the Actuarial Society, by Mr. Thompson, of the Mutual Life, that seems to confirm very largely the principles that I have enunciated; and if I read that paper correctly, it would seem that we will have to practice a much more severe selection in regard to lightweights and personal and family histories of tuberculosis.

Of course, there is a question of how far you can go in that selection since you cannot handicap your agency force too much in refusing the disability benefits where you accept the case for standard insurance.

In the case of family history of insanity (and we always consider suicide as an insanity case) and where there is some mental or nervous instability, we adopt this same principle of trying to adapt the numerical mortality rating. If we rate +15, we again refuse the disability benefits, although we accept the case for standard insurance. In our company this means that one case of insanity in the family would not bar a man from disability benefits, but two or more cases would.

The question of occupation is brought up in this same paper of Mr. Thompson's to which I have referred. He shows that many low mortality occupations have a high disability rate.

Take the case of farmers, for instance. There he shows—well, I can't remember just what the ratio is, but it is quite unfavorable. He attributes that very largely to the fact that the accidental disablement rate is about two-and-a-half times the normal.

Our experience has also been unfavorable among farmers, but not for the same reason that the Mutual Life's experience has been. We find our accidental rate about the same as the normal; but we have had an unduly large number of claims from nervous and mental and cardiac diseases among farmers.

In analyzing those claims among farmers, I have been rather struck with this fact: that a farmer, if he is prevented from following his own particular occupation, is also prevented, by reason possibly of his education and training, from following any other occupation. If this is true generally, it means, I think, that we are issuing to farmers an "own occupation" clause. That is true, I think, of a number of other occupations; and the thought has just occurred to me whether or not it would be correct practice for us to consider those occupations as own occupations and charge an extra premium accordingly.

MR. C. H. BECKETT:

"The Considerations Necessary When Underwriting Disability" is a subject upon which it is difficult to generalize; and the Chairman has just stated that he wanted us to particularize.

There are so many subdivisions of labor in even one important industry, as, for example, the metal trades, that it is impossible to provide by a general classification for all the varying degrees of probable disablement.

I have also been interested in the paper by Mr. Thompson referred to by Mr. Hunter, and particularly in the table, "Influence of Occupation on Disability." Under "Cause of Disability—Pulmonary Tuberculosis," we find some curious results, for example, the rate per 100,000 for teachers is 75; for physicians, 43. In fairness to Mr. Thompson it should be stated that he calls attention to the limits of his investigation but states that it indicates the type of investigation which it seems to him will be most useful as a guide in underwriting risks and adjusting claims in the future. It is apparent from Mr. Thompson's paper that he has given very careful study to the experience under certain phases of disability risks. Therefore, I was particularly impressed with his concluding sentence, which was to the effect that an investigation of a sufficiently extensive disability experience, along the lines which he indicated, would be of great benefit.

In the President's address he referred to the desire to have more homogeneous statistics as a basis for things we are trying to do; in Mr. Thompson's paper he made the same plea. That is what I want to talk about a bit this morning, even at the chance of being considered a crank on this subject, for I have been talking about this for a good many years.

You will recall that at a joint meeting of the Institute and the Actuarial Society at French Lick Springs we were given the new code for physical impairments. There was a strong plea made there by the Chairman of the Joint Committee, followed by the Chairman of the Committee of this Institute and one or two other gentlemen who spoke to that subject, urging companies to prepare the new forms of cards that were suggested.

The absence of an occupational code undoubtedly caused many companies to defer keeping these cards. There is now available for the first time in the history of our business a suitable general occupa-

tional code. Therefore, I wish to urge with all possible emphasis that all actuaries interest themselves in the preparation of these very necessary data and without further delay. I know of no other means by which more light can be thrown upon some of the perplexing problems of disability and extra accidental hazard within a reasonable time than through joint examinations of the data gathered by many companies.

MR. OLIVER W. PERRIN:

The Penn Mutual takes into account the mortality factor in granting Disability benefits. It may be interesting to you to outline some of the features of our practice as related to Disability benefits on substandard lives. We use the numerical system and rate up for both occupation and physical impairment. When the risk is such that we rate it at 25 per cent extra mortality, the charge for Disability benefits is, generally speaking, one-and-one-half times the regular. When the risk is such that we rate it at 50 per cent extra mortality, the charge is twice the regular. When over 50 per cent, we do not grant Disability benefits on any terms.

This, of course, is done upon the general theory that where the mortality of the group is higher than the average, the company is entitled to some extra charge for Disability benefits.

We are particularly careful about granting any Disability benefits whatever to certain occupations and impairments even though the mortality risk would be acceptable at standard rates or rated up to 25 or 50 per cent extra. A few examples of such occupations are furriers, glass-workers, dyers and bleachers, and Fire Department employees; some examples of physical impairments of this sort are light weight combined with a family history of tuberculosis, history of nervous breakdown or infantile paralysis, repeated frequent attacks of chronic asthma, syphilis, and cases with a history of any kind of heart disease.

MR. E. McCONNEY:

Some day, I suppose, we shall have some combined statistics and then we shall learn to distrust them as much as we now distrust the table on which we endeavor to base those premium rates. That has been due chiefly to the fact that each company has various slight differences in its underwriting and its method of handling claims that make a big difference in the disability rate.

For some time we have used a ninety-day clause. In the application we have asked for the income of the applicant and as to what disability coverage he carries in life companies and in accident and health companies.

When we come down to particular cases, in the farmers we find from our own experience that they are roughly about one and one-third as bad a risk as all the other occupations put together. I don't see how we can make much distinction in farmers; they are such a huge class of the country. But we do know that their mortality is considerably better than the average. So I suppose we offset the loss in disability against the gain in mortality, the statutes of anti-discrimination notwithstanding.

In the railroad trades, the accident factor is such a big thing that we do not give disability except to a few men who are in the train itself, the mailman, and the office help.

There has been considerable improvement in the death-rate of trainmen due to the safety campaign, and I suppose that is affecting the disability somewhat, but we don't know enough about it.

In the metal trades, there are very few that we grant disability to because of the heat and the fumes and the dust and the low class of labor.

In the mining trades, they are practically all rated for life insurance anyway and they don't get disability.

When we come to the physical impairments in cases with one case of tuberculosis in the family history, if the applicant is 90 per cent of average weight or better, we grant him disability. Of course there are so many factors that enter in there, the association of the applicant, the occupation of the applicant, his home environment, etc., that it is difficult to particularize. In case of two cases of tuberculosis in the family history, the applicant must be 110 per cent of average weight or better. That is not based on any statistics that we know of. It is just a sort of a handy office rule that we have invented for ourselves.

MR. J. M. LAING:

In regard to the agricultural trades, I think probably there is a point, at least we have noticed in our small experience with farmers, that while they are subject probably to a higher rate of disability, they are also rather doubtful risks from the standpoint of

the length of the claim. I think there is likely to be, or can be, quite strong selection against the company.

I recall two cases in our small experience of claims on farmers where we could not satisfy ourselves that the man was really totally disabled, and yet the evidence produced for total disability was of such a nature that we could not very well refuse the claim. The man was disabled to a certain extent, but he was still managing his farm, a large farm in each case. He had to have help in any case, even had he been well, and his farm was still running smoothly. He was able to manage it. I think that is a point that has to be considered in regard to farmers. I do not think it is a safe practice to grant them a very large disability benefit. The farmer who is getting, say, a $10,000 policy, $100 a month—I think that is probably too much to grant to even the best farmers. I am inclined to think it should be cut down below that.

MR. G. F. HOLLENBERG:

I cannot particularize as to occupation, but I might say that the Union Central has this rule: Whenever we find it necessary to charge an extra premium on account of occupation, regardless of the fact that the risk may be standard from a purely medical standpoint, we will not grant disability.

MR. R. A. HOHAUS:

One of the previous speakers pointed out that the same standards of selection do not necessarily apply for disability as do for insurance. Our underwriting procedure for insurance is in effect a numerical system. Theoretically, the same procedure should be used, probably separately, for disability, but this is not practical at the present time on account of the lack of definite information about disability.

The Metropolitan issues disability coverage with both ordinary business, which is standard, and intermediate business, which is the substandard group used for 130–50 per cent mortality.

In effect we used a modified form of separate underwriting for disability. In the schedule showing the insurance ratings for occupations and impairments another column appears which is headed "Disability" and in which one of three notations will appear, either "Yes" or "No" or "No, if more than ____ per cent."

The "Yes" does not mean that the applicant will get disability. "Yes" means that, for the particular occupation or impairment under consideration, he will be rated in the same class for disability as for insurance.

"No" means that the applicant is disqualified for disability coverage on account of the occupation or impairment under consideration.

"No, if more than ____ per cent" means that if rating for that particular impairment is greater than ____ per cent, no disability will be granted. It may so be that ordinary insurance will be granted for a case with a 25 per cent rating for the particular impairment, but that the disability will be refused because the rating is "more than 15 per cent."

Disability is refused if a single "No" appears on the rating sheet, no matter how many times "Yes" may be noted.

For certain special or unusual cases no notation will appear but instead statement that the case will receive special consideration by the underwriters.

Substandard disability is not granted in conjunction with standard insurance. As a matter of fact, we have so far found only a very few cases where it might be granted on this basis. Our present practice is either to grant disability in the same class as the insurance or to refuse disability coverage entirely. The practical advantages of this need no elaboration.

As to occupation, the schedule of occupational ratings lists the appropriate notation for each individual occupation in each industry. The question relates to treatment by industries, and the following will indicate in a general way our practice as to the individual occupations in the various industry groups.

As to the agricultural group, the notation is usually "Yes" except for the unskilled labor.

As to the railroads, it is ' No" for the highly dangerous occupations, and for the others it is usually "Yes" so that the disability coverage generally falls in the same class as the insurance.

As to the metal trades, it is usually "No" for the "grinding" occupations, while for most of the others it is "Yes," although it should be noted that the life insurance rating is generally such that it precludes disability coverage.

As to mining, the life insurance rating likewise usually precludes

disability. Outside and surface workers will be granted both insurance and disability in the intermediate class. Superintendents may, of course, be granted standard insurance or disability if conditions of their employment are favorable.

As to the general industries, it is "No" for those trades where there is a hazard from dust, heat, fumes, unsanitary working conditions, or "grade." "Grade" means the possibility of a moral hazard, such as exists in certain professions.

If the working conditions or the safety appliances are above the average, or if the hazard in the particular industry is less than normal, evidence to that effect may be submitted and will be considered in connection with the granting of insurance and disability coverage.

As to family history, "No" will appear for one or more cases of tuberculosis or two or more cases of insanity. For two or more cases of Bright's disease, apoplexy, or cancer, a smaller rating will disqualify for disability than for insurance. For overweights the disability underwriting is in close conformity with that for insurance, while for lightweights it is stricter.

The underwriting of females, while not covered directly by the scope of the question since it can hardly be regarded as an impairment, requires special consideration. The question is usually brought up in every discussion on disability; and some of us feel from the experience we have seen in group health insurance and under retirement plans that the disability rates are about twice as high for females as for males. In our own company a special underwriting is used for females. The individual risk is studied much more carefully and, further, the maximum amount of insurance with which disability will be granted is set at $10,000 instead of the $25,000 limit for males.

MR. V. R. SMITH: I want to ask a question, if I may. Perhaps it is a little outside the scope of the question as set out.

The accident companies maintain, as I understand it, a bureau somewhat the same as the one maintained by the life companies known as the M.I.B. Is it the general practice of the life companies on this side of the line to subscribe to the accident bureau so that they may obtain reports on the risk from the standpoint of the accident or benefit hazard?

MR. A. J. MCANDLESS: I think Mr. Smith wishes to know

whether any of the American life insurance companies have joined the bureau which distributes information in regard to claims paid under accident and health policies for the purpose of protecting themselves against an applicant applying for insurance carrying disability benefits, after he has had policies terminated by an accident and health company.

I do not believe that any of the American life companies have joined that bureau for this purpose. A number of companies which have accident and health departments take advantage of information contained in their accident and health files in connection with applications for life insurance.

MR. J. D. WILLIAMSON:

In regard to occupations, we have first of all the agricultural class. Our company accepts farmers who are proprietors at standard rates for the benefits, but hired help, if employed on the farm all the year round, are charged one-and-one-half times. Hired help, part-time in farm work (in Canada we have many employed on farms in the summer who go to the bush in the winter and work in the lumber camps), are taken at twice the standard rates.

In regard to railroad service—those employed in yards or along tracks are usually denied the disability benefit, but a few such as signalmen in towers, track supervisors, yard-office clerks, flagmen, and gatemen are accepted at one-and-one-half times. Employees in repair shops are charged one-and-one-half times, while employees in roundhouses are denied the benefit. Employees on trains—baggagemen, conductors on passenger trains, express clerks, mail clerks, stewards and waiters (select) in dining-cars—are accepted at one-and-one-half times, while brakemen, engineers, and firemen are denied the disability benefit. Office employees in most cases are taken at standard rates, but station baggage masters, station agents, and train masters must pay one-and-one-half times, as they occasionally are required to go about the yards.

Metal trades.—A previous speaker has referred to shop-workers, such as buffers, finishers, burnishers, polishers, etc. These classes are charged two-and-one-half times the standard rate for disability. In the hot-iron and steel industry, rates are charged at one-and-one-half times or twice, depending on the nature of the work; for instance, boss-rollers, core-makers, cranemen, forgemen, foundry-

men, and molders are taken at one-and-one-half times, while blast-furnace employees, catchers, doublers, Open Hearth and Crucible Steel workers are accepted at twice the standard rate. Ordinary laborers are denied the benefit.

In the mining industry surface workers, as a rule, are taken at one-and-one-half times, but employees on trains, that is, on the narrow-gauge railroads common to surface mines, are denied the disability benefit. In underground mines, miners are, of course, denied total disability. Engineers and superintendents of metal and quartz mines who frequently go underground are accepted at one-and-one-half times, while in coal mines the class is accepted at the standard rate.

In regard to physical impairments our practices follow pretty closely those already mentioned here today.

Mr. V. R. Smith: I think Mr. Williamson might have added that the schedule he mentions is the recommendation of the Actuaries' Club of Toronto, and anyone who is interested in it may obtain a copy of the entire list by applying to Mr. Williamson, who is president of the Actuaries' Club this year.

MR. A. T. MACLEAN:

I haven't anything very particular to say because my company is not engaged in substandard business except in the sense that we have been put into it by the fact that there are some cases that are substandard for disability and not substandard for life insurance.

I think there is one point that has to be kept in mind in this connection; i.e., that the "accident" rate or the accident hazard for the disability benefit is a very different thing from the accident hazard as viewed by an accident insurance company.

An interesting illustration of that is our own experience on railroad engineers. We have granted disability benefits, or, rather, we have granted waiver of premium to engineers for about twelve years, and up to the time I left the home office, as far as I could discover, we hadn't had a single disability claim on the group. This would seem to show that the type of accident hazard to which engineers are subject does not produce disability claims of the type covered by the life insurance contract.

Again, I think the experience of any company is in the long run going to be reflected more by its liberality in the payment of claims,

its policy with reference to those claims, than any other one subject. That is where the difference in experience has arisen. Until we arrive at some homogeneity of treatment in this respect, I do not believe we will ever get a disability experience that will be of any general application.

I might say, too, incidentally, that we follow the same rules that Mr. Hohaus has mentioned in the matter of mining superintendents, etc., who occasionally go underground. We have taken them for disability waiver only, and our experience has been very light indeed.

POLICY SETTLEMENT PROVISIONS

a) Extent of their use
   1. By the insured
   2. By the beneficiaries

b) Company practice
   1. Instructions to agents
   2. Advice tendered the insured
   3. Limitations, legal and practical, imposed on beneficiaries

c) Effect on
   1. Size of policy written
   2. Persistency rate

MR. PERCY H. EVANS:

I suppose the general purpose of this topic was to set before the Institute as clearly as possible a picture of the weight or importance of the existing system of policy settlement provisions.

The company that I am with has had the settlement provisions as a regular part of all its policies since 1900. As you doubtless know, the underwriting practices of the Northwestern are limited. Consequently, the features of the settlement provisions have, for salesmanship purposes, been emphasized to a degree. There has been a very determined effort to train agents to bring the settlement provisions into their selling arguments. The benefits of the settlement options have been made the subject of a great deal of printed matter and of instruction and discussion at agency meetings. Consequently, I suppose the experience of the Northwestern and the results accomplished under this particular phase of the business might be taken to represent almost the maximum of weight that these provisions have secured in the business.

The extent of the use of settlement provisions is covered by two items in the question, "by the insured" and "by the beneficiaries." I think perhaps it might have been pertinent to add "the use of settlement provisions by the agent," because, of course, the agent is quite a large factor in determining the use of these features.

Before submitting data as to the extent to which settlement provisions are used, I should say, as a background, that these provisions are so drawn that if the insured directs the payment of the policy to be made under one or more of the options—i.e., if the insured designates some method of settlement—the beneficiary is bound thereby and cannot make any change in that form of settlement unless the insured also specifically provides that the beneficiary may change the mode. That results in tying up the form of settlement as far as the beneficiary is concerned.

The number of death claims and endowments paid during the six years, 1920–25, averages about 10,000 per annum. There have been 60,581 death claims and endowments paid during that period. The number of these settlements put under one of the income options has increased from only 283 cases out of 9,590 settlements in 1920 to 939 out of 10,445 in 1925, or an increase from 3 per cent in 1920 to 9 per cent in 1925. That, of course, is out of a total volume of insurance in force at the end of 1925 of some 896,000 policies for $3,100,000,000. It is apparent that the proportion of policies settled in that way is still very small in spite of the educational efforts to train agents to talk settlements.

I find that of 753 death claims settled under options in 1925, 54 per cent were selections by the beneficiary; i.e., cases where the insured had made no designation but the beneficiary made an option settlement. In that connection, bearing on the question of "instructions to agents" or "advice tendered the insured," our Claim Department always calls attention of the beneficiaries under the policy to the privilege they have to put the proceeds under one of these settlements. Of course, the agents are doing that constantly. Many of the general agents tell me of cases where they have induced the widow to put the insurance under one of the options. Many of these cases are a continuous performance. Every time an instalment payment comes due the general agent has to re-educate the beneficiary to keep her money there; she wants to take it out and put it into some other investment that she thinks will pay her better returns.

Usually they do so and consequently the persistency of settlements selected by the beneficiary is very low.

As regards settlements directed by the insured, the beneficiary has not the privilege of change. While there is no legal obstacle in the way of a modification or commutation of the benefit as far as the beneficiary and the company are concerned, the company does not in practice consent to those changes. We get a small number of cases where the beneficiary wishes to change the settlement so as to enjoy the proceeds in cash. Usually she submits some more or less plausible argument about it, but in only perhaps two or three cases out of several thousands has the company been willing to become a party to such a change, and then only where it was perfectly evident that a serious hardship would otherwise ensue. We feel that the company is morally obligated to resist any change in the directions of the deceased insured.

The secretary's department reports that of the new insurance issued about 4 per cent by policies and 6 per cent by amounts is issued with some settlement indorsement. The bulk of the settlement indorsements is made after the policies are issued. For example, in 1924 the secretary made indorsements on 14,080 policies; in 1925, on 18,436 policies; but even those large numbers amount to only 2 per cent of the policies in force.

I have noticed one interesting thing about the demand for these settlements in the case of endowments. Our endowments are payable to the insured at the end of the endowment period, the beneficiary having only a death interest in the policy. There is a very marked difference related to current financial conditions in the number of endowments put under the options by the insured at the end of the endowment period. For example, in 1913, $3\frac{1}{2}$ per cent of the endowment settlements were put under one of the options; but in 1921 that percentage dropped to seven-tenths of 1 per cent, 1921 being a year when ready cash was quite in demand. Since then the percentage has increased so that in 1925 nearly $5\frac{1}{2}$ per cent of the endowment settlements were placed under the options.

Concerning "instructions to agents" I have only to say again that we endeavor to train our agents to specialize in the settlement features of the business because we feel that our success depends upon that kind of personal service to policyholders. We have quite a paternal attitude of mind toward the parties to the contract. We

give a great deal of specialized attention to the formulation of settlement indorsements; endeavoring, of course, to adhere to the text of the policy contract and not to make forms of settlement outside of it. But where there seems to be a special and legitimate need that is not merely a salesman's idea, we go pretty far in accepting and providing for special settlements. For example, we do not have in the policy that form of settlement which provides for the payment of a fixed amount monthly for as long as the proceeds of the policy will last, depending on future rates of interest in excess of 3 per cent. The reason is that we think so long as there is an uncertainty under a participating system, it is better for the parties that the uncertainty should be as to the exact amount of each instalment over a guaranteed sum than to have the uncertainty relate to the number of months during which the income will be paid. We have not adopted that particular form of settlement for that reason; nevertheless, if the parties have a valid reason for wanting that feature attached to an existing policy we meet their requirements. We go pretty far in meeting what seem to be the legitimate needs of our people.

"Limitations, legal and practical, imposed on beneficiaries": I don't know exactly what that means. Of course, we proceed on the theory that the rule against perpetuities applies to this type of contract. We have a system of beneficiaries in succession, but do not go beyond the second class, i.e., the direct beneficiary and one or more contingent beneficiaries. At the death of the last surviving contingent beneficiary, the proceeds have to be turned over in cash to the remainder interests.

Then there is a matter of remarriage which is quite an interesting question and illustrates the latitude that we take where a legitimate need appears to exist. These remarriage contingencies are, we think, quite dangerous, but we are willing to put a remarriage clause in these settlements provided it is qualified in such a way that the question of validity cannot be raised and any payments made prior to notice of a remarriage cannot be attacked.

On the third section, "size of policy written" and "persistency rate," we have no data. Of course, I have, as all of you have, a theory about it. It seems perfectly evident to us that if an agent talks about a $10,000 policy he creates an impression of size that is a very different impression than would be created if he talked about an income of $652 a year to the beneficiary. Therefore, we think

that the emphasis on settlement provisions tends to increase the size of the policies. I don't see how it could be otherwise.

So far as the persistency rate is concerned, there also we have a theory. The company has quite a low lapse rate. Last year the lapse rate for policies during the first year was only 6.78 per cent by amounts. We think that policyholders who have adopted some sort of a program of insurance under these option settlements, and buy their insurance with that end in view, are less likely to lapse than those who buy haphazard, and that option sales talk tends toward persistence; but, of course, as I have pointed out, the use of option settlements is still in its infancy. The percentage of policies issued with an option settlement is so small that it probably has little effect on persistency rates at this time.

MR. E. W. MARSHALL:

There are some interesting parallels between what I have been intending to say and what the preceding speaker has stated. The company with which I am connected also emphasizes to its agents the value of the settlement option. Our experience has been, however, that while the settlement option is used extensively as a talking point by means of which the prospect may be shown his need for increased insurance expressed in terms of income, in many cases the idea is not crystallized into a policy indorsement.

It is estimated that about 8–9 per cent of our 1925 new business, by amounts, was indorsed with a settlement option. This figure is not far different from that mentioned by the preceding speaker. On our old business about 1–2 per cent, by amounts, has been indorsed. About 3.6 per cent of our current death claims, by amounts, is settled under an option selected by the insured during his lifetime and about 0.7 per cent under an option selected by the beneficiary, making a total of 4.3 per cent. The corresponding figure on matured endowments is about 4 per cent.

There is an increasing tendency today for insurance which otherwise would be settled under a company option to be diverted to an insurance trust. I reside in a suburb of Philadelphia, and in the last six months I have received from four different local trust companies about ten direct-by-mail circulars, which were most attractively gotten up, emphasizing the usefulness of life insurance and suggesting that any life insurance taken should be placed under a trust

agreement. Naturally, such a campaign will have some effect on the number of option settlements made by life insurance companies. It will not harm the life insurance business; it will help it because it will increase the public realization of the need for life insurance in larger amounts than many persons heretofore have carried. Thus while it is not a disadvantage it will divert some business from the settlement options.

The practice of the Provident has been affected by the fact that for over fifty years, until it became a mutual company in 1922, the Provident Life and Trust Company, as it was known, had a Trust Department transacting an extensive trust business. What seems to have been the first insurance trust in the country was created by that Trust Department in 1877. The Provident has had some practical experience in the carrying out of trusts planned years before, and as a result the company is rather careful in the type of settlements which it is willing to handle. The Trust Department experience has shown that there are certain kinds of settlement which may cause great difficulty, expense, and disappointment if carried out under an inflexible insurance contract. Hence our agents are instructed fully regarding the types of settlement to be suggested to policyholders or prospects. The need of keeping these settlements of a simple nature has been stressed repeatedly in company literature, and complex survivorship forms, particularly among the secondary beneficiaries, are avoided. Also the irregular settlements which require special individual treatment in valuation, etc., are discouraged as tending to increase the expense rate unduly.

The contingency of remarriage is not covered, and in fact, wherever the complexities of the proposed settlement are such that they would make a life insurance company assume the special functions and organization of a trust company, they are not approved. We instruct our agents to place such complicated business with some good local trust company, feeling that the trust company is organized to give better personal service in such instances, has discretionary powers to meet changed conditions encountered by the beneficiary, and possesses certain legal safeguards which a life insurance company, carrying out a simple contract, cannot have.

Our rules are kept well within the 1923 *Recommendations of the Committee of the Association of Life Insurance Counsel*, with which you are doubtless familiar.

A policy-settlement arrangement is as important as a will, and is subject to revision to meet changed circumstances. We have found that some of our agents who have specialized in rather involved policy settlements have had to give much time to those revisions, although they do not involve new insurance. The agent feels that he must give this time in order to maintain the standard of service which he previously had stressed. This, of course, is excellent, but the danger is that the agent's time may be unduly taken in such matters, to the detriment of his production of new business, particularly where the settlement is of a complicated nature. The simple forms of settlements such as those for protection of wife and family and for educational purposes are greatly to be desired. They are the ones that our company feels should be stressed.

The advice to the insured or beneficiary regarding the use of settlement options is given through the agent, although we do have some descriptive literature on the subject.

We do not know the average amount of insurance per policy issued with settlement option, although we presume that the use of the income idea in the sale increases this average amount considerably. Under our income policies with deferred survivorship annuity feature, the average amount of insurance per policy is about two-and-one-third times as large as for other policies.

We have no lapse data on policies with a settlement provision, but from observing the lapse rates in our individual agencies we have the impression that properly programmed insurance persists much better than insurance which is sold on a haphazard basis, or on the basis of an appeal which is but temporary in its nature.

MR. F. J. OWEN:

In our company we find that in the settlement of death-loss claims 97 per cent of these claims are paid in full. In 1925, of a gross loss of $1,880,000, only six claims, aggregating less than $50,000, were settled under the instalment certain, instalment certain and continuous, or trust-fund agreement. The experience of 1926 to date is practically the same.

This condition with us can, I believe, be accounted for by reason of the small coverage where the insured specifically indicates that the claim be paid under one of the several instalment options, although practically in all of our contracts this right is available to

the beneficiary. Our experience seems to substantiate the fact that, if choice be left to the beneficiary, the income options in the policy contracts are ignored and demand is usually made for payment of claims in one sum. There is, of course, another fact which undoubtedly has a direct bearing, and that is the size of the policy. It seems reasonable to expect, however, that with the advent of the so-called "program insurance salesman" there will be a wider diffusion of the income idea to the insuring public, and companies that have but few agents in the million-dollar producing class and do not make a direct drive on income policies will, in some measure, increase the size of the policy sold, and possibly will have the effect to increase the amount payable under settlement options.

Our company gives no instructions to the agent as to the form of policy to be sold, leaving the choice of plan and amount wholly with the agent and the client. In other words, we have made no office effort to sell income insurance nor have we brought any pressure to bear upon the beneficiary as to method of settlement of his or her claim. It may be that we are wrong in ignoring that phase of life insurance, for probably at no period in the history of the policy is expert advice more needed, and oftentimes necessary, than at the settlement of the claim. When we consider the absolute safety of the investment, and the good interest return, it seems that a life insurance company is not going outside its province in assuming the responsibility to encourage the choice of one of the income options in the settlement of at least a portion of the insurance estate. The company helped to create that estate during life, and should be properly concerned in some measure with the method of its disbursement.

MR. LAWRENCE M. CATHLES:

Here is an extraordinary situation. According to the percentages which have been quoted, in spite of the deliberate attempts by companies over a period of ten or more years to educate their agents and policyholders into appreciating the benefits, the self-evident benefits of instalment settlements, practically a negligible percentage of claims is settled by instalments. I am wondering whether the growth of the income policy would place a greatly different complexion on that apparent situation. I have always considered that the real reason why we could not get policies settled in this beneficial way was the difference between the interest rate which was

assumed in calculating the amount of the instalments to be realized and the interest rate at which the insured thought he could invest when he was considering the proposition and which the beneficiary thought should be realized when she was receiving payment of the claim.

Perhaps there is another reason. I sympathized with the last speaker when he said he found it practically impossible to get a beneficiary to take an instalment settlement. I found the same, and I tried for years to discover the reason for it. I became convinced that it was a very natural suspicion on the part of the beneficiary. Here she had a definite promise to pay, say, $10,000 in cash. When she went to collect it, they suggested that they pay her only a thousand dollars or less this year and the same next year, etc., and she thought perhaps the company was busted and could not pay the money all at once.

I would be very much interested if we could develop some other reason for the lamentable failure of our companies to make these instalment settlements more popular.

MR. W. P. COLER: I have been thinking of arranging to have my personal insurance paid in instalments either directly from the insurance companies or by a trust company.

Is there any information available as to the tax burden which arrangements of this kind are apt to invite? In other words, at the present time are trust estates of this kind taxed as personal property?

If there is such a tax, how will the net proceeds of the trust-company method compare with the net proceeds at present available under instalment settlement options arranged directly with the insurance companies?

MR. J. M. LAING:

I think a ruling has been given by the Income Tax Department in Canada that amounts left with the company to be payable in instalments are not taxable; no part of the annuity is taxable. Whether that is responsible for an apparent increase in the number of optional settlements in our company and probably in the other leading companies, I don't know.

We have a comparatively small number of claims—when I left we had only 57 optional settlement claims in force. Of that number

15 had been selected by the assured with instructions to pay to the benificiary; 24 of them had been selected by the beneficiary to be paid to the beneficiary, that was, at the time of claim; and 18 had been selected by the assured payable to himself.

Of those 57, 31 had been selected this year; and of those 31, 13 were selected by the assured payable to himself, 13 selected by the beneficiary payable to herself, and 5 selected by the assured payable to the beneficiary.

I am inclined to think that in the case of our own company an increase in the rate of interest this year on amounts left on deposit or left under optional settlements has had something to do with this increase in the number.

As compared with regular income policies, we have 130 claims in force and only 4 of those have occurred this year.

In connection with the general subject of policy settlements, the Model Insurance Act, the Uniform Act rather, of Canada, contains two provisions which were not in the previous acts of the province.

The first provision provides:

Where the insurance money is payable in installments and the contract, or a subsequent instrument in writing signed by the insured and delivered to the insurer, expressly provides that the beneficiary shall not have the right to commute the installments or to alienate or assign his interest therein, the insurer shall not commute the installments or pay them to any person other than the beneficiary, and the installments shall not, in the hands of the insurer, be subject to legal process except in an action to recover for necessaries supplied to the beneficiary or his or her infant children.

Then that is qualified by section 2, which provides:

2. Notwithstanding anything contained in subsection 1,

a) the insured may, by an instrument in writing signed by him and delivered to the insurer, declare that the beneficiary shall have the right to commute, or alienate or assign, as the case may be;

b) the Court may, upon the application of the insurer or the beneficiary, upon at least ten days' notice, declare that in view of special circumstances the beneficiary shall have the right to commute, or alienate or assign, as the case may be;

c) after the death of the beneficiary his personal representatives may commute any installments payable to them.

3. In this section the word "installments" includes insurance money or any part thereof held by the insurer under the provisions of the next following section.

Then a further section gives the companies a definite privilege of leaving the money on deposit. It reads as follows:

Subject to the provisions of this part relating to preferred beneficiaries, where it is so expressly provided in the contract or by any subsequent agreement in writing, the insurer may hold the insurance money or any part thereof after maturity of the contract subject to the order of the beneficiary, or upon such trusts or other agreements for the benefit of the beneficiary as may be provided in the contract or agreement, allowing and paying to the person entitled to such insurance money, or any part thereof, interest thereon at a rate not less than that specified in the contract or agreement for the term during which the insurer retains such insurance money or any part thereof.

That provision under which the court may declare that the beneficiary may commute is a rather useful provision. In our old policies there was no provision for commutation of the privilege, and, like the Northwestern Mutual, we believed that the instructions of the assured should be carried out strictly, and we refused to commute instalments even though the beneficiary insisted. It has worked hardships. For instance, we had a claim under which there was a considerable indebtedness, and it required six years of the regular instalments to eliminate that indebtedness, and then the beneficiary died before the seventh instalment was paid.

This new provision gives the court the right to intercede on behalf of the beneficiary and give her some relief.

We do not permit directions that, for instance, in case the beneficiary remarries the instalments are to cease. While we do not agree with the direction in the first place, we think it is a rather difficult direction to keep account of, so we do not permit that.

We try to get the direction definitely settled as to the privilege of commutation or otherwise, and we always try to have a definite beneficiary named, in case the first one named dies, so that the later questions may not be too involved.

We have no data on the size of the policy written or on the persistency rate.

MR. A. T. MACLEAN:

In regard to Mr. Coler's question, as far as the United States is concerned, the 1926 Finance Act makes all of the usual instalment payments exempt except the interest on moneys left with the company. The Treasury Department recognized that there was interest included in some of the other instalments, but they figured the amounts involved would be small; so they deliberately made these tax exempt. That is not very clear from the law or from the regulations, but you will find that more clearly stated in the *Senate Finance Committee Report* on the Act itself.

We have put that question up to several authorities, and they are all agreed. Compared with a trust company, of course, the only way the beneficiary could avoid tax would be to have the fund put in tax-exempt securities. That, of course, would involve accepting a lower rate of interest and also the charge by the trust company for the services.

Our company, as far as instalment settlements are concerned, is in somewhat the same position as the Northwestern as outlined by Mr. Evans. We have emphasized this feature of our contracts very much indeed in the last few years, and in fact we have done so for a great many years.

Last year approximately 16 per cent of our business was on the instalment settlement plan; and a curious sidelight on that is that we have practically eliminated the regular income policy.

I might give the comparison of the number of instalment accounts arising through policy claims for the years 1921–26. In 1921 we had 494 instalment accounts. In 1922 the number was 566, an increase of 72, or 15 per cent. In 1923 we had 671, an increase of 105, or 19 per cent. In 1924 we had 865, an increase of 194 accounts, or 29 per cent. In 1925 we had 1,044, or an increase of 179, which represents a percentage increase of 21.

However, in answer to the question as I understood Mr. Cathles to put it, I would say that we have not been doing this long enough to produce sufficient death claims payable under the options. We shall probably know more about that during the next few years.

The records of our company, as I have just stated, show that the number of such accounts has increased. This shows that the number of individual beneficiaries receiving instalment settlements has increased from 494 in 1921, as I said, to 1,210 in the current

year, up to the time I left the office, which is a percentage increase of 16.

As regards the amounts of the policies, we took out a group of policies involving over $80,000,000 of insurance issued in 1925. We found that the average policy on the lump-sum plan was $4,500 and on the instalment plant it was $6,800, a difference of over 50 per cent, which, I think, is an element attributable to the sales effectiveness of the instalment plan.

We found in that same year that the "not taken rate" of the single-sum policies was a little over twice that of the instalment policies.

It is rather hard to trace the effect of persistency or lapse, whatever you call it; but we took a group of policies in the same year, and we found that the loans on policies were 2 to 1 in favor of the instalment policies. We know that loans have a tendency to increase lapsation. The indications are that the single-sum policy is more likely to lapse than the other.

We have always recommended the simpler form of option, not because we are afraid of the complicated one, but because, like Mr. Evans, we realize there are certain cases where certain complications are unadvisable.

There is no record of court decisions on this subject, however, and we think it is necessary to be careful, because there is a great danger of covering too much in one clause.

We find that of the instalment options the interest option is the most popular. Of the business taken under the options, 80 per cent of the cases were taken under the interest options; i.e., 80 per cent of all cases where an instalment option was chosen. But I think that will be changed because of the tax provision that I mentioned.

As I have said, we find that the interest option is the most popular; and that where payments are made to the widow, then on the death of the widow the balance is paid to the children in one lump; or upon the death of any child the balance to be paid to issue in one sum; interest to sons or daughters until they attain a certain age.

We have made it a hard-and-fast rule that we will not continue unpaid instalment payments to unnamed grandchildren of insured, always requiring such beneficiaries to take their share immediately as a one-sum payment.

Where unnamed beneficiaries are involved we require the provision:

It is inderstood and agreed that all decisions upon questions of fact arising at any time during settlement of the policy made by the company in good faith, based on proof by affidavit or other evidence satisfactory to the company, shall be conclusive and fully protect the company in acting in reliance thereon.

We find a great use of the instalment policy in the matter of education, and we take the beneficiary's affidavit that he or she is then in attendance at college or university, making that affidavit sufficient evidence that he or she is entitled to the money.

We prefer under educational benefits to have a separate policy for each child. We find that mixing them up is troublesome. We always felt that a policy that is for educational purposes shall be limited to that use alone and not be mixed up with other benefits, except that we do provide that in the event of the death of the father, say, before the boy or girl has attained college age, the interest shall be payable to the mother in the meantime.

We have a clause that may be of interest to some of you which was created in order to cover the case where the wife and the husband die at the same time, as in an accident. We suggest an interest settlement in favor of the wife, with the right of withdrawal on her part at any time after the insured's death, with the further provision that if she predeceases the insured, or dies subsequently to him without having exercised her right of withdrawal, the proceeds are to revert to the children.

Under this arrangement, if the wife survives the insured for any length of time, she could elect a one-sum payment in her favor by exercising her right of withdrawal; but if she should survive the insured by only a short time, then she would not have had an opportunity to exercise her right of withdrawal, and the proceeds would revert to the children.

We do not feel, as I take it Mr. Evans feels, any objection to the remarriage clause. We will allow that on any option excepting an option involving the life of the beneficiary—the option that provides for so many months certain and the life of the wife thereafter, with the provision that if she marries the benefits revert to the children -such an option would involve us in finding out whether the widow was living after the period certain.

Outside of that single condition, we will accept the remarriage clause with the provision that a statement from the widow or her affidavit shall be sufficient evidence that she has not remarried.

I think that briefly summarizes the general practice of our company.

I want to say briefly that I am satisfied that the instalment option method in any of its forms is one of the greatest things that the field force has today to help them to sell insurance, and also one of the best things the company has in order to provide the kind of insurance that stays on the books and creates good advertising for the company.

I can't help but believe that the constant sending out of monthly checks to beneficiaries is one of the best advertising campaigns that we could indulge in.

MR. R. A. HOHAUS:

The probable answer to Mr. Cathles' question is the apparent reluctance of the beneficiary to exercise the option to a larger extent.

For the claims paid in 1925, our experience was very similar to the others quoted, as $1\frac{1}{2}$ per cent by number and $3\frac{3}{4}$ per cent by amount were settled under one of the optional modes.

However, in the policies issued prior to January 1, 1907, no provision was included which permitted the insured to select an optional mode of settlement. Consequently, for those policies it has been the practice of the company to require the beneficiary to join with the insured in selecting a mode of settlement.

I have no figures showing how many of the claims were on policies issued prior to 1907, but no doubt there were quite a few, under all of which it was necessary to have the consent of beneficiary in order to have the proceeds payable by one of optional modes of settlement.

I think we all agree with Mr. Maclean as to the desirability of the optional mode of settlement, because through it the policyholder can be assured that the beneficiary will receive a certain income. The Metropolitan has a slightly different agency problem than most companies and is approaching in a slightly different manner the question of extending the use of income settlements in the sale of insurance. Instead of issuing the customary policy form with an optional mode of settlement designated by means of a rider or

supplemental agreement, we have brought out a comprehensive set of policy forms. These include various types of income policies, providing income for different guaranteed periods in addition to customary continuous monthly income forms.

Our agents' instructions explain when a lump-sum settlement is desirable and when income settlement is desirable. If the latter is desirable, we attempt to educate him to sell an income policy, instead of using the optional mode of settlement clause in the regular policy to obtain the desired coverage. There is not sufficient experience as yet to know if this method will be satisfactory.

As to company practice concerning settlement agreements, we belong to the "conservative" group and adhere very closely to the recommendations given in the reports of the Association of Life Insurance Counsel.

As to the size of the policy, our average income policy runs about two-and-a-half times the size of the average of the regular policy. However, I do not think that this is entirely due to the fact that it is an income policy, but is partly due to the fact that the applicant would have taken a policy for an amount larger than the average, even if it were not an income policy.

As to the persistency rate, we have not investigated our experience as yet; but, arguing a priori, there is little doubt that there is a better persistency rate on the income type of policy than on the general type. However, by no means do I feel that this is due entirely to the type of policy, but rather in a considerable degree to the type of man who takes this type of policy. When an applicant elects to take a policy providing for an income settlement, it is not unreasonable to assume that he is trying to get the best type of protection for his family and is the kind of insured who, from his nature, is not likely to lapse. Hence I feel that a great part of the low persistency rate which may be expected on these policies may be said to be due, not to the type of coverage, but to the type of the insured.

MR. W. H. McBRIDE:

I want to confine my remarks to *b*) (3), "limitations, legal and practical, imposed on beneficiaries," and to speak briefly about the danger that I believe is present whenever the policy is written as an instalment policy, on the face of it, either certain or continuous.

In a somewhat recent decision of the Kansas City Court of

Appeals this danger came to my notice. It was so overshadowed by the main issue of the case that I think it attracted little attention.

The issue before the court was whether the company was liable for a reduced amount of paidup insurance or for the full amount under extended insurance. The court decided quite properly, I think, under the somewhat peculiar Missouri law, that the company was liable for the full amount under extended insurance. Here is where it went astray: That was an instalment-certain policy, and the court held that the failure of the company to pay the first instalment was a default of a series of instalments promised which made the subsequent instalments due immediately. Say it was a policy promising $1,000 a year for twenty years—the company was compelled to pay the sum of the twenty annual instalments promised or $20,000, instead of the real liability or commuted value of about $17,500.

Not only did that decision compel the company to pay $2,500 more than the insurance purchased by the premiums paid, but it defeated the purpose of the insured in providing an income certain for his beneficiary.

I don't know what the remedy for that may be. The best remedy we have been able to think of is that even where settlements are elected at the time of application the policy should be written as a lump-sum policy and the desired instalments provided by indorsement.

THE ADJUSTMENT OF REINSURANCE CLAIMS

a) Overstatement of age
b) Understatement of age
c) A disability claim where the disability benefit provides for the payment of the sum insured in instalments
d) Where the principal company collects a true premium more frequently than annually but pays the reinsurance premium annually

MR. M. W. TORREY:

The question does not differentiate between coinsurance and yearly renewable term reinsurance, and I shall, therefore, take up each separately.

Subdivisions a) and b)—coinsurance.—The reinsuring company should follow the practice of the principal company and pay such

amount as the reinsurance premium paid would purchase at the correct age of the insured at the time of issue.

Yearly renewable term.—The reinsuring company should pay such amount of yearly renewable term reinsurance as the reinsurance premium paid by the principal company would purchase at the correct attained age of the insured for the policy year of death.

Subdivision c)—coinsurance.—The reinsuring company should pay the then amount of the conisurance in the same number of instalments as the principal company, the same entries being made on the books of the reinsuring company as are made in connection with one of its own disability instalment policies where the insured became disabled. There will, of course, be no regular waiver of premium as under a disability instalment policy, the premiums simply ceasing at disability and the policy becoming payable in instalments.

Yearly renewable term.—The same general remarks will apply in the case of yearly renewable term reinsurance with disability instalments as in the case of coinsurance—the reinsuring company paying to the principal company the then net amount of the yearly renewable term reinsurance (initial amount less reserve) in the same number of instalments as the direct writing company; the yearly renewable term reinsurance premiums ceasing.

Taking up now subdivision *d*)—I do not think that this question is quite as clear as it might be.

Subdivision d)—coinsurance.—In the case of coinsurance, the reinsuring company is really a partner and should follow the practice of the principal company, and if the principal company does not deduct the deferred premium, if any, then the reinsuring company should refund to the principal company such pro rata portion of the annual reinsurance premium received by the reinsuring company as is represented by the deferred premium which is not deducted by the principal company.

Yearly renewable term.—In connection with yearly renewable term reinsurance, the conditions are very different. The general practice of the companies as far as I have been able to find out is not to make any refund of the unearned portion of the yearly renewable term reinsurance premium. In yearly renewable term reinsurance, the reinsuring company is not a partner and the premiums charged are simply to cover the net death loss with a small

loading for expenses, and are not sufficient to warrant a refund of that portion of the yearly renewable term premium represented by the deferred premium waived by the principal company.

MR. R. B. STURTEVANT:

The topics covered in this question are usually not written specifically into a reinsurance treaty, although the provisions that are in the usual reinsurance agreements might readily infer the proper procedure to be followed. My remarks on this topic are confined to the annual renewable term plan of reinsurance rather than to coinsurance.

In the case of overstatement of age, i.e., a policy issued at age thirty, let us say, to an applicant actually aged twenty-nine, many companies have provisions in their policies that would not require them to pay a greater sum when death occurs than the face of the policy. The provision in such case would provide that only in the event of understatement of age would there be payable an amount differing from the face of the policy. Where, however, an overstatement of age does require a greater amount payable than the face it seems logical that the original company should not hold the reinsurer liable for more than the reinsurance policy originally called for, particularly when there is no agreement for the original company's entire excess to be reinsured with one single reinsurer.

In the case of understatement of age, i.e., a policy issued at age thirty to an applicant who, when death occurs, is found to have been actually aged thirty-one at the date of issue, it is customary for an adjustment to be made in the amount payable as a claim. Let us assume that of a policy of $10,000, the original company is carrying its regular limit of $5,000, and the excess, supposed at date of issue to be $5,000, is carried by the reinsurer. On account of the adjustment in age, let us assume that the claim is settled for only $9,500; under such conditions it seems logical that the provisions of the reinsurance treaty with respect to reduction of liability from any cause should apply. In other words, the policy has really been in effect from its beginning for only $9,500, and as the original company has agreed to carry at its risk $5,000 throughout the course of the contract, then the reinsurance should be reduced as soon as the discrepancy is discovered to $4,500.

Of course, in the case of either overstatement or understatement

of age, proper adjustment of the reinsurance premiums should be made on account of correction in the age.

As regards item *c*), let us assume a case of $1,000 reinsured on the A.R.T. plan, on which an instalment disability claim occurs. In the year in which disability claim is made, let us say that the reinsurance has been reduced to $800, the original company having in its hands a reserve of $200. It can be seen that the two together will provide a sufficient amount to pay the liability to the disabled policyholder. The original company takes the proper share of its reserve and, added to the proper share received from the reinsurance, makes its instalment payments to the disabled policyholder. Of course, no further reinsurance premiums are payable as the policy under such conditions is in effect a paidup instalment endowment policy.

Item *d*) should be a matter of contract entirely. If a reinsurance contract is on the annual renewable term plan, then in my opinion refunds are not payable to the original company.

MR. TORREY: With reference to what was just said regarding yearly renewable term reinsurance adjustments where the age was misstated and the true age at time of death was found to be thirty-five instead of thirty, I would state that when we first started writing reinsurances, we had under consideration the question of making the kind of adjustment which has just been referred to and which involves the question of interest. We decided, after taking everything into consideration, that the simplest and best way to adjust the matter was to pay such amount of reinsurance as the premium actually would purchase at the true age at time of death. This method has been found to work very satisfactorily.

MR. S. E. ALLISON:

I agree with Mr. Torrey in *a*) and *b*) on coinsurance and on a strict interpretation of a contract on the yearly renewable term. I think Mr. Sturtevant is possibly right; but I can't conceive of any company in this age, when they are all driving for reinsurance, sticking to it very rigidly. I think they most likely would pay a proportionate part.

Now, in regard to item *c*), I think that question is very interesting to us at the present time because we have such a case under consideration, the waiver of premium and payment in instalments.

Payment in instalments is really a waiver of premium and waiver of interest. We have a man disabled and we are paying him the instalments. The policy provides for twenty instalments of $50 each per $1,000 of insurance. And the question was raised whether or not we should collect the instalments from the reinsuring companies. Some, as a matter of policy, I would say, agreed to it; others thought that it was wrong.

My own belief is that while we should like to get the instalments, what they really should pay us is the interest on the instalments, because there is this problem that confronts us: In case of recovery during disability our clause provides that the instalments paid shall either remain as a lien against the policy and the man shall pay us interest thereon, or he can reduce the amount of insurance.

Now, what would happen if the policy had, say, a $2,000 cash value and we had paid the man $1,000 of instalments and the man recovered? And suppose he got mad at something and took the cash-surrender value of this policy? We had collected part of those instalments from the reinsuring company. Would we repay those? They have no cash value to their policy. It is a question in my mind as to what their action would be.

Mr. Torrey: There are two different forms of disability instalments; under one form in case of recovery, the policy of the principal company is revived for the face amount less the instalments paid and the premium is reduced accordingly; under the other form, in case of recovery, the policy of the principal company is revived for the original amount with the instalments paid charged as a lien with interest against the policy.

In case of yearly renewable term reinsurance under the first form, I think it is much better for the reinsuring company to pay its then net amount of yearly renewable term reinsurance in instalments, and if the insured should recover after the principal company had paid, say, one-half of its policy in instalments, then for the reinsuring company to revive the initial amount of its reinsurance for one-half of the original amount. It should be borne in mind that by the initial amount of reinsurance I mean the first-year reinsurance, and this initial amount is, of course, reduced each year as the reserve on the initial amount increases.

In case of yearly renewable term reinsurance under the second form referred to, I think it is also much better for the reinsuring

company to pay its then net amount of yearly renewable term reinsurance in instalments, and if the insured should recover and the principal company revives its policy with the instalments paid charged as a lien with interest, then the reinsuring company should revive its reinsurance for the full initial amount and the principal company should return to the reinsuring company the instalments paid.

Any other method than that outlined above involves the question of interest. If, however, interest is taken at the same rate at which the instalments are calculated, it does not make any difference mathematically which form of settlement is made.

MR. E. A. PORTER:

Coverage in reinsurance of instalment disability policies was one of the first actuarial problems I ever tackled, and it is interesting now to find it still up for discussion. It has seemed to me to be almost unavoidable to treat the case, after disability claim has been allowed, as coinsured rather than reinsured;—when claim is allowed, the insuring company to pay over to the reinsuring company the proper part of the original policy reserves and in the future the companies to stand side by side on a coinsurance basis.

Mr. Allison (I believe) has pointed out how arrangements to reimburse for lost interest lead to difficulties when recovery from disability occurs. There is equal confusion after the instalments are all paid if the insured still lives. It may be virtually impossible to determine the date at which he dies and payment of interest should cease.

Adoption of some variation of the coinsurance principle is almost unavoidable.

MR. G. B. PATTISON:

I believe the difficulty under question *c*) is largely occasioned by a loosely drawn reinsurance agreement in the first place. I, personally, have never consented to reinsure any kind of a disability clause, unless I had it definitely understood exactly what the coverage was. If you are going to reinsure a disability clause which provides for the payment of the face of the policy in instalments, then you must have an understanding with the insuring company as to what they expect you to pay in the event of a claim; and you will

find that they always expect you to pay your proportionate share of the instalments.

The reinsuring company should collect a sufficient premium to cover the cost of reinsuring the benefit to the extent of the payment of its proportion of the instalments in the event of the disability of the insured. I think if the reinsuring company collects such a premium, and if it is prepared to pay its proportionate share of such instalments, that that covers the reinsurance required.

I think to reinsure, or attempt to reinsure, such a clause and be only prepared to pay the waiver of interest would not be reinsuring the clause at all. The reinsuring company would be giving the insuring company very little relief.

You should not attempt to collect a reinsurance premium for a reinsurance benefit that does not protect the insuring company.

I have been very careful with the reinsurance of disability benefits. I hate to reinsure disability benefits, just like everybody else, because I never get enough for it.

Mr. Torrey: Our reinsurance agreements are drawn in the way Mr. Pattison has outlined. We have special clauses in the agreement to cover disability benefits and we specify in the agreement what we will pay in case of disability. We charge what we hope is an adequate rate.

Mr. Pattison: I like the word "hope." This is beside the point. Most disability reinsurance premiums are hopelessly inadequate.

With reference to the other questions, particularly *a*) and *b*) in this informal discussion, my interpretation of them in practice has been perhaps more liberal than that expressed here today.

My thought was that, where the terms of the contract did not clearly cover the particular question at issue, the company reinsuring on the yearly renewable term basis was in effect going into partnership with the insuring company, just the same as though it were on a coinsurance basis.

So that if the insuring company insured a man at age thirty for $100,000 and only retained $5,000 of that amount and reinsured the other $95,000, and then when the insured subsequently died they found he was forty and not thirty, and they were only paying actually about $80,000 in the way of total claim, that the reinsuring company's liability would be proportionate. And the same thing

would hold true if he had stated his age to be forty and subsequently discovered it to be thirty and they were actually carrying $130,000 on that risk. I think the reinsuring company's liability would still be proportionate. I think you are morally, if not by contractual liability, entering into partnership with the insuring company when you reinsure a policy.

I think a reinsurance policy, no matter what plan it is under, should be interpreted as a partnership contract. I admit that this is a liberal interpretation, but I believe it is the only practical interpretation in handling the reinsurance business. I think we are all familiar enough with the action of the different reinsuring companies to know that they are very liberal in the handling of these matters. Usually they go farther than the contract calls for. Very rarely do we find a reinsuring company that stands on the exact letter of its contract. The tendency is to be rather overgenerous. I am not talking particularly about ourselves at all. I think that is generally true in the reinsurance business.

MR. ARTHUR W. LARSEN:

I am interested in this question if for no other reason than the fact that our company has reinsured the claim cited by Mr. Allison. Where a policy has been coinsured, no difficulty, of course, arises. However, under treaties of reinsurance existing at the time this type of disability benefit was commonly offered, the amount of disability coverage was as a rule based on a level amount of insurance, while the life benefit was carried on the yearly renewable reducible term basis. Although various policy provisions have been in existence in the past undertaking to mature the policy by disability, they are fundamentally the same, and I should like to present a method of arriving at a settlement taking the more or less usual provision for the payment of the face of the policy in ten equal annual instalments commencing one year from the receipt of proofs.

The reinsurance arrangements are those that I have just outlined, the reinsuring company coinsuring the disability portion for a level amount and reinsuring the life portion on the annual renewal reducible term basis.

Assume the case of a whole-life policy issued at age x, where the life reinsurance is for an initial amount of $1,000 reducing annually by the reserve on that $1,000 and where disability coverage is based

on a flat amount of $1,000 for the entire period of the contract. Assume further that disability occurs at the end of s years and the first instalment is paid by the original company at the end of $s+1$ years.

First, what is the obligation of the reinsuring company during the ten years following disability? The original company suffers a loss of the premium waived together with the loss of interest on the instalment payments made to the insured. The instalment payments themselves are not to be considered a disability loss as such payments represent rather the loss which would eventually be incurred as a death claim. There may be some question as to the rate of interest to be used in calculating the loss, but theoretically it seems to me the rate should be that used in the calculation of the disability premium rate. The reinsuring company may expect to reimburse the original company, therefore, by an annual payment in advance as follows:

1. The annual premium $1{,}000P_x$ on $1,000 of whole-life insurance
2. Interest in the amount of $100 \times d \times n$ where n is the number of instalments paid, $100 being the reinsuring company's pro rata part of each instalment on which interest is lost
3. Less the same annual renewable term premium that would be due had the life continued active

Under this arrangement the reinsurance of the life portion remains unaffected by the disability so that if death occurs during the ten-year period the reinsuring company's further obligation to the original company would be the amount of reinsurance in effect at the date of death.

The question now arises, What should be the reinsurance settlement if the policyholder survives the ten-year period? He passes out of the company's history at that point, and hence reinsurance settlements must undergo revision since the date of death will not be known. Is it not logical and mathematically correct, therefore, to assume that, from that point, the disabled life will be subject to the mortality provided by the disabled-life table? This leads to the conclusion that the lump-sum settlement of the reinsuring company to the original company at the end of ten years from the occurrence of disability should be the present value, on the basis of the disabled-life table, of future annual payments calculated in the manner already set forth, together with the single premium, based on the

disabled-life table, which will purchase the decreasing renewable term insurance. For this calculation, it is more satisfactory, I believe, to use net instead of gross premiums in items 1 and 3.

The present values of the three items entering into the annual settlement are as follows:

Net annual premiums waived ($1{,}000\pi_x$):

$$1{,}000\pi_x \, a^{i}_{x+s+10} \,. \tag{1}$$

Interest in advance on $1,000:

$$1{,}000d \, a^{i}_{x+s+10} \,, \tag{2}$$

Less annual renewable term premium on net amount at risk:

$$\sum_{t=s+10}^{\infty} (1-{}_{t+1}V_x)\pi^{1}_{\overline{x+t\,1}|}\frac{D^{i}_{x+t}}{D^{i}_{x+s+10}} \,. \tag{3}$$

The single premium for the decreasing insurance benefit is

$$\left.\begin{aligned} &\sum_{t=s+10}^{\infty} 1{,}000(1-{}_{t+1}V_x)\frac{C^{i}_{x+t}}{D^{i}_{x+s+10}} \\ &\qquad = \sum_{t=s+10}^{\infty} 1{,}000(1-{}_{t+1}V_x)\pi^{i\,1}_{\overline{x+t\,1}|}\cdot\frac{D^{i}_{x+t}}{D^{i}_{x+s+10}} \,. \end{aligned}\right\} \tag{4}$$

The net lump-sum settlement, ten years after the occurrence of disability, is therefore

$$1{,}000\left\{(\pi_x+d)\,a^{i}_{x+s+10} + \sum_{t=s+10}^{\infty}(1-{}_{t+1}V_x)(\pi^{i\,1}_{\overline{x+t\,1}|}-\pi^{1}_{\overline{x+t\,1}|})\cdot\frac{D^{i}_{x+t}}{D^{i}_{x+s+10}}\right\} \,.$$

or

$$1{,}000\left\{\frac{a^{i}_{x+s+10}}{a_x} + \sum_{t=s+10}^{\infty}(1-{}_{t+1}V_x)(\pi^{i\,1}_{\overline{x+t\,1}|}-\pi^{1}_{\overline{x+t\,1}|})\,\frac{D^{i}_{x+t}}{D^{i}_{x+s+10}}\right\} \,.$$

This formula may be considered too difficult of application to be resorted to in practice but I have applied it to several particular cases, and it may be of interest to you to know the results. I have assumed that the reinsurance policy is for an initial amount of

$1,000 reducing annually by the whole-life reserve on $1,000, American Experience 3 per cent basis. The disability clause, including waiver and payment of face in instalments, is coinsured for a face amount of $1,000 for the life of the contract. The age at issue has been taken as thirty-five.

Table I shows the results for each of four assumptions regarding the time of occurrence of disability, and the policyholder is presumed to outlive the ten-year period during which the face of the policy is payable. While general conclusions cannot be drawn from these figures, the indication seems to be that the settlement required at the end of the ten-year period does not differ by a large percentage from the amount of life reinsurance in effect at that time. This is exactly as one would expect.

TABLE I

Disability Occurs at Age	Final Settlement 10 Years Later	Life Reinsurance in Effect at Date of Final Settlement
40	766.70	748.32
45	672.40	653.00
50	574.49	554.84
55	477.09	458.06

My purpose in presenting this discussion was to indicate what appears to me to be a scientific method of arriving at the end in view. The rarity of cases occurring in practice precludes the necessity for extensive calculations, and approximate methods undoubtedly serve the purpose just as well. The subject has, however, an academic interest, and its understanding may help to clarify other closely related questions which sometimes occur in practice.

MR. W. L. CRAWFORD:

I think the settlement with a reinsuring company of a disability claim where the disability benefit provides for payment of the sum insured in instalments will appear clearer if we assume that the reinsurance is placed with two different companies, say, A and B. Suppose Company A reinsures the death risk only, on the annual renewable reducible term plan, and suppose Company B reinsures the disability risk only. In order that the principal company be completely covered, Company B must guarantee in the event of disability:

1. To pay the principal company, while the life insured is alive and disabled, the amount at risk in a fixed number of equal instalments
2. To pay to the principal company, while the life insured is alive and disabled, the premium for the death reinsurance, which the principal company must pay to Company A, and
3. Company B must, if the insured should recover and again become an active life, pay to the principal company a sum sufficient to bring up to the proper amount—the reserve which the principal company must hold for the reduced sum insured, according to the then duration, plan, and age at entry of the policy

I believe it is usual for the company reinsuring the death risk to carry also the disability risk, and if we combine Company A and Company B into one company, I think it is clear that when disability occurs, the reinsuring company should pay to the principal company the amount at risk at date of disability, in annual instalments, and should further charge its disability losses and credit its premium account, with the annual premiums on the death risk, which will be reduced each year because of a disability instalment but not by an increase in reserve.

If we consider separately, as I have suggested, the reinsurance of the death benefit and the disability benefit, I think that all questions, such as increase in the principal company's retentions, will be found easy of solution.

It has been suggested that the principal company should transfer to the reinsuring company, in the event of disability, the death reserve which it held, but, by doing so, the principal company would lose the interest profit it might otherwise make, and it appears to me that this method would increase the accounting. Further, the reserve would require, presumably, to be retransferred to the principal company, if the life insured should recover.

PERFORATED CARDS

a) Accounting records.
b) Agency records.
c) Investment records.

MR. F. R. JORDAN:

There has certainly been a great deal of interest in the last year or two in connection with the Hollerith cards and the use of those cards and machines for accountancy purposes. If a company is

merely trying to get the figures required in the annual statement it is my own feeling that the Hollerith machines do not effect any material saving in, say, arriving at the usual items of income and disbursements, except in the case of premiums and commissions. However, if a company is required to segregate, we will say, the payments to policyholders into different funds, such as participating and non-participating, or if a company wants to make investigations by year of issue in connection with any one of those funds, then the Hollerith system is of great advantage.

Furthermore, in the first-year and renewal premiums and the first-year and renewal commissions, the Hollerith system is a great advantage in this respect: The cashier's office can make the usual record of the collection of the premiums and total the same at the end of the day, and the cash book slips which are made in our office at the same time the premium notices and receipts are made are made from the addressograph stencils, and are turned over to the Statistical Department, where cards are punched also for payment of commissions.

Now, then, the Hollerith machines make all the distributions of those premiums by states for taxation purposes and into funds or any other distribution we might need. They also sort the cards by general agencies; and then run these cards in the printer tabulator, and this printed slip is given to the agency Accounting Department, which credits on this slip the commission due. This procedure refers to renewal premiums.

This printed slip is then returned to the Hollerith Card Department and the commission is punched on the cards, it is run through the printer tabulator, and statements are furnished to the general agent each month showing the commissions credited to his account or paid him in cash. Then these cards can be sorted, and the commissions paid by funds or by year of issue can be readily obtained.

In reference to *b*) "agency records," I want to state that we make rather extensive records on our Hollerith cards, consisting of the following: (1) business received, issued, not taken, paid for, and insurance accounts which are kept for general agents and individual agents; (2) the persistency and lapse records; (3) all club-year records; (4) all other contests.

In refe ence to *c*) "Investment records": First, we get the accrued-interest statements at any time during the year. We have

not devised any method of arriving at the accrued interest that in any way compares with the method which makes use of the Hollerith cards. Second, other information relative to the company's investments is obtained and classified according to state or kind of loan. I think that briefly covers the questions here.

MR. JOHN S. HALE:

This subject is such a large one that it is only possible to emphasize one or two points in each of the subdivisions offered for discussion.

It is very difficult to give an explanation of a perforated-card installation, or its application to any particular office procedure, without giving some idea of the office routines leading to the statistical room. And, in my opinion, the office routines are almost as important as the statistical work itself—a fact we found out through very sad experience.

Our first installation was on the insurance account and valuation; we had a beautiful system set up, all the cards punched, but the statistical operator was like the bride "waiting at the church"—the stuff did not come to him! We therefore had to revise all of our office routines.

In accounting records we attempted to use the perforated cards as a direct accounting medium, and attempted to get our premium income from perforated cards by punching the cards in advance. At the same time the actuarial card was punched we punched a premium card, the first twenty-five columns of the two cards being identical. We set up the premium cards in a file and attempted to pull them as the remittances were received and run them. This procedure failed, due chiefly to the fact that so many premiums were paid on a different basis from the notice, and to the large number of premium extensions granted.

We also attempted to do our agency accounting by a great deal the same method that Mr. Jordan outlined, by punching a card for all policy issues, showing policy nets and commissions, and then running those cards as remittances were received. We found that it could be done quite readily by making the sorts and tabulations daily, but on account of the extremely small units it did not pay to run them daily. We tried to tabulate once a month, and found that if at any time during the month we wanted to know the status

of an agent's account, it was necessary to make a special run of the cards; and, in addition, it put a peak load on the tabulating units at the end of the month. Those were our only two attempts to use the perforated cards as a direct accounting medium.

We now punch a card for all new issues. It is really an accounting card. We use it for control and distribution purposes and also for getting the total amount of commissions paid to agents for income-tax purposes.

We are starting to instal a new method of handling our renewal premiums, by punching a card as the premium is received. The remittance slips are distributed to the various desks in which we keep our premium-record cards. The bookkeeping entries are all made from a blotter; and then the distribution is taken care of by a punched card from which we secure the premiums by states, the premiums divided into life, disability, and accident, direct and reinsurance, and reinsurance premiums, received by companies, net premiums for the gain and loss, deferred premiums for the annual statement, and also a control on our renewal commissions.

I am going to pass over the second subtopic—agency statistics—because we are doing nothing unusual in that respect.

On investments we have, I think, a rather unique method of handling policy loans. When a policy loan is closed and ready to go through the office, the entries for the general books are made on a blotter which is a combination voucher and check register; and the check is in the form of a voucher-check made in duplicate; a third copy is made which is a perforated card.

The perforated card is then punched from the information actually given on the card, and it is punched in duplicate—one for a control file and the other one for what we call our "active file." This punched card is the only record of policy loans that we keep in the office.

When we installed this method, we did away with the usual policy-loan card, which was a typewritten card, an index card, and an interest-tickler card. From the control file we keep track of the changes in the policy loans by reason of new loans and old loans canceled and lapses or surrenders. The active file is tabulated on the fifteenth of each month and is balanced with the figures on the general ledger. Our policy-loan agreements themselves are filed in numerical order. We did away with a special loan number and use

the policy number as the loan number. An adding-machine tape of the actual policy loans is balanced against a run of the cards.

This system has been in operation for nearly two years, and for the first time in our history our policy-loan account is quickly and easily balanced.

From the policy-loan card, we ascertain the amount of the annual interest for control purposes, and also the advance interest as of December 31, for the annual statement.

On the other investment records, bonds, and mortgages, we use a perforated card. On the bonds and mortgages we do not attempt to use the card except for control and statistical purposes. There was too much information required on a mortgage-loan card to permit the use of a perforated card. We therefore use that for control purposes—the total loans on the punched cards must be in balance with the books; and from a run with those cards we make a physical audit of our securities periodically. We ascertain from the cards the accrued interest at the end of the year for the statement, and the same method is used on bonds.

We are using one dual card—the dividend card. It worked well for a while but I find now that the constant use of the cards in entering the amount of the dividend and recording payments has worn the edges, and we are having a little trouble with them jamming in the sorter as they go through.

Mr. Jordan: I want to make one thing clear in connection with the commissions. We do not run these cards through daily; we run them through once a month.

We have also considered punching up a premium card at the time the policy was issued, but due to the frequent changes in methods of premium payment, as mentioned by Mr. Hale, we have come to the conclusion that it is much better to punch a card at the time the premium is actually collected and show on that punched card the method by which the premium was paid, whether annually, semiannually, or quarterly, or part cash or part note.

We also have a dual card for all note payments, and that is the only note record we have in the office. We do not have any other written record of the note. We find that is very convenient because we can sort them in the machine and get the due dates of the notes. And while that is a dual card, yet, as a general rule, the notes are not extended beyond six months, and the cards are used generally

for a period not exceeding six months. Therefore, the objection Mr. Hale has to a dual card would not hold in the case of that particular card.

MR. W. A. P. WOOD:

In the case of the Canada Life, the use of the perforated cards on the tabulating machines is specially advantageous in the accounting work, because we have a number of different funds to deal with. We have our old deferred dividend fund, quinquennial and annual dividend funds, and also the non-participating fund. We have, therefore, several funds to deal with. Our general system of handling business is the branch system. Formerly in the statements which came in from the branch, say, under the heading of "renewal premiums," we would have deferred dividend renewal premiums, quinquennial dividend renewal premiums, non-participating renewal premiums, etc. Now we are able to have just the renewal premiums, and the work of subdivision is taken care of by the perforated cards and the tabulating machines. In this respect it is a great saving in our work. It has reduced the number of ledger accounts very materially.

Another section of the accounting work in which the company has found use for the perforated cards to considerable advantage is the preparation each year of the individual agent's income from commissions for federal, provincial, and municipal tax returns. Formerly, we depended largely on the work done in our branches. Now we have it done more expeditiously and more accurately in the home office by means of these machines.

The use of the cards assists in the rapid preparation of various statements from the same data through sorting and resorting and tabulating of the cards. This facilitates our arrangements for the collection of premiums by simplifying the routine required at our branch offices when premiums are collected at the branch office nearest the place of the assured's residence instead of the branch through which the policy originated. This method of collection is what we call the "reciprocal arrangement for collection of premiums." A man may take out a policy in our Detroit office and he may move to Chicago and pay his premium through the Chicago branch, although the credit for the commissions goes to the original agent. We have found that the tabulating machines are of assistance to us in taking care of work of this nature.

In regard to the outstanding and deferred premiums, I may say

that our company for several years used the tabulating-machine method for getting at the outstanding premiums but we found that to punch up cards at the end of the year for the outstanding premiums made too great an overload at that time of the year when the staff handling the card work was already working at its maximem capacity. We therefore reverted to Burroughs machines for the listing of these premiums at the close of the accounts each year. But in the preparation of the deferred-premium figures our company has found the use of perforated cards superior to the methods previously used. They are prepared and maintained as a part of our insurance accounting work with the resulting minimum of labor.

In planning work of this nature one has, of course, to study out whether the machines will handle the work at the end of the year when there is the peak load. You have got to make your arrangements accordingly. If you are not able to handle it at that time by this method, then you have to take some other way of handling all these things. We have found on the whole that the perforated cards and the tabulating machines are of great assistance to us in our accountancy work. I might add we have used them but very little in the investment section.

MR. JAMES S. ELSTON:

We use various Hollerith cards besides those for mortality purposes. Our deferred-premium card is especially valuable because we wish to keep the deferred premiums on regular business and on several types of wholesale separate, and because we also obtain our separation for Canada from them.

We also have a statistical card that is mainly used for listing the reserve, and for numerous analyses of new business by state, type of substandard business, size of policy, type of disability provision, additional indemnity provision, form of policy, non-medical business, etc. Most of these analyses are made by branch office. We also obtain our disability premiums and Schedule Q premiums from these cards. Space is left for enough cancellation data, so that they will eventually be used for mortality purposes on wholesale and non-medical business where comparatively few data as to impairments are available.

We have an agency "amount" card from which we obtain issued and paid-for business by agents and by branches. Cards are also

punched for not taken, rejected, and postponed, so these can be analyzed by branch office and further as to whether standard or which type of substandard. All these records show the policy number so that, for instance, an agent's business can be listed by machine.

Our distribution of premiuns between first year and renewal, and between regular and wholesale, not only for the whole business but by branches, comes from an agency "premium" card. This card is the same as is correspondingly used for each of the casualty and fire lines, so that these cards furnish agent's premiums for each line and also all his commissions, not only for agency purposes, but for schedules G and H, and for federal-income-tax purposes. Our premiums by states for taxation purposes also come from these cards. The branch offices are furnished with monthly tabulations of the business by lines covering the important agents.

Our policy-loan cards have supplanted the handwritten cards in our policy-loan division. The debits and credits are balanced daily and a monthly report made to the Finance Committee, listing the individual loans made during the month, showing the individual policy number, amount of loan, and cash value of the policy. Once a year an individual list of about eighty thousand loans in force is made for the company audit. Analyses are made from the cards according to the rate of interest, and between regular and wholesale. An annual separation by states is made, and analyses may be made as to whether loans made are original, increases, or restorations, and as to whether cancellations are made by repayment, maturity, death, or lapse of the policy.

MISS ELOISE B. KOCH:

The application of perforated cards to accounting has long been a hope of mine, but we have felt some hesitancy about applying them because we have not been able to find any other company who used them to the complete extent that I had planned to do it, and we consequently lacked the confidence to proceed.

My plan—this may be a bit out of order because I can't say that I have demonstrated it, but I hope to—my plan is this: To prepare a premium card as many other companies do, to be used for distributing the premiums by states and by life, double indemnity, and disability; but also to combine with this card what

might be called an "accounting card." On this card would be one field for debits and one field for credits, and also, of course, a field for the account number and a field for the voucher number as well as fields for descriptive data.

Our present bookkeeping system is on a voucher basis, i.e., every item is prepared in some department other than the Bookkeeping Department, and is then sent to the Bookkeeping Department where a clerk selects the proper sheets from the loose-leaf ledger and makes the entries by means of a bookkeeping machine.

I plan, when a remittance is received, to have the premium cards pulled from the file and sent with any other vouchers to the Statistical Department which would prepare the accounting card.

To give a specific example, let us assume that a premium is to be settled by means of a premium lien note. The entry on the accounting card would require a punch in the debit column for the lien note and a corresponding punch in the account column, indicating the account number for premium lien notes. It would then be necessary to punch an additional accounting card for the interest, which will of course appear in the credit column. The total premium on the premium card is punched in the credit column.

The data common to all cards would be punched on to the accounting card by means of the duplicating key-punch from the premium card. All cards, both the accounting and the premium, would then go to the Statistical Department where a preliminary list would be made to add the total debits and total credits. Of course, if there is a balance, then we know that the cards have been properly prepared.

They would then be sorted by account number, the proper loose-leaf ledger sheet for each account number inserted into the lister, and the total of the items for that particular account number would be listed on the proper ledger sheet. The original list by policy numbers could be kept as a record for later reference, if necessary; but the permanent record that we are after is the listing on the loose-leaf ledger sheet.

The use of perforated accounting cards such as these should prove valuable for an auxiliary purpose, namely, to verify asset accounts such as policy indebtedness, because they may easily be selected and sorted at any time after having been used for making the original ledger entries. For instance, the accounting cards used

for entries under "Policy Loans" could be selected at the end of a given period, say one month, and sorted by policy numbers and added and listed in small groups. The difference between the totals of the debits and the credits of each group could very easily be compared with the difference between the total loans in force at the end and at the beginning of the period in the same numerical group for the purpose of locating errors in the loan records or the ledger entries.

There may be several reasons why many companies have not adopted this method; it may be because of the branch-office system, where I believe the branch offices do a great deal of the accounting for the home office. Of course, in that case there would not be much advantage to the home office in this method, or it may be that it is a matter of opinion that this is not the most economical way of handling the question. I would be very much interested to know whether any other companies have used this method or anything similar to it.

MR. G. F. HOLLENBERG:

In answer to Miss Koch, I might say that I have here a series of cards that the Union Central has used in connection with accounting. The cards are open to her and anyone for inspection. I also might say that the Union Central has carried the perforated-card system into every detail where it can be applied, of course, efficiently.

I don't know whether the President would want me to go through these cards and tell you the various headings; but here is one card for accounting purposes for premiums and dividends. We have the agency and state and premium, first-year, premium renewal, dividends used to pay premiums, dividends used to purchase additions, and dividends left at interest. That is all on one card. We have another card for lapsed and surrendered policies which shows the agency and state and the debit and credit items for cases handled through the auditor's journals. We have a similar card for dividends paid in cash. We have one for items affecting the lapsed and surrendered policy account other than through agency reports, on which we have columns for the agency and the state and the year and the amount of cash, and the dividend additions surrendered, and the cancelled premium notes and interest, and the cancelled policy loans and interest.

We also have a card for deferred premiums. Then we have a card for cross-footings for our Auditing Department. Under the heading of "Investments" we have a card for policy loans and interest. That has the policy number and the state and the agency and the due date and the amount of interest and the number of years interest paid.

We have another policy-loan card that shows the number and the state and the agency, and we go to the subagency on that and give the date and also the amount of loan and the accrued interest. We have several other policy-loan cards.

For our mortgage-loan experience we have a card that provides for the number of the loan, the state and the agency and the county, the rate and discount, and class and date and the amount and the cost of acquiring the mortgage.

We have a card for real estate purposes and also for our mortgage-loan delinquency-interest record. We have a card for the mortgage-loan accrued interest.

The company, as perhaps some of you know, has recently gone into the city loan business. We have a card taking care of it, showing the state and county and the city, the agency, the year made and year of maturity, the pay dates, and the amount and rate of interest, etc.

Then we have a card for appraisement purposes, showing the items that would come in under that heading, which would be the land and the building and the fire insurance, and tornado insurance.

Under the heading "Policy Reserve Valuation" we, of course, have our greatest number of cards. Under that heading we place the card that we prepare in connection with the cancellation of a policy, and these cards are passed to the various divisions that are interested, who can sort them in almost any way because the card shows every detail. I might say right here that this card is also used for agency purposes, to determine the lapse rate in each agency. It shows how the policy is canceled, and each agency receives, at the end of the quarter, an entire survey of all its cancellations during those three months. I think it has had quite an effect on our lapse rate.

I will not take any more time, but if anyone is interested in these cards I shall be glad to have him look at them later.

MR. W. B. KIEFT:

Our company has not had as great an experience in the matter of punched-card records as some of the older companies, but we have tried to make as great use of it as is possible, and I shall state briefly our experience with it.

ACCOUNTING RECORDS

In determining the superiority of one method of accounting over another, two points must be considered, viz., accuracy and cost.

Generally speaking, machines, past the experimental stage, are more accurate and faster than the "hand" methods. For certain kinds of work one machine may answer the purpose better than another.

Our company has found the punched card and the tabulating and sorting machinery valuable aids in accounting work. For example, we punch a card to obtain a premium distribution of both gross and net rates. By punching in a classification it is possible to sort by first year and renewals and then add six columns at a time at the rate of 150 per minute. This means a saving in time, is less laborious, and produces a greater degree of accuracy than if an adding machine were issued.

A system may be devised whereby the premium distribution cards may be used year after year. This saves the cost of punching a new card and eliminates chances of error by reducing the number of operations.

For bookkeeping a card may be punched from the original entries reaching the Bookkeeping Department from the various departments. Entries may be made on the journals and cashbook from the original entries and balanced against the punched cards.

Sorted by account numbers and printed, the bookkeeping cards make it a simple matter to post to ledger accounts daily, semi-monthly, or monthly. This does away with the wide sheets with headings for the various ledger accounts and permits the use of a sheet which will fit in an ordinary long-carriage typewriter.

The punched card used in connection with bookkeeping makes it possible to obtain a distribution of premiums and dividends by states for the purpose of making tax statements with the same cards used for the ledger-account distribution.

AGENCY RECORDS

For agency records the punched card is valuable in that it may be used to obtain information for several different purposes. For example, if a card has the state, plan, agency number, policy number, date of policy, amount of insurance, premium paid, charges, and commissions punched in it, it is possible to obtain the amount of business produced by states or by plans of insurance. The amount of insurance written by a particular agency can be obtained by sorting the cards by agents. A commissions statement can be made from the same card.

INVESTMENT RECORD

The principal advantages in using a punched card in connection with the investment record is in making a financial statement. The punching on these cards usually includes the office number of the investment, state, classification, multiplier, anniversary month, amount of loan or bond, accrued interest to first of month, monthly increase in interest, due date, etc.

By sorting the cards by interest-due-date and then listing the interest-due-date, accrued interest to first of month and monthly increase, the accrued interest may be obtained for a financial statement.

We do not have the difficulty which some companies experience with relation to first-year premium loans or notes as we do not use the premium-note system. If an agent for the company writes an application and makes settlement by note, we require that he shall remit to the home office the amount of the net premium and take care of the note personally. So this feature of the records is eliminated from our plan of operation.

MR. HENRY W. BUTTOLPH:

There are just two things I want to say.

We regard our premium card as an actuarial card rather than an accounting card. If it had not been for what Mr. Wood said about what they believed to be the uselessness of attempting to determine outstanding premiums from these punched cards, I would not have mentioned it.

We have a premium card, kept in the Actuarial Department, which is used for many purposes, one of which is as a check on the

Accounting Department. It is used for determining the premiums received in the different states. It has both the gross and the net premiums shown so that the loading is a matter of accurate computation for gain and loss purposes.

But one of the uses of that card which we regard as of very great advantage is in the determination of the outstanding and deferred premiums. Ours is a small company as compared with that of Mr. Wood's; but I think that within an hour's time at the end of any month of the year I can tell the exact amount of outstanding and deferred premiums, not only gross but also net.

Another card which we use is somewhat similar to that described by Mr. Jordan as their agency card, but we go considerably farther than he described.

We have an agent's record card which shows all of the things mentioned by Mr. Jordan and many others. We have, I think, ten classifications as to the general character of the agent's business, large or small policies, rural or city business, male or female, information in regard to whether the insurance seemed to be well within the income of the applicant, etc. From memory I can't give you a very good description and do not think it is necessary; but we have all these classifications of the general character of the business done by an agent. We can take out at almost a moment's notice the business of an individual agent and tell how much business he has sent in to us which was never completed by a medical examination, how much business was returned for cancellation as not taken, the amount of commissions, the date when he sent in his last application, and all of those other details to which I have referred, and then we make a real use of all of this information.

We keep in our New Business Department a card which is not a perforated card. It is a large card that we call our "agent's picture card." All of this information gathered from the perforated agent's record card is summarized in a general sort of a way and entered on this card. This picture card is revised so as to keep it up to date, is kept in the New Business Department, and the New Business Department at all times has a picture of that agent's business, his general behavior, etc. We believe, while it requires considerable work to keep up this agent's record, that it is a very valuable one.

In the matter of investment cards we have the policy-loan

records on perforated cards and also the information regarding our mortgage loans, from which, as Mr. Jordan stated, we can compute very readily and quickly at the end of any month the exact amount of accrued interest, and we can get all of the classification of those investments necessary for statistical purposes—farm or city mortgages, state, county, the correspondent through whom we handle the business, and all of the general information that is desired from time to time in regard to the investments of the company.

MR. JORDAN: I want to make one other statement. Miss Koch said that she had not been able to devise a card that would take care of all the accounting work. I think it is a mistake to try to take care of all of your accounting work on the Hollerith system. As far as disbursements are concerned, we found it advisable to take care of disbursements to policyholders and commission disbursements. With the other items it is better to use your Burroughs bookkeeping machine and handle it in that way.

We have one ledger account in the general ledger—disbursements to policyholders. Whenever a voucher is drawn, a duplicate voucher is made by carbon copy and the carbon copy goes to the punch-card operators and they split up that payment to policyholders in the different accounts that appear in the annual statement. At the end of the month those are summarized and entered, merely the totals of those subdivisions, into the general ledger; but it simplifies the work of the operator of the Burroughs bookkeeping machine. I don't think we should wait until we can carry all of our accounting system but simply use such portions of it as may seem advisable.

MR. E. M. McCONNEY:

There was one use on which we stumbled the other day. I don't know whether the other companies have found it out or not. On our mortgage cards we found that by sorting them by counties and running them through the printer a lot of time was saved in the Treasurer's Department when he went to furnish a list of mortgages to the different county officials to see whether taxes had been paid on those pieces of property. The Treasurer's Department used to sort the cards up by hand and make a rather long job of it. It saved them putting their mortgage cards out of order by giving them a printed list of the mortgage cards.

MR. W. H. McBRIDE:

Many of the speakers this morning have emphasized the way peak loads limit the application of perforated cards and the uses to which we should like to put them.

I wonder why they have all omitted mention of a very effective expedient for avoiding peak loads. I have in mind what we call a "group total card." I think that conveys the idea with sufficient clearness so that it will not be necessary to consume time in elaborating it. By that expedient, in our office, millions of cards concentrate themselves into a few thousand cards, and they are handled at the required time with very little congestion.

MR. EDWARD W. MARSHALL:

The Provident Mutual has developed its perforated-card systems to a considerable degree, its experience being that these systems are not only efficient and economical, but make it possible to obtain information which would otherwise be dispensed with because of the great amount of time and labor required to secure it. A brief description of the perforated-card systems used by the Provident Mutual, together with a summary of the information obtained from them, may be of interest. These systems have demonstrated their value to our company in actual performance over a number of years. The cards in use will be considered one by one.

ACCOUNTING RECORDS

Valuation card.—The attained-age valuation card and the collateral information obtained therefrom have been described in *TASA*, Volume XXV, page 219, and need not be discussed further here. To that list should be added the valuation of policies on the lives of Massachusetts residents.

Disability card.—This card was also described in the above-mentioned article and the information derived from the system stated. The disability experience obtained is now subdivided by sex. A condensed policy exhibit of disability provisions is also secured from this system together with a list of policies on which the disability coverage expires or changes during the coming year.

Seriatim valuation card.—A dual card is used, the data perforated including year of issue, policy number, office year of birth, plan, year maturing or becoming full paid, amount of insurance, and

net premium. The written matter includes year of valuation and reserve, which is computed individually. The card is used for joint insurance, deferred survivorship annuity feature on income policies, etc. The card may be sorted by attained age and the reserve totaled by adding machine. Thus the cost of insurance and net premium data may be obtained.

Mortality card.—This is a card for making formal mortality investigations by impairments, occupation, build, etc., and while different from the more recently designed mortality card recommended for use in various companies, it serves somewhat the same purpose.

Premium settlement card.—This is a true accounting card prepared daily from the premium report sheets received from various agencies. It has proved exceedingly valuable in practice. It includes agency, state, county, premium reported, amount of dividend if any and how applied, code for first-year premiums, and day of record. By using this card, the cash premiums received and dividends used to reduce premiums or paid in cash may be subdivided monthly by states for tax purposes with a minimum of effort. The agency premium report sheets are audited by means of this card and a subdivision into first-year and renewal premiums made.

AGENCY RECORDS

Paid-for new-business card.—These cards are prepared daily as the paid-for new business is reported. The card includes year, month and day of issue, agency, special agent, policy number, state, county, amount of insurance, gross yearly premium, plan, mode of exit, and duration. A daily statement of total paid-for new business is obtained, also monthly report of new business by agencies and by county and state. The quarterly production by individual special agents is obtained from this system. The system is maintained for two years after issue in order that the first- and second-year lapse rates of the individual special agent may be obtained. This gives a valuable insight into the quality of the special agent's work. By the use of perforated summary cards the production and lapse rates of the special agent may be studied and comparison made between new and old agents, large and small producers, etc.

Agency card.—The data on this card include policy number, agency, state, county, age, plan, amount of insurance; gross yearly

premium, mode and time of exit. This card forms the basis of a miniature policy exhibit by agencies and the "agency record" sheet for each agency produced every four months. This sheet gives a bird's-eye view of the operation of the agency, including an analysis of its production, not only as to quality but as to relative amount, not taken business, term conversions, etc., and shows separately the first-year, second-year, and after-second-year lapse rates. These figures are scheduled beside similar rates for the company appearing on the same sheet, and have been found valuable in agency guidance. The card is also used for obtaining data on old policyholders, transferred business, etc. The gross premium on this card furnishes totals which supplement those now taken from the valuation card mentioned above, although, as stated, it is planned to have the valuation card self-supporting in this regard.

Some of these systems have been created especially for the main purpose described. In other cases the system has been enlarged to furnish additional data not originally contemplated when the system was inaugurated. For this reason the systems may not be as ideal in every respect as if they had been created *ab initio* but they meet the company's needs for definite information in the many important lines covered.

INVESTMENT RECORDS

Although the subject has been given repeated consideration, as yet it has not been found worth while to prepare a perforated card for use in connection with any of the company's investments. The advantages of such a card have been realized, but so far they have not been considered of sufficient moment to warrant the installation of another system.

BOOK REVIEWS AND NOTICES

Advanced Life Insurance. By CHARLES K. KNIGHT, PH.D. New York: John Wiley & Sons, 1926. Pp. xi+426.

In the Author's Preface he has an apology for the title *Advanced Life Insurance.* The reviewer believes that the title is a misnomer from the standpoint of the generally accepted interpretation of the title, although he believes that no reader looking for the true "advanced life insurance" would regret having purchased the book on the basis of the title.

The reviewer sincerely believes that a title *Up-to-the-Minute Life Insurance*, although lacking in dignity, would be an excellent descriptive title. It would be difficult to name a subject matter, even though apparently insignificant, which has not received the attention of the author. The presentation of all such matter makes the reader feel that he is being given the very latest and most reliable information. The reader with much experience in the technical features of life insurance will naturally recognize this point of view, mention of which is here made that the student may have confidence in what he gleans from this book.

The reviewer does not know the extent of the practical experience of the author and if he (the author) has not had a fulness of such experience he has disguised the fact, as the book has all the evidence of the experienced hand.

The author refers in the Preface to conforming to the principles of pedagogy, and no doubt that is the reason the book is so readable but withal instructive. The student gropes around for some verbal support in his attempt to fathom a new subject matter couched in mathematical formulas. Verbal and symbolic formulas are best when joined together. This book furnishes the "word solution" in a most excellent manner, and should be a collateral study in connection with the mathematical study of life insurance.

Many topics are of value to those interested in life insurance other than from an actuarial viewpoint, and the book is fully recommended to such.

The book will rank high wherever it is necessary to impart the fundamentals of life insurance.

J. C. S.

OTHER RECENT PUBLICATIONS

Alcohol and Longevity. By RAYMOND PEARL. New York: Alfred A. Knopf, 1926. Pp. 260.

Compensation Insurance Rates as a Measurement of Accident Prevention in Mines. By U. S. MINES. Washington: Government Printing Office, 1926. Pp. 6.

Compound Interest and Annuity Tables. By FREDERICK C. KENT AND MAUDE E. KENT. New York: McGraw-Hill Book Co., 1926. Pp. 214.

Fees and Taxes Charged Insurance Companies. By INSURANCE DEPARTMENT. New York, 1926. Pp. 58.

Fire Insurance Accounting. By W. B. WIEGAND. New York: Ronald Press Co., 1926. Pp. 108.

Industrial Life Insurance. Spectator Co., 1926. Pp. 196.

Interpolation. By J. E. STEFFENSEN. Baltimore: Williams & Wilkins Co., 1927. Pp. ix+248.

Life Insurance Accounting. By C. E. MATHER. New York: Ronald Press Co., 1926. Pp. 119.

Making of the Fire Insurance Rate. By EDWARD R. HARDY. New York: Spectator Co., 1926. Pp. xiv+344.

Mathematics of Finance. By L. L. SMAIL. New York: McGraw-Hill Book Co., 1925. Pp. 310.

Occupation Code and Alphabetical Index to Occupations. By JOINT COMMITTEE ON MORTALITY ASSOCIATION OF LIFE INSURANCE MEDICAL DIRECTORS AND ACTUARIAL SOCIETY OF AMERICA. New York, 1926. Pp. vii+651.

Origin and Early History of Insurance. By C. F. TRENNERY. London: P. S. King & Son, 1926. Pp. xiv+330.

Principles and Practice of Personal Accident Disease and Sickness Insurance. By J. B. WELSON. London: Sir Isaac Pitman & Sons, Ltd., 1926. Pp. x+123.

Selected Articles on Unemployment Insurance. By ALLEN B. FORSBERG. New York: H. W. Wilson Co., 1926. Pp. cvii+487.

Valuation of Liabilities under Industrial Pension Plans. By J. H. WOODWARD. New York: L. W. Lawrence, 1925. Pp. 37.

Workmen's Compensation Act of Great Britain. London: H. M. Stationery Office, 1925. Pp. 52.

Workmen's Compensation Law of Michigan. Lansing: Browning Publishing Co., 1925. Pp. 91.

MINUTES OF THE SEMIANNUAL MEETING
of the
AMERICAN INSTITUTE *of* ACTUARIES

HELD AT THE EDGEWATER BEACH HOTEL, CHICAGO, ILLINOIS
NOVEMBER 4 AND 5, 1926

The Institute was called to order by the President at 10:30 A.M., November 4. The following Fellows, Associates, and Nominees of Contributing Members were in attendance:

FELLOWS

S. E. Allison
E. C. Ames
A. E. Babbitt
C. H. Beckett
J. P. Bowerman
J. A. Budinger
H. W. Buttolph
E. R. Carter
L. M. Cathles
R. B. Coit
W. P. Coler
H. R. Corbett
H. W. Curjel
J. S. Elston
P. H. Evans
James Fairlie
E. G. Fassel
J. E. Flanigan
A. C. Good
C. E. Herfurth
R. A. Hohaus
A. J. Huenergard
R. G. Hunter
H. H. Jackson
D. M. Jamison
R. E. Kennon
E. B. Koch
J. M. Laird
A. W. Larsen
A. T. Lehman
J. F. Little
E. W. Marshall
E. L. Marshall
L. R. Martin
H. C. Marvin
A. J. McAndless
F. B. Mead
P. V. Montgomery
W. O. Morris
R. E. Moyer
J. H. Nitchie
J. G. Parker
G. B. Pattison
O. W. Perrin
S. L. Phelps
W. F. Poorman
E. A. Porter
H. L. Rietz
J. C. Rietz
J. C. Seitz
V. R. Smith
A. A. Speers
B. J. Stookey
M. W. Torrey
R. M. Webb
H. J. Williams
F S. Withington
W. A. P. Wood

ASSOCIATES

T. L. Anderson
H. G. Brunnquell
D. E. Buckner
H. L. Clark
C. F. Cross
C. C. Dubuar
A. C. Eastlack
J. H. Evans
H. C. Fetsch
C. H. Folz
R. N. Griswold
J. S. Hale
W. J. Hiller
W. H. Huehl
W. A. Jenkins
L. J. Kalmbach
W. D. MacKinnon
J. D. Maynard
W. H. McBride
W. O. Menge
Merlin Oates
G. L. Pardieck
R. B. Robbins
C. O. Shepherd
L. L. Stevens
F. D. Strudell
R. B. Sturtevant
Margaret Walker
H. G. Walton
A. C. Wellman
J. R. Young

NOMINEES OF CONTRIBUTING MEMBERS

G. D. Babbitt.....Bankers Reserve Life Company
Henry Bauer.....Continental Assurance Company
W. H. Bain.....Manufacturers Life Insurance Company
P. F. Bourscheidt.....Peoria Life Insurance Company
S. B. Bradford.....Central Life Insurance Company of Illinois
R. M. Brown.....Continental Assurance Company
F. R. Brunner.....Security Mutual Life Insurance Company
O. J. Burian.....Missouri State Life Insurance Company
L. C. Cox.....National Life and Accident Insurance Company
James Craw.....Springfield Life Insurance Company
W. L. Crawford.....Great Southern Life Insurance Company
T. C. Denny.....Central Life Assurance Society of the U.S.
C. R. Dent.....Confederation Life Association
E. F. Estes.....Bankers Reserve Life Company
W. A. Fairlie.....Mutual Life of Illinois
T. H. Galbraith.....Central Life Insurance Company of Illinois
D. H. Garver.....Maryland Life Insurance Company
A. O. Graeser.....Ohio National Life Insurance Company
W. D. Haller.....Royal Union Life Insurance Company

M. N. Hanson.....Wisconsin Life Insurance Company
H. G. Hewitt.....Western States Life Insurance Company
Kenneth Hittle.....State Life Insurance Company
G. F. Hollenberg.....Union Central Life Insurance Company
Peter Hondorp.....Central Life Assurance Society of the U.S.
H. J. Hornberger.....Great Northern Life Insurance Company
L. H. Howe.....John Hancock Mutual Life Insurance Company
J. N. Jamison.....Reliance Life Insurance Company
W. B. Kieft.....Springfield Life Insurance Company
J. M. Laing.....Mutual Life Assurance Company of Canada
W. W. Lane.....LaFayette Life Insurance Company
G. L. Lutterloh.....Peoples Life Insurance Company of Illinois
R. M. Malpas.....Reinsurance Life Company of America
E. McConney.....Bankers Life Company
J. L. McLachlin.....Confederation Life Association
R. Merriman.....Scranton Life Insurance Company
Carl Mitcheltree.....Columbus Mutual Life Insurance Company
H. E. Mizener.....Register Life Insurance Company
C. E. Nelson.....Missouri State Life Insurance Company
F. J Owen.....Berkshire Life Insurance Company
F. H. Pakes.....Guaranty Life Insurance Company
A. E. Pequegnat.....Mutual Life Assurance Company of Canada
J. G. Phipps.....Peoples Life Insurance Company
H. G. Pinger.....Illinois Life Insurance Company
T. C. Rafferty.....International Life Insurance Company
J. B. Russell.....Peoria Life Insurance Company
H. B. Sturtevant...Old Line Life Insurance Company of America
V. V. Veatch.....Merchants Life Insurance Company
A. C. Washburne.....Berkshire Life Insurance Company
Ethel Welsh.....National Life and Accident Insurance Company
J. F. Williams.....Illinois Life Insurance Company
J. D. Williamson.....Canada Life Assurance Company
Douglas Wood.....Missouri State Life Insurance Company
B. H. Zahner.....American Life Reinsurance Company

Mr. A. D. Watson, actuary, Department of Insurance, Ottawa, Canada, was also in attendance.

The President delivered an address as printed in No. 32 of *The Record*.

The Minutes of the meeting held June 3 and 4, 1926, as printed in No. 31 of *The Record*, were approved.

In summarizing the business of the Board since the last meeting of the Institute not mentioned in the address by the President, the Secretary announced that George W. Geddes, Frederick B. Gerhard, M. Louis Johnson, Lawrence L. Stevens and Byron Wright had been admitted to Associateship without examination; and that the Mutual Life Insurance Company of New York and the Connecticut Mutual Life Insurance Company had been admitted to Contributing Membership.

The Secretary reported that Edward W. Marshall was duly elected to Fellowship without examination; also that constitutional request has been made for amendments of Article V of the constitution and Article I of the by-laws, constitutional notice of which was given to the Fellows of the Institute on October 5, 1926.

On motion of B. J. Stookey, seconded by E. L. Marshall, it was resolved to amend Article V of the constitution by inserting the matter in italics and by omitting the matter within brackets, as follows:

The Board of Governors shall consist of the Officers [the ex-Presidents, if Fellows, and six other Fellows] *six other Fellows and the former Presidents who have held that office within six years; provided that the President elected at the annual meeting in 1926 and his predecessors shall be members of the Board for life.*

Motion carried unanimously.

On motion of P. H. Evans, seconded by S. L. Phelps, it was resolved to amend items 5–10, inclusive, of Article I of the by-laws by the addition of an item numbered 11, by inserting the matter in italics, and by omitting the matter within brackets, as follows:

5. Announcement by Presiding Officer of time of election of Officers and Governors.
[5.] 6. Candidates for Fellowship considered and voted upon.
[6.] 7. Reports of Committees and Officers.
[7. At annual meeting, election of Officers and Governors.]
8. Unfinished business; new business.
9. Reading of papers prepared for presentation to the Institute.
10. Discussions of papers presented.
11. *Discussion of topics of current interest.*

Motion carried unanimously.

A formal paper was then presented, after which the Institute entered upon the discussion of the papers presented at the June meeting.

The morning session adjourned at 12:30 P.M.

The Institute reconvened at 2:20 P.M. and continued the discussion of papers.

The President announced that a cable message had been received from Sir Joseph Burn, president of the Institute of Actuaries (London), as follows: "Greetings all to all. Co-operation for International Congress London heartily desired." L. M. Cathles stated that the message referred to an International Congress of Actuaries which is expected to meet at London in June or July, 1927, and on his motion, duly seconded, it was voted to cable the following message in reply: "American Institute appreciates greetings and assures you of its interest and hearty co-operation International Congress."

The informal discussion was entered upon and continued until adjournment of the afternoon session at 4:40 P.M.

Friday, November 5, the Institute reconvened at 10:05 A.M. and resumed the informal discussion.

Mention was made of the serious operation recently undergone by Mr. T. A. Phillips, and the Institute gave direction to send an appropriate message to him.

The President announced that the next meeting of the Institute will be held on June 2 and 3, 1927, in Toronto, Canada.

The Institute adjourned at 12:50 P.M.

CONSTITUTION *and* BY-LAWS
OF THE
AMERICAN INSTITUTE *of* ACTUARIES

CONSTITUTION

NAME

ARTICLE I. This organization shall be called "American Institute of Actuaries."

OBJECT

ART. II. The object of the American Institute of Actuaries is to advance the Science of Insurance Mathematics and the knowledge of the theory and practice of Life Insurance and related interests by associating together persons of like interests.

MEMBERSHIP

ART. III. The membership of the Institute shall be composed of three classes, Fellows, Associates, and Contributing Members.

OFFICERS

ART. IV. The Officers shall consist of a President, a Vice-President, a Secretary, a Treasurer, a Librarian, and an Editor of *The Record*.

GOVERNORS

ART. V. The Board of Governors shall consist of the Officers, six other Fellows, and the former Presidents who have held that office within six years; provided that the President elected at the annual meeting in 1926 and his predecessors shall be members of the Board for life.

DUTIES OF OFFICERS AND GOVERNORS

ART. VI. The duties of the officers shall be such as usually appertain to their respective offices or as may be prescribed from time to time by the Board of Governors. The duties of the Board of Governors shall be in general to manage the affairs of the Institute. It shall have the power, if the Institute be not then in session in annual meeting, to fill all vacancies occasioned by death or resignation of Officers or Governors, such appointees to serve until the

next annual meeting of the Institute and until their successors be elected. Appropriate subcommittees may be appointed by the Board of Governors for such matters as from time to time may be found desirable.

ELECTION OF OFFICERS AND GOVERNORS

ART. VII. The President, Vice-President, Secretary, and Treasurer shall be elected at each annual meeting to serve until their successors be elected. The President and Vice-President shall not be eligible for election to the same office at three successive annual meetings. The Librarian and the Editor of *The Record* shall be elected by the Board of Governors, subject to confirmation by the Institute at each annual meeting, and shall serve until their successors be elected. At each annual meeting two Governors shall be elected to serve until the third succeeding annual meeting and until their successors be elected. They shall not be eligible for election to succeed themselves at the same meeting at which their terms expire. Any vacancies existing on the Board shall be filled by election at each annual meeting, the Fellow so elected to serve during the unexpired term of his predecessor. All elections shall be by ballot.

MEETINGS

ART. VIII. The annual meeting of the Institute shall be held between the fifteenth day of May and the fifteenth day of June in each year on a date and at a place to be designated by the Board of Governors. Forty days' notice of the time and place of meeting shall be given the membership by the Secretary. Other meetings may be called by the Board of Governors or shall be called by the President upon the request in writing of not less than ten Fellows and not less than twenty days' notice shall be given of such meetings.

ASSOCIATES

ART. IX. Anyone at least twenty-one years of age pursuing actuarial studies and looking forward to a future Fellowship may be enrolled as an Associate under the following conditions: that he shall be nominated by at least two Fellows of the Institute; that a confidential notice of such nomination shall be given by the Secretary to each Fellow of the Institute; that this nomination shall be approved by the Board of Governors not less than twenty days after

such notification of nomination has been given; that he shall pay such examination fee and pass such preliminary examination as shall be prescribed by the Board of Governors. If the Board of Governors finds that a candidate has passed an equivalent examination required by another recognized society of actuaries, or has had such practical experience as in its opinion renders him qualified for Associateship, it may waive the examination herein required. An Associate shall be entitled to all the privileges of the Institute except those of voting and holding office.

ADMISSION TO FELLOWSHIP

ART. X. Any Associate whose application shall be approved by the Board of Governors may become a Fellow if at least twenty-five years of age under the following conditions: Every member of the Board of Governors shall have confidential notice of the application at least twenty days prior to the meeting at which it shall be acted upon, and the applicant shall pay such fee and pass such final examination as shall be prescribed by the Board of Governors. Otherwise no one shall be admitted as a Fellow unless recommended by a duly called meeting of the Board of Governors followed by a ballot of the Institute either in person or by mail. Confidential notice of such recommendation shall be mailed by the Secretary to every Fellow, and after twenty days, if not more than seven negative votes and affirmative votes of not less than half the total number of Fellows have been cast, the applicant shall become a Fellow.

CONTRIBUTING MEMBERS

ART. XI. Contributing Members shall be legal reserve Life Insurance Companies applying for such membership and elected thereto by a majority vote of the Board of Governors. Contributing Members shall have the use jointly with other Members of the Library under the rules of the Institute governing the same. Any person nominated by a Contributing Member shall have the privileges of Associates at any of the annual or special meetings of the Institute except during executive sessions.

QUORUM

ART. XII. Four members of the Board of Governors shall constitute a quorum; at any duly called meeting of the Institute fifteen Fellows shall constitute a quorum.

EXPULSION OR SUSPENSION OF MEMBERS

ART. XIII. The Board of Governors shall receive and hear all complaints preferred by any Fellow of this Institute against any other Fellow or Associate for misconduct in his relations to the Institute, or in his profession, or the practice thereof, or affecting the interests of the actuarial profession, provided such complaints shall be in writing plainly and specifically stating the matter complained of, and subscribed by the complainant.

The proceedings of the Board of Governors upon such complaint shall be deemed confidential and kept secret except so far as written or printed reports of the same shall be necessarily and officially made in the Institute.

Each party may appear personally and by counsel. Witnesses shall vouch for the truth of their statements on their word of honor. The Board of Governors shall make its decision, and if it finds the complaint to be true and of sufficient importance, it will so report to the Institute with its recommendations.

The Institute will take such action on the report as it shall see fit; but no member shall be expelled or suspended unless by a vote of at least two-thirds of the members present and voting.

ART. XIV. No resolution expressive of opinion shall be entertained at any meeting.

AMENDMENTS

ART. XV. Upon the written request of five Fellows of the Institute, the Secretary shall forthwith transmit by mail to the Fellows of the Institute any proposed amendment to this Constitution, and such amendment shall become effective provided it shall receive the favorable vote of two-thirds of the Fellows present and voting at the duly called meeting of the Institute next succeeding the mailing of such notice by the Secretary, and held not less than thirty days after the mailing of such notice.

BY-LAWS

ARTICLE I. *Order of business.*—At a meeting of the Institute the following order of business shall be observed, unless the Institute votes otherwise for the time being:

1. Calling of the roll.
2. Address or remarks by the President.
3. Minutes of the last meeting.

4. Summary of the business transacted by the Board since the last meeting of the Institute, presented for its information and approval.

5. Announcement by Presiding Officer of time of election of Officers and Governors.

6. Candidates for Fellowship considered and voted upon.

7. Reports of Committees and Officers.

8. Unfinished business; new business.

9. Reading of papers prepared for presentation to the Institute.

10. Discussions of papers presented.

11. Discussion of topics of current interest.

The Board of Governors' meetings shall proceed in this order as far as it may apply to them.

ART. II. *Meetings of the Board of Governors* shall be called whenever the President or four members of the Board of Governors so request, but not without sending notice to each member of the Board ten or more days before the time appointed. Such notice shall state the subjects intended to be brought before the meeting; should other matters be passed upon, any member of the Board shall have the right to reopen the same at the next meeting.

During any interim between meetings of the Board of Governors the President may, and upon written request by three members of the Board, the President or Secretary shall submit to the Board by personal communication in writing, mailed or delivered to each member of the Board, any matter of business requiring consideration or action by the Board. The votes of the Board, communicated in like form and manner to the President or Secretary, shall have the same effect as though voted at a regular or called meeting, provided no member of the Board votes against such action or in favor of postponement to a subsequent meeting of the Board.

ART. III. *Duties of officers.*—The President, or in his absence the Vice-President, shall preside at meetings of the Institute and of the Board. At the Institute meetings the presiding Officer shall vote only in case of a tie, but at the Board meetings he may vote in all cases.

The Secretary shall keep a full and accurate record of the proceedings at the meetings of the Institute and of the Board, send out calls for the said meetings, and, with the approval of the President, carry on the correspondence of the Institute. Subject to the direc-

tion of the Board, he shall have immediate charge of the office and archives and other chattels of the Institute.

The Editor of *The Record*, under the general supervision of the Board, shall have charge of all matters connected with editing and printing the Institute publications. There shall be a publication known as *The Record*, which shall contain only the proceedings of the meetings, original papers presented by Fellows and written by Fellows or Associates, and such discussions or other matters as may be expressly authorized by the Board of Governors.

The Treasurer shall send out calls for annual dues and acknowledge receipt of same; pay all bills approved by the President for expenditures authorized by the Board of the Institute; keep a detailed account of all receipts and expenditures, and present an abstract of same at the annual meetings after it has been audited by a Committee of the Board.

ART. IV. *Dues.*—Each Fellow shall pay fifteen dollars and each Contributing Member twenty-five dollars upon entrance and a like amount on the date of each annual meeting thereafter, except in the case of Fellows not residing on this continent who shall pay seven dollars and fifty cents annually at the time stated. Each Associate shall pay five dollars upon entrance and a like amount on the date of each annual meeting thereafter until five such payments in all shall have been made. Beginning with the sixth annual meeting after the admission of an Associate as such the dues of any Associate heretofore or hereafter admitted shall be the same as those of a Fellow and shall be payable at the times when dues of Fellows are payable.

It shall be the duty of the Treasurer to notify by mail any Fellow, Associate, or Contributing Member whose dues may be six months in arrears, and to accompany such notice by a copy of this article. If such Fellow, Associate, or Contributing Member shall fail to pay such dues within three months from the time of mailing such notice, the Treasurer shall report the delinquent to the Board, by whom the name of such member shall be stricken from the rolls, and such member shall cease to be a Fellow, Associate, or Contributing Member of the Institute. Such Member may, however, be reinstated by vote of the Board and upon payment of arrears of dues.

And also except that any person who shall have attained the age of seventy years while a Fellow of the Institute, having been a

Fellow on the date of the first annual meeting or otherwise for a period of at least twenty years, shall enjoy all the privileges of membership without further payment of dues.

ART. V. *Designation by initials.*—Fellows of the Institute are authorized to append to their names the initials F.A.I.A., and Associates are authorized to append to their names the initials A.A.I.A.

ART. VI. *Amendments.*—Upon the written request of five Fellows of the Institute, the Secretary shall forthwith transmit by mail to the Fellows of the Institute any proposed amendment to the By-Laws, and such amendment shall become effective provided it shall receive the favorable vote of two-thirds of the Fellows present and voting at the duly called meeting of the Institute next succeeding the mailin of such noticeg by the Secretary, and held not less than thirty days after the mailing of such notice.

REQUIREMENTS *for* ADMISSION

TO THE

AMERICAN INSTITUTE *of* ACTUARIES

ASSOCIATESHIP

Any person pursuing actuarial studies and looking forward to a future Fellowship may be nominated for Associateship by two Fellows of the Institute on a blank form that will be furnished by the Secretary on request. The nomination paper must state which parts of the examinations the candidate will write at the next examination session and be mailed to the Secretary with the examination fee **not later than January 31** preceding the examination date. If the nomination is approved by the Board of Governors the candidate will be notified of the place and time of examination. If not approved, the fee will be refunded.

On passing one or more parts the candidate will be listed as a Student and his name continued on the list as long as he has at one of the last three examination sessions passed one or more parts.

Upon passing the Associateship examinations the candidate subject to final approval by the Board will be enrolled as an Associate of the Institute upon payment of dues for the current year provided he has reached the age of twenty-one years and has had such practical experience as in the judgment of the Board qualifies him for such enrolment. If at the time of passing the final examinations the candidate is less than twenty-one years of age he cannot be enrolled until attaining that age and until the Board of Governors has again approved his application. If on the date of passing the last of the Associateship examinations the candidate has not had such practical experience as in the judgment of the Board of Governors qualifies him for enrolment he cannot be enrolled as an Associate unless nor until he shall have completed such experience within three years and his application has again been approved by the Board of Governors. The experience usually required is at least one year of actuarial work of a kind to be approved by the Board.

FELLOWSHIP

Any Associate may apply for Fellowship on a blank form that will be furnished by the Secretary. This application must state which parts of the Fellowship examinations the candidate will write at the next examination session and be mailed to the Secretary with the examination fee **not later than January 31** preceding the examination date. Upon passing all of the Fellowship examinations the candidate may be admitted as a Fellow by final action of the Board and payment of the dues for the current year except that if less than twenty-five years of age such final action will not be taken until the candidate has attained that age.

EXAMINATIONS

Examinations are held unless otherwise ordered on the second Tuesday and the following Wednesday in April of each year at points within as convenient reach of the candidates as is possible. The schedule of examinations is as follows:

Tuesday, A.M.	Parts 1 and 5 of the Associateship and Part 9 of the Fellowship.
Tuesday, P.M.	Parts 2 and 6 of the Associateship and Part 10 of the Fellowship.
Wednesday, A.M.	Parts 3 and 7 of the Associateship and Part 11 of the Fellowship.
Wednesday, P.M.	Parts 4 and 8 of the Associateship and Part 12 of the Fellowship.

Candidates nominated for Associateship must designate in the nomination paper the parts they intend to write at the next examination. Similarly Associates applying for Fellowship must designate in the application the parts they intend to write at the next examination. At other times candidates desiring to write examination papers are required to file a written request with the Secretary **not later than January 31** preceding the examination date, specifying by number the parts the candidate desires then to write, accompanied by the examination fee.

The several parts of the examination must be taken in sequence. No candidate may present himself for any part unless he has previously passed or is then presenting himself for all previous parts. For example, a candidate who passed Parts 1, 3, and 4 but

failed in Part 2 may sit at the next examination for Parts 2 and 5 but not for 6 because of the conflict in time, and not for 7 because out of sequence. No candidate may take any part of the Fellowship examinations before he has passed all of the Associateship examinations. The Board has the right to deny the privilege of examination in its discretion to any one at any time.

EXAMINATION FEES AND MEMBERSHIP DUES

The examination fee required to accompany the nomination for Associateship, the application for Fellowship or an interim request for examination shall be $2 for each part but not less than a minimum fee of $5; thus, for one or two parts $5; for three parts $6 and for four parts $8. Examination fees will not be refunded as to any parts requested to which the candidate is admitted and duly notified of the place and date of examination unless a written request for refund is received by the Secretary on or before February 28 preceding the examination date.

The membership dues are, Fellows $15; Associates $5; payable on admission as such and a like amount at each annual meeting thereafter, except that beginning with the sixth annual meeting after the admission of an Associate as such the dues of any Associate then and thereafter shall be the same as those of a Fellow; provided, however, that for Fellows not residing on this Continent the dues shall be one-half of the amount otherwise payable.

SYLLABUS *of* EXAMINATIONS
OF THE
AMERICAN INSTITUTE *of* ACTUARIES

SUBJECTS

ASSOCIATESHIP

Part 1. Elementary Algebra
Plane Geometry
Part 2. Plane Trigonometry
Plane Analytical Geometry
Part 3. Advanced Algebra
Probability
Part 4. Differential and Integral Calculus
Finite Differences
Statistics
Part 5. Compound Interest
Accounting
Part 6. Life Contingencies Involving Single Life Functions Only
Part 7. Life Contingencies Involving Joint Life Functions
Part 8. Construction of Actuarial Tables
Multiple Decrement Tables
Disability and Accident Contingencies
History of Life Insurance

FELLOWSHIP

Part 9. Mortality and Disability Investigation
Selection of Risks and Treatment of Substandard Risks
Disability and Accident Contingencies. (Required only of candidates that passed Part 8 prior to 1926)
Part 10. Gross Premiums
Valuation of Liabilities
Non-Forfeiture Values and Change or Reinstatement of Life Insurance Contracts
Part 11. Analysis and Distribution of Surplus
Life Insurance Accounts
Investments and Valuation of Assets
Banking and Finance

Part 12. Life Insurance Law
Preparation of Policy Forms
Pension Funds
General Topics of Actuarial Interest

SCOPE OF EXAMINATIONS AND RECOMMENDED COURSE OF STUDY

ASSOCIATESHIP

Part 1

Elementary Algebra

—Hall and Knight: *Algebra for Colleges and Schools* (also *Answers*, printed as a separate pamphlet), chaps. 1–35, and 37.

Plane Geometry

—Phillips and Fisher: *Elements of Geometry*, Books 1–3.

Part 2

Plane Trigonometry

—Hall and Knight: *Elementary Trigonometry*, chaps. 1–16.

Plane Analytical Geometry

—Smith and Gale: *New Analytic Geometry*, chaps. 1–5.

Part 3

Advanced Algebra

—Hall and Knight: *Higher Algebra*, chaps. 1–17, 21, and 29.

Probability

—Hall and Knight: *Higher Algebra*, chap. 32; Henry: *Calculus and Probability*, chap. 21.

Part 4

Differential and Integral Calculus

—Granville: *Elements of the Differential and Integral Calculus*, chaps. 1–6 (to end of Sec. 65), 7, 8, 11, 13 (to end of Sec. 107), 15, 17, 18 (to end of Sec. 147), 22, 24–27 (to end of Sec. 199), and 28 (to end of Sec. 210); Henry: *Calculus and Probability*, chaps. 1, 10–14, and 16–19.

Finite Differences

—Henry: *Calculus and Probability*, chaps. 2–9, 15, and 20; Rietz and others: *Handbook of Mathematical Statistics*, chap. 3 (omitting "Graduation"); *J.I.A.*, L, 15–33 (D. C. Fraser: "Notes on Finite Differences").

Note.—Where the Recommended Course of Study refers to papers, the discussions thereof are included.

Statistics

Does not include graduation or curve fitting.

—Yule: *An Introduction to the Theory of Statistics*, chaps. 6–17; Rietz and others: *Handbook of Mathematical Statistics*, chaps. 1, 2, 5 (omitting "Test of Goodness of Fit"), 6 and 8–12; *R.A.I.A.*, XIII, 214–250 (H. L. Rietz: "Certain Applications of Statistics").

PART 5

Compound Interest

—Rietz, Crathorne, and Rietz: *Mathematics of Finance*, chaps. 1–6; Todhunter: *Institute of Actuaries' Text-Book, Part I*, chaps. 1–7 and 10.

Accounting

—Bogle: *Comprehensive Bookkeeping;* Dickinson: *Accounting Practice and Procedure*, chaps. 1–8.

PART 6

Life Contingencies Involving Single Life Functions Only

—Spurgeon: *Life Contingencies*, chaps. 1–4, 5 (omitting Arts. 24, 25), 6–9, 10 (from Art. 16), 11 (to Art. 13) and 12; Robertson and Ross: *Actuarial Theory*,* in Part II such sections of chaps. 1–16 and 18 as do not involve joint life functions, omitting Art. 29 and examples 24–26 of chap. 7; *J.I.A.*, LIV, 123–149 (P. H. McCormack: "Damaged Lives and Options").

PART 7

Life Contingencies Involving Joint Life Functions

—Spurgeon: *Life Contingencies*, chaps. 13–19; Robertson and Ross: *Actuarial Theory*,* in Part II such sections of chaps. 1–16 and 18 as involve joint life functions.

* Where *Actuarial Theory* in Part II refers to the "Text-Book," the *Institute of Actuaries' Text-Book, Part II*, is meant; inasmuch as the latter has been replaced by Spurgeon: *Life Contingencies*, the references are to be ignored.

PART 8

Construction of Actuarial Tables

—Todhunter: *Institute of Actuaries' Text-Book, Part I*, chap. 8; Spurgeon: *Life Contingencies*, chap. 20.

Multiple Decrement Tables

—Spurgeon: *Life Contingencies*, chap. 21.

Disability and Accident Contingencies

Only such disability and accident benefits as are included in life insurance policies.

—*T.A.S.A.*, XXIV, x–xii ("Disability Symbols"); Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, pp. 18–22 and 32–56; *T.A.S.A.*, XII, 66–71 ("Hunter's Disability Table"†); XVII, 211, 212, and

214–240 (M. Davis: "Disability Benefits"); XXIV, 92, and 99–125 (E. H. Hezlett: "Disability Benefits").

—For the theory of accident contingencies relating to benefits included in life insurance policies the published material is meager. Candidates should learn the practice of several companies in calculating double indemnity net premiums and reserves for individual policies on the common plans of insurance. Some help may be derived from *T.A.S.A.*, XIX, 82, 83, and 85 (W. A. Hutcheson: "Double Indemnity"); *R.A.I.A.*, VIII, 119 (H. W. Buttolph: "Death from Accident"); *T.A.S.A.*, XXVI, 37–51 (A. Hunter: "The Accidental Death Feature in Life Insurance Policies").

† It is important to change the headings of this table as indicated in Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, p. 36.

History of Life Insurance

—*T.A.S.A.*, XXI, 331–364 (W. A. Hutcheson: "Evolution of Life Insurance"); Maclean: *Life Insurance*, pp. 342–367 ("Historical Development of Life Insurance in the United States").

FELLOWSHIP

Part 9

Mortality and Disability Investigation

The principles to be observed in the investigation of mortality and disability experience and in the compilation and graduation of tables derived therefrom; also the sources, methods, and characteristics of the principal investigations that have been made and of the corresponding tables.

—Mortality. —Investigation of Experience and Compilation of Tables: Murphy and Papps: *Construction of Mortality Tables from the Records of Insured Lives;* Wolfenden: *Population Statistics and Their Compilation; J.I.A.*, XLVIII, 294–303 (G. King: "Abridged Mortality Table"); LIII, 104–166 (G. King: "Short Method of Constructing Select Mortality Tables").

—Graduation: Rietz and others: *Handbook of Mathematical Statistics*, pp. 58–70, 78–81, and 92–119; Hardy: *The Theory of the Construction of Tables of Mortality, etc ;* Henderson: *Graduation of Mortality and Other Tables; J.I.A.*, XXVI, 77–120 (T. B. Sprague: "Graphic Method of Adjusting Mortality Tables"); *T.A.S.A.*, XIV, 256–276 (J. B. Maclean: "Graduation by the Summation Method"); *J.I.A.*, L, 251–259 (W. P. Elderton: "Graduation by Makeham's Hypothesis"); *T.A.S.A.*, XXIV, 398–413 (F. B. Gerhard: "Makeham's Formula by Graphic Method").

—Principal Investigations and Tables: Moir: *Sources and Characteristics of the Principal Mortality Tables; Medico-Actuarial Mortality Investigation*, Vols. I–V; *American-Canadian Mortality Investigation, 1900–1915*, Vols. I and II; *T.A.S.A.*, XXI, 178–206 (V. Howell: "Two Graduations of American-Canadian Mortality Experience"); *United States Life Tables, 1890, 1901, 1910, and 1901–1910*, pp. 17–21 and 203–239; *T.A.S.A.*, XXIII, 269–273

("Review of U.S. Life Tables"); XXI, 157–177 (A. Hunter: "Mortality of American Annuitants"); *J.I.A.*, LIV, 43–90 and 269–284 (W. P. Elderton and H. J. P. Oakley: "Mortality of British Offices' Life Annuitants," 1900–1920); LV, 144–159 ("Mortality of British Government Life Annuitants," 1900–1920) and 160–177 (D. C. Fraser: "Notes on Recent Reports on the Mortality of Annuitants"); *T.A.S.A.*, XVIII, 311–338 (H. Moir: "Mortality Graphs"); XXIII, 408–434 (H. Moir: "Recent Mortality"); *J.I.A.*, LVI, 199–202 ("Joint Mortality Investigations in America"); *R.A.I.A.*, XIII, 348–353 (Discussion); *Proceedings of the Seventh International Congress of Actuaries*, Vol. I, 527–552 (J. Burn and W. C. Sharman: "Changes in the Rates of Mortality amongst Assured Lives during the Past Century"); *T.A.S.A.*, XXII, 209–221 (J. B. Maclean: "Swedish Mortality Investigation"); XXI, 207–211 (S. A. Joffe: "Mortality Experience of Life Insurance Companies in Germany") and 378–391 (W. A. Hutcheson: "Mortality Experience of Mutual Life"); XXIV, 53–91 (M. Davis: "Mortality Experience of Equitable Life"); XXII, 466–482 (F. H. Johnston: "Convertible Term Business"); XXIV, 304–311 (P. C. H. Papps: "Mortality Experience under Term Policies and under Policies Issued at Old Ages") and 325–333 (E. B. Morris: "Comparison of Mortality Elements between Group and Regular Life Insurance"); XXVI, 332–374 (E. E. Cammack: "Combined Group Mortality Investigation"); XIX, 53–59 (P. H. Evans: "Mortality by Order of Birth"); XXVI, 58–67 (C. D. Rutherford: "A Note on Mortality Variations") and 37–51 (A. Hunter: "The Accidental Death Feature in Life Insurance Policies").

—Disability.—Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, pp. 1–32; *T.A.S.A.*, XII, 44–55 (A. Hunter: "Waiver of Premium on Permanent Disability"); XXII, 320–327 (W. A. Hutcheson: "Account of Friendly Societies"); XIX, 219–226 (A. Hunter: "Claims for Disability Benefits"); XXII, 200–208 (J. S. Thompson: "Causes of Total and Permanent Disability") and 442–465 (A. Hunter: "Claims for Permanent Total Disability"); XXIV, 92–99 (E. H. Hezlett: "Disability Benefits"); Watson: *An Account of an Investigation of the Sickness and Mortality Experience of the I.O.O.F. Manchester Unity, 1893–1897*, pp. 1–80; *Report of Committee on Disability Experience* (*May, 1926*).

Selection of Risks and Treatment of Substandard Risks

—Blank Application and Medical Examiner's Report of Several Companies; Maclean: *Life Insurance*, pp. 148–166 ("Selection"); *T.F.A.*, VIII, 103–162 (L. P. Orr: "Selection"); *T.A.S.A.*, XII, 1–17 (A. Hunter: "Selection"); XX, 273–332 (A. Hunter and Dr. O. H. Rogers: "Numerical Rating"); XXIII, 322–346 (R. D. Murphy: "Selection").

—*T.A.S.A.*, XVII, 281–289 (A. Hunter and Dr. O. H. Rogers: "Need of Standards for Border-Line Risks"); Maclean: *Life Insurance*, pp. 167–182 ("Substandard Risks"); *T.A.S.A.*, XVI, 64–82 (A. Hunter: "Underaverage Lives"); *R.A.I.A.*, XI, 158–186 (F. B. Mead: "Substandard Insurance") and 187–204 (A. Hunter: "Underaverage Business"); X, 32–38 (A. Coburn: "Substandard Life Insurance"); *T.A.S.A.*, XXIII, 114–160 (J. E. Hoskins: "In-

cidence of Substandard Mortality"); XXV, 11–28 (V. Howell: "Substandard Practice"); *R.A.I.A.*, XIV, 42–67 (E. E. Cammack: "Substandard Business").

—*T.A.S.A.*, XXI, 365–377 (A. Hunter: "Is the Average the Same as the Normal?"); XV, 62–76 (A. Hunter and Dr. O. H. Rogers: "Interpretation of Results of M. A. Investigation"); XXI, 16–63 (A. Hunter and Dr. O. H. Rogers: "Occupation"); XX, 353–363 (Discussion of "Overweight"); XXII, 512–531 ("Joint Committee Report on Height and Weight") and 342–384 (A. Hunter and Dr. O. H. Rogers: "Ratings for Principal Impairments"); XX, 83–100 (A. Hunter and Dr. O. H. Rogers: "Heart Murmurs"); XXII, 532–537 ("Joint Committee Report on Heart Murmur and Albuminuria"); XXV, 210–218 (A. Hunter and Dr. O. H. Rogers: "Heart Impairments"); XXII, 538–540 ("Joint Committee Report on Family History"); XXIII, 347–367 (A. Hunter and Dr. O. H. Rogers: "Inflammatory Rheumatism, Tuberculosis and Blood Spitting"); XXI, 151–156 (O. W. Perrin and Dr. H. Toulmin: "Asthma, Renal Colic and Pleurisy") and 452–477 (J. D. Craig: "Colored Lives"); *R.A.I.A.*, XIV, 86–89 (W. F. Poorman: "Aviation Hazard"); *T.A.S.A.*, XIX, 235–238 (P. H. Evans: "Habits"); *Blood Pressure* (*Report of Joint Committee, 1925*); *T.A.S.A.*, XXVI, 290–300 (A. Hunter and Dr. O. H. Rogers: "Albuminuria, Glycosuria") and 301–318 (W. M. Strong and Dr. F. S. Weisse: "Overweight").

—Maclean, *Life Insurance*, pp. 255–257 ("Selection for Disability Benefits"); *T.A.S.A.*, XIX, 226 and 227 (A. Hunter: "Selection for Disability Benefits"); XXIII, 394–396 (J. M. Laird: "Disability Underwriting"); XVII, 264–267 and 271–273 (W. J. Graham: "Selection for Group Insurance"); XXII, 120–139 (D. E. Kilgour: "Insurance without Medical Examination") and 140–144 (A. Hunter: "Savings in Expense Compared with Expected Extra Mortality under Insurance without Medical Examination"); *R.A.I.A.*, XIV, 116–140 (Discussion of "Non-Medical Business"); *T.A.S.A.*, XXVI, 273–289 (V. R. Smith and Dr. H. C. Scadding: "Life Insurance without Medical Examination in Canada").

Disability and Accident Contingencies

Required only of candidates that passed Part 8 prior to 1926.

—See Part 8.

Part 10

Gross Premiums

Does not include substandard premiums.

—Maclean: *Life Insurance*, pp. 80–86 ("Premiums"); Spurgeon: *Life Contingencies*, Arts. 24 and 25 of chap. 5; *T.F.A.*, II, 207–259 and 262–268 (H. Moir: "Office Premiums"); VII, 262–273 (W. Denham and C. K. Granger: "Office Premiums"); *T.A.S.A.*, XX, 379–409 (E. E. Cammack: "Premiums for Non-Participating Life Insurances"); *J.I.A.*, XLVIII, 45–64 (F. J. Cameron: "Office Premiums for Whole Life Non-Profit Assurances"); *T.A.S.A.*, XVI, 272–278 (E. E. Rhodes: "Premium Loadings and Expense Limitations"); XX, 23–71 (A. Hunter: "Should the American Men Mortality Table Be the Basis for Premiums and Reserves?"); XXI, 175–177 (A. Hunter: "Premiums on Basis

of New Annuity Rates"); XXII, 49–52 (J. D. Buchanan: "Industrial Premiums"); *J.I.A.*, XXXVI, 51–81 (A. Hunter: "Effect of Using Lapse Element"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 97 ("Limitation of Expenses") and 85 ("Actual Premium Less than Net Premium"); insurance laws of Wisconsin, Sec. 1950*m*, *n*, *o*, and *p* ("Limitation of Expense Provision in Premiums and of Expenses").

Valuation of Liabilities

—Method of at least one company in determining each item of liability in the "convention blank" annual statement; Maclean: *Life Insurance*, pp. 225–227 ("Liabilities"); Wolfe: *The Examination of Insurance Companies*, chaps. 10, 11, 14, and 15; Spurgeon: *Life Contingencies*, chaps. 10 (to Art. 15) and 11 (from Art. 14); *T.A.S.A.*, XXV, 219–245 (E. W. Marshall: "Office Methods of Obtaining Policy Reserve"); IX, 150–158 (H. Moir: "Group Valuation and Mortality Register"); XI, 144–162 (R. Henderson: "Valuation by Attained Age"); XXII, 43–47 (J. D. Buchanan: "Industrial Valuation").

—Maclean: *Life Insurance*, pp. 91–96 and 101–123 ("Reserves"); *J.I.A.*, XXXVII, 91–99 (W. A. Hutcheson: "American Valuation Tables"); *T.A.S.A.*, X, 179–200 (H. Moir: "Valuation"); XXIV, 312–324 (R. B. Robbins: "Reserve Standards for Life Insurance"); XXI, 125–133 (A. A. Welch: "Preliminary Term Valuation"); *T.F.A.*, VII, 103–126 (A. T. Maclean: "Valuation with Allowance for Initial Expense"); *R.A.I.A.*, VIII, 22–25 (H. W. Buttolph: "Individual Reserves in Life Insurance"); XIV, 281–293 (Discussion of American Men Table as a Basis of Valuation).

—Hunter: *Total Permanent Disability Benefits in Relation to Life Insurance*, pp. 57 and 58; *T.A.S.A.*, XXV, 80–94 (F. A. Shailer: "Approximate Methods of Valuation").

—Baldwin: *New York Insurance Law Annotated 1925*, § 84 ("Valuation").

Non-Forfeiture Values and Change or Reinstatement of Life Insurance Contracts

—Maclean: *Life Insurance*, pp. 126–136 ("Policy Provisions Relating to the Payment of Premiums"); *R.A.I.A.*, XI (Part I), 12–38 (P. H. Evans: "Surrender and Non-Forfeiture Values"); *J.I.A.*, XXXV, 199–241 (F. W. Fulford: "Surrender Values"); XLVIII, 71–81 (F. J. Cameron: "Surrender Values for Whole Life Non-Profit Assurances"); L, 313 and 314 (A. Fraser: "Surrender Values of Altered-Class Policies"); *T.A.S.A.*, XIII, 259–272 (J. F. Little: "Modern Surrender Values"); *R.A.I.A.*, VII, 87–98 (Discussion of "Surrender Values in Policies on Substandard Lives"); XIII, 251–282 (F. S. Withington: "Using Standard Values for Substandard Policies"); *T.A.S.A.*, X, 608 (E. E. Rhodes: "Conclusions from Mutual Benefit Experience under Extended Insurance"); XXIV, 386–397 (W. P. Barber: "Automatic Premium Loans"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 88 ("Surrender Value of Lapsed or Forfeited Policies") and 92 ("No Forfeiture without Notice").

—*T.F.A.*, II, 259–262 (H. Moir: "Changes in Policies"); *T.A.S.A.*, XI, 465–496 (W. Young: "Alteration of Life Insurance Contracts"); XXIII, 204–

222 (J. S. Thompson: "Changes in Policy Contracts"); XXV, 253–271 (J. E. Hoskins: "Office Practice on Changes in Policy Form").

—Maclean: *Life Insurance*, pp. 136 and 137 ("Reinstatement"); *T.A.S.A.*, XI, 296–303 (H. Moir: "Reinstatement of Policy on Impaired Risk"); XVI, 284–296 (A. Hunter: "Reinstatement"); XVIII, 246–259 (R. D. Murphy: "Reinstatement").

Part II

Analysis and Distribution of Surplus

—Maclean: *Life Insurance*, pp. 183–199 ("Dividends"); *R.A.I.A.*, XI (Part I), 106–141 (J. C. Rietz: "Methods of Surplus Distribution"); XII, 35–65 (A. Coburn: "Distribution"); *T.A.S.A.*, XI, 49–54 (P. C. H. Papps: "Original Contribution Plan"); X, 200–223 (H. Moir: "Surplus Distribution") and 359–361 (Discussion); IX, 310–326 (R. W. Weeks: "Practical Rule for Calculating Annual Dividends"); IX, 344–346 (M. M. Dawson: "A Distribution Formula"); *J.I.A.*, XLIV, 415–452 (H. J. Rietschel: "Expenses"); LIII, 167–191 (C. H. Maltby: "Effect of Valuation Bases upon Surplus"); *T.A.S.A.*, XXIII, 83–113 (J. S. Elston: "Expense Investigations"); XXIV, 14–52 (A. Hunter: "Methods of Distributing Annual Dividends") and 291–298 (R. Henderson: "Equity in Surplus Distribution"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 83 ("Distribution of Surplus to Policyholders") and 87 ("Contingency Reserve").

—*T.A.S.A.*, XV, 40–48 (D. P. Fackler: "Annuities with Participation") XVI, 279–283 (D. P. Fackler: "Annuities with Participation"); XV, 354–363 (C. C. Ferguson: "Participating Annuities").

—*R.A.I.A.*, XIV, 21–28 (E. G. Fassel: "Excess Interest under Annuity-Certain").

Life Insurance Accounts

—Method of at least one company in determining each item of the "convention blank" annual statement with schedules; Maclean: *Life Insurance*, pp. 218–235 ("Annual Statement"); *T.A.S.A.*, XV, 328–353 (R. D. Murphy: "Life Insurance Bookkeeping and Annual Statements"); Glover and Wightman: *Life Insurance Accounting;* Wolfe: *The Examination of Insurance Companies* chaps. 6–8, 16–18, and 20; *Proceedings of the Insurance Institute of Toronto, 1922–1923*, pp. 84–128 (W. G. Reburn: "Life Assurance Bookkeeping"); *R.A.I.A.*, XIV, 68 85 (V. Howell: "Financial Statements"); IV (Part II), 79 88 (Discussion of "The Policy Exhibit"); VI, 104 and 106–111 (Discussion of "Excess Credits over Reserve"); XIV, 146–159 (Discussion of "New Gain and Loss Exhibit"); *T.A.S.A.*, XX, 101–111 (J.W. Tebbetts: "Gain and Loss on Annuity Business") and 112–133 (E. G. Fassel: "Formulae Used by Various Companies in Computing Expected Mortality"); XXI, 134–150 (J. S. Elston: "Treatment of Disability Claim Transactions in the Annual Statistics"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 44, 45, and 103 ("Reports").

Investments and Valuation of Assets

—Jordan: *Investments*, chaps. 1–19 and 21–27; Maclean: *Life Insurance*, pp. 200–217 ("Assets"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 16–20*a* and 100 ("Investments"); Wolfe: *Examination of Insurance Companies*, chaps. 2–5 and 9; *T.F.A.*, IX, 1–38 (R. Murrie: "Investments of Insurance Offices"); *J.I.A.*, LVI, 121–152 (C. R. V. Coutts: "Life Assurance Investment Policy"); *R.A.I.A.*, XII, 96–150 (A. T. Maclean: "Course of Rate of Interest"); *T.F.A.*, VII, 171–213 (H. Moir: "Amortization of Bonds and Valuation of Assets"); *T.A.S.A.*, XXI, 95–117 (J. B. Gibb: "A Method of Valuing Bonds of a Life Insurance Company").

Banking and Finance

—Shearman: *Practical Economics;* Holdsworth: *Money and Banking* (Fourth Edition), chaps. 1–22 (omitting Appendix); Dewing: *Corporation Finance*, chaps. 1–5, 14–19, and 23–26.

PART 12

Life Insurance Law

—Maclean: *Life Insurance*, pp. 323–341 ("Legal Aspects of Life Insurance"); *Corpus Juris*, §§ 1–756 of article on "Insurance," §§ 1–368 of article on "Life Insurance," §§ 1–54 of article on "Annuities," §§ 204–211 of article on "Bankruptcy," §§ 1–210 of article on "Accident Insurance" and § 195 of article on "Infants," omitting footnotes and matter that does not apply to the business of fixed premium life insurance companies; *T.A.S.A.*, XXVI, 375–442 (W. M. Strong: "Disability Benefits—The Contract and the Courts"); *Legal Notes* in last five volumes of *T.A.S.A.*, noting that in some cases the decisions do not accord with generally accepted doctrines or the law in other jurisdictions.

—Maclean: *Life Insurance*, pp. 307–322 ("Regulation and Taxation"); Baldwin: *New York Insurance Law Annotated 1925*, §§ 1–15, 21–26*a*, 28–43, 46–82, 86, 89–91, 94–96*a*, 98, 99, 101*a*, 101*c*, 101*d*, 102, 104, 106 and Appendix (Tax Law, §§ 187–197), omitting all matter that does not apply to the business of fixed premium life insurance companies; *Regulations 65 Relating to the Income Tax under the Revenue Act of 1924*, §§ 242–247 and Arts. 661–693.

—At the option of the candidate, **either:** Matter relating to the business of fixed premium life insurance companies in the insurance laws of Illinois, Minnesota, and Ohio; **or:** *Proceedings of the Insurance Institute of Toronto, 1924–1925*, pp. 185–210 (H. J. Sims: "The Uniform Life Insurance Act, Canada"), *T.A.S.A.*, XXVI (Part I), "Address of the President" (A. B. Wood: "Uniform Life Insurance Act"), and matter relating to the business of fixed premium life insurance companies in the insurance laws of Ontario and the Dominion of Canada.

In the case of New York, a thorough knowledge of the laws is required, but for Illinois, Minnesota, and Ohio, or for Ontario and the Dominion of Canada, only a general knowledge is necessary.

Preparation of Policy Forms

—Insurance and annuity policy forms on the principal plans of at least one company that incorporates disability and double indemnity benefits in life policies, and the more important indorsements and riders used at issue or on subsequent change; Maclean: *Life Insurance*, pp. 124–147 ("Terms of the Policy"); *R.A.I.A.*, X, 118–126 (Discussion of "Corporation and Partnership Insurance"); *T.A.S.A.*, XII, 175–191 (H. Moir: "Liberality of Modern Policies"); XVIII, 63–71 (O. W. Perrin: "On Modes of Settlement under Policy Provisions") XVII, 263, 264, 267, and 268 (W. J. Graham: "Group Insurance"); XXII, 52–56 (J. D. Buchanan: "Industrial Policy Contract"); Baldwin: *New York Insurance Laws Annotated 1925*, §§ 101, 101*b* and 101½ ("Standard Provisions").

Pension Funds

—*T.F.A.*, IV, 195–228 (J. J. M'Lauchlan: "Fundamental Principles of Pension Funds"); *J.I.A.*, XLV, 149–229 (H. W. Manly: "On Staff Pension Funds"); XXXIX, 129–206 (G. King: "On Staff Pension Funds"); *T.F.A.*, VII, 215–258 (J. J. M'Lauchlan: "On the Construction of Salary Scales and the Valuation of Prospective Pensions"); *J.I.A.*, LI, 106–117 and 233–244 (W. J. H. Whittall: "Pension Problems in America"); *T.F.A.*, II, 307–342 (V. Marr: "Widows' Funds"); *J.I.A.*, LII, 1–37 (C. E. Howell: "Widows' Funds"); *R.A.I.A.*, III, 33–52 (H. L. Rietz: "Status of Certain Current Pension Funds"); VIII, 1–12 (H. L. Rietz: "Pensions for Public Employees"); X, 1–14 (H. L. Rietz: "Pensions for Insurance Company Employees"); *T.A.S.A.*, XVI, 109–144 (D. A. Walker: "A Staff Pension Fund"); XXI, 77–94 (J. B. Maclean: "Problems of Small Pension Funds"); XV, 306–314 (M. M. Dawson: "Mortality and Remarriage Tables"); XXVI, 480–506 (R. A. Hohaus: "Reinsurance of Retirement Plans").

General Topics of Actuarial Interest

—Riegel and Loman: *Insurance Principles and Practices*, chaps. 1–4 and 10–23; Maclean: *Life Insurance*, pp. 1–31 ("Life Insurance Systems and Companies") and 259–306 ("Group Insurance, Industrial Insurance and Internal Organization"); *T.A.S.A.*, XXII, 317–339 (W. A. Hutcheson: "Review of Disablement Insurance"); *J.I.A.*, LIV, 150–203 (P. N. Harvey: "National Health Insurance"); *T.A.S.A.*, XVII, 263–273 (W. J. Graham: "Group Insurance"); XXIV, 168–219 (J. D. Craig: "Unemployment Insurance"); *R.A.I.A.*, XII, 1–34 (M. Gunn: "Unemployment Insurance"); *T.A.S.A.*, XXIII, 27–66 (W. N. Bagley and J. M. Laird: "Life Reinsurance") and 67–75 (M. W. Torrey: "Reinsurance"); XXII, 75–119 (A. A. Rydgren: "Value of Business Reinsured in Bulk"); XXV, 5–10 (A. Hunter: "Reinsurance and Transfer of Business in Foreign Countries"); XXII, 36–74 (J. D. Buchanan: "Industrial Life Insurance"), 385–417 (E. B. Morris: "Some New Problems Affecting Life Insurance") and 13–35 (W. R. Halliday: "Problems of Smaller Companies"); *R.A.I.A.*, XIII, 283–347 (M. A. Linton: "Returns under Agency Contracts"); current issues of two or three well-known insurance journals; latest volumes of

R.A.I.A., *T.A.S.A.*, *J.I.A.*, *T.F.A.*, and such items of actuarial interest as may be found in the lastest volumes reporting the proceedings of the Association of Life Insurance Presidents, National Convention of Insurance Commissioners, and American Life Convention.

PUBLICATIONS REFERRED TO IN THE RECOMMENDED COURSE OF STUDY

ACTUARIAL JOURNALS

R.A.I.A.—Record of the American Institute of Actuaries.
T.A.S.A.—Transactions of the Actuarial Society of America.
J.I.A.—Journal of the Institute of Actuaries.
T.F.A.—Transactions of the Faculty of Actuaries.

AUTHOR, TITLE, AND PUBLISHER OF TEXT-BOOKS

Baldwin, W. E., *New York Insurance Law Annotated 1925*, Banks Law Publishing Co., New York.

Bogle, A. M., *Comprehensive Bookkeeping*, The Macmillan Co., New York.

Dewing, A. S., *Corporation Finance*, Ronald Press Co., New York.

Dickinson, A. L., *Accounting Practice and Procedure*, Ronald Press Co., New York.

Glover, J. W., and Wightman, E. C., *Life Insurance Accounting*, Edwards Brothers, Ann Arbor, Mich.

Granville, W. A., *Elements of the Differential and Integral Calculus*, Ginn and Co., Boston.

Hall, H. S., and Knight, S. R., *Algebra for Colleges and Schools* (also *Answers* printed as a separate pamphlet), The Macmillan Co., New York; *Elementary Trigonometry*, Macmillan and Co., London; *Higher Algebra*, Macmillan and Co., London.

Hardy, G. F., *The Theory of the Construction of Tables of Mortality, etc.*, C. & E. Layton, London.

Henderson, R., *Graduation of Mortality and Other Tables* ("Actuarial Studies No. 4"), Actuarial Society of America, New York.

Henry, A., *Calculus and Probability*, C. & E. Layton, London.

Holdsworth, J. T., *Money and Banking* (Fourth Edition), D. Appleton and Co., New York.

Hunter, A., *Total Permanent Disability Benefits in Relation to Life Insurance* ("Actuarial Studies No. 5"), Actuarial Society of America, New York.

Jordan, D. F., *Investments*, Prentice-Hall, Inc., New York.

Maclean, J. B., *Life Insurance*, McGraw-Hill Book Co., New York.

Moir, H., *Sources and Characteristics of the Principal Mortality Tables* ("Actuarial Studies No. 1"), Actuarial Society of America, New York.

Murphy, R. D., and Papps, P. C. H., *Construction of Mortality Tables from the Records of Insured Lives* ("Actuarial Studies No. 2"), Actuarial Society of America, New York.

Phillips, A. W., and Fisher, I., *Elements of Geometry*, American Book Co., New York.

Riegel, R., and Loman, H. J., *Insurance Principles and Practices*, Prentice-Hall, Inc., New York.

Rietz, H. L., and others, *Handbook of Mathematical Statistics*, Houghton Mifflin Co., Boston.

Rietz, H. L., Crathorne, A. R., and Rietz, J. C., *Mathematics of Finance*, H. Holt and Co., New York.

Robertson, W. A., and Ross, F. A., *Actuarial Theory*, The Spectator Co., New York.

Shearman, H. P., *Practical Economics*, McGraw-Hill Book Co., New York.

Smith, P. F., and Gale, A. S., *New Analytic Geometry*, Ginn and Co., Boston.

Spurgeon, E. F., *Life Contingencies*, C. & E. Layton, London.

Todhunter, R., *Institute of Actuaries' Text-Book, Part I*, C. & E. Layton, London.

Watson, A. W., *An Account of an Investigation of the Sickness and Mortality Experience of the I.O.O.F. Manchester Unity, 1893–1897*, I.O.O.F. M.U., 97 Grosvenor St., Manchester, England.

Wolfe, S. H., *The Examination of Insurance Companies*, Insurance Press, New York.

Wolfenden, H. H., *Population Statistics and Their Compilation* ("Actuarial Studies No. 3"), Actuarial Society of America, New York.

Yule, G. U., *An Introduction to the Theory of Statistics*, C. Griffin and Co., London.

OTHER PUBLICATIONS

American-Canadian Mortality Investigation, 1900–1915 (in two volumes), Actuarial Society of America, American Institute of Actuaries and National Convention of Insurance Commissioners.

Blood Pressure (*Report of Joint Committee, 1925*), Acturial Society of America, New York.

Corpus Juris. (This is a voluminous reference work that may be consulted in any modern law library.)

Medico-Actuarial Mortality Investigation (in five volumes), Association of Life Insurance Medical Directors and Actuarial Society of America.

Proceedings of the Insurance Institute of Toronto, Apply to C. E. Elvins, Imperial Life Assurance Co., Toronto, Canada.

Proceedings of the Seventh International Congress of Actuaries (in two volumes), Roe Loffzen—Huebner & Van Santen, Amsterdam.

Regulations 65 Relating to the Income Tax Under the Revenue Act of 1924, Government Printing Office, Washington, D.C.

Report of Committee on Disability Experience (*May, 1926*), Actuarial Society of America, New York.

United States Life Tables, 1890, 1901, 1910 and 1901–1910, Government Printing Office, Washington. D.C.

The insurance laws of Illinois, Minnesota, Ohio, and Wisconsin, or of Ontario and the Dominion of Canada, may generally be consulted in company libraries and may be obtained from the respective insurance departments.

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WENDELL P. COLER — ARTHUR W. LARSEN
ARTHUR J. C. HUENERGARD — ROSS E. MOYER
WILBUR M. JOHNSON — ELDER A. PORTER
RALPH E. KENNON — J. CHARLES RIETZ

JOHN F. REILLY

PROGRAM COMMITTEE

REGINALD C. MCCANKIE, *Chairman*

ELGIN G. FASSEL — A. J. MCANDLESS

PRESS COMMITTEE

ELDER A. PORTER, *Chairman*

A. THOMAS LEHMAN — WILLIAM F. POORMAN

COMMITTEE ON REVIEWS

ARTHUR W. LARSEN, *Chairman*

EARL O. DUNLAP — J. CHARLES RIETZ

COMMITTEE ON STATUTORY VALUATION

PERCY H. EVANS, *Chairman*

OSWALD J. ARNOLD — JOHN M. LAIRD
LAWRENCE M. CATHLES — HENRY MOIR
ROBERTSON G. HUNTER — EDWARD B. MORRIS

COMMITTEE ON SYLLABUS

THE CHAIRMAN OF THE EXAMINATION COMMITTEE, *Chairman*
THE VICE-CHAIRMEN OF THE EXAMINATION COMMITTEE
THE SECRETARY
(*All ex-officio*)

PAST PRESIDENTS

	Term of Office	
Lucius McAdam	1909–10	Died 1913
Henry Wright Buttolph	1910–12	
Oswald James Arnold	1912–14	
Joseph Howard Nitchie	1914–16	
Albert Garfield Portch	1916–18	Died 1918
Charles Harrison Beckett	1918–20	
George Graham	1920–22	
Lawrence M. Cathles	1922–24	
Percy H. Evans	1924–26	

SCHEDULE OF MEMBERSHIP—DECEMBER 31, 1926

Particulars	Fellows	Associates	Company Members	Total
Membership—December 31, 1925	121	84	122	327
Increase:				
By examination	11	14	.....	25*
By election	1	7	5	13
	133	105	127	365
Decrease:				
By death	1	.....	.....	1
By other withdrawals	.....	.....	1	1
Transfers from Associate to Fellow by examination	.....	11	.....	11
Membership—December 31, 1926	132	94	126	352

*Includes transfers from Associate to Fellow.

LIST *of* MEMBERS *and* STUDENTS

December 31, 1926

(KEY: † Charter Member at Organization in 1909; * Member by Examination)

FELLOWS

ADMITTED

1915 ALLISON, SINCLAIR EDWARD, A.A.S., Actuary, Pan-American Life Ins. Co., New Orleans, La.

1914 ALLSTROM, HENRY WILLARD, A.A.S., Secretary and Actuary, Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1915 AMES, ERNEST CAPRON, A.B., LL.B., Vice-President and Actuary, Bankers Life Ins. Co. of Nebraska, Lincoln, Neb.

† ANDERSON, LEWIS ALBERT, B.L., Madison, Wis.

1921 ANGELL, CHARLES HART, A.B., A.A.S., Third Vice-President and Actuary, Massachusetts Mutual Life Ins. Co., Springfield, Mass.

† APPLEBY, TROY WILSON, A.M., President, The Ohio National Life Ins. Co., Cincinnati, Ohio.

† ARNOLD, OSWALD JAMES, B.S., President, Northwestern National Life Ins. Co., Minneapolis, Minn.

1924 *BABBITT, ALBERT E., A.M., Actuary, Lamar Life Ins. Co., Jackson, Miss.

1920 BAGLEY, W. NELSON, B.S., F.A.S., Assistant Actuary, The Travelers Ins. Co., Hartford, Conn.

† BARNARD, WILLIAM FRANCIS, 1402 Midland Ave., Syracuse, N.Y.

† BARNETT, SAMUEL, A.B., C.E., Insurance Lawyer and Consulting Actuary, 502 Forsyth Building, Atlanta, Ga.

† BECKETT, CHARLES HARRISON, A.B., MEM. A.M.S., Actuary, State Life Ins. Co., Indianapolis, Ind.

1912 BOWERMAN, JUDAH PHILIP, Actuary, George Washington Life Ins. Co., Charleston, W.Va.

1917 BUCK, GEORGE BURTON, LL.B., F.C.A.S., Consulting Actuary, 25 Spruce St., New York, N.Y.

ADMITTED

1926 *Budinger, Joseph Aloysius, F.A.S., Assistant Actuary, Franklin Life Ins. Co., Springfield, Ill.

1913 Burton, Russell Conklin, A.B., Secretary and Actuary, New World Life Ins. Co., Spokane, Wash.

† Buttolph, Henry Wright, A.B., A.M., A.A.S., Actuary, American Central Life Ins. Co., Indianapolis, Ind.

1914 Cameron, John Campbell, M.A., F.F.A., A.A.S., Vice-President and Actuary, Great Southern Life Ins. Co., Houston, Tex.

1925 Cammack, Edmund Ernest, A.I.A., F.A.S., Vice-President and Actuary, Aetna Life Ins. Co., Hartford, Conn.

† Campbell, Dr. Donald Francis, A.M., Ph.D., Mem. A.M.S., Consulting Actuary, Suite 1832, 160 N. La Salle St., Chicago, Ill.

1920 Cann, Francis Edgar, M.A., F.A.S., Vice-President and Secretary, Jefferson Standard Life Ins. Co., Greensboro, N.C.

† Carter, Edwin Richmond, Actuary, National Life Ins. Co. of the U.S.A., 29 S. La Salle St., Chicago, Ill.

1913 Cathles, Lawrence Maclagan, F.F.A., President, North American Reassurance Co., 250 Park Ave., New York, N.Y.

1921 *Coates, Barrett Nelson, B.S., A.A.S., F.C.A.S., Consulting Actuary, 354 Pine St., San Francisco, Cal.

1920 Coburn, Arthur, F.A.S., A.I.A., F.F.A., Vice-President, North American Reassurance Co., 250 Park Ave., New York, N.Y.

1914 Coit, Ralph Bolles, Actuary, Jefferson Standard Life Ins. Co., Greensboro, N.C.

1926 *Coler, Wendell Phillips, Actuary, The Maccabees, 5065 Woodward Ave., Detroit, Mich.

† Corbett, Henry Richmond, B.Sc., Ph.M., Consulting Actuary, 175 West Jackson Blvd., Chicago, Ill.

1918 Craig, James Douglas, F.A.S., F.C.A.S., Actuary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

1918 Curjel, Harald Worthington, M.A., F.I.A., A.A.S., Actuary, Illinois Life Ins. Co., 1212 Lake Shore Drive, Chicago, Ill.

ADMITTED

1926 *Davis, Malvin Edward, B.A., F.A.S., A.C.A.S., Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

† Draper, Frederick Adams, Actuary, National Life Association, Des Moines, Iowa.

1911 Dunlap, Earl Oglebay, F.C.A.S., Assistant Secretary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

1924 *Ehrenclou, Orvar A., F.A.S., Actuary, Northern Life Ins. Co., Seattle, Wash.

1918 *Elston, James Strode, A.B., F.A.S., F.C.A.S., A.A. Suisse, Cor. Mem. A.A. Belg., Mem. A.M.S., Assistant Actuary, Life Dept., The Travelers Ins. Co., Hartford, Conn.

1914 Epsteen, Saul Robinson, B.S., Ph.D., F.C.A.S., 232 Coronado Bldg., Denver, Col.

1919 Evans, Percy H., Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1914 Fairlie, James, M.A., A.A.S., Vice-President and Actuary, Abraham Lincoln Life Ins. Co., Springfield, Ill.

1922 *Fassel, Elgin George, A.I.A., F.A.S., Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1919 Flanigan, James Edward, F.A.S., F.C.A.S., Agency Manager, Bankers Life Co., 220 Broadway, New York, N.Y.

1915 Fondiller, Richard, LL.B., F.C.A.S., Consulting Actuary, Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y.

1917 Gibb, James Burnett, F.F.A., A.I.A., F.A.S., Actuary, Penn Mutual Life Ins. Co., Philadelphia, Pa.

† Glover, James Waterman, Ph.D., Mem. A.M.S., F.A.A.S., F.A.S.A., F.C.A.S., Professor of Mathematics and Insurance, University of Michigan, 3014 Angell Hall, Ann Arbor, Mich.

1926 *Good, Alexander Caldwell, 1416 Chemical Bldg., St. Louis, Mo.

1926 *Goodrich, Clarence Robertson, Central States Life Ins. Co., St. Louis, Mo.

† Gould, William H., M.A., A.I.A., A.A.S., F.C.A.S., 75 Fulton St., New York, N.Y.

1913 Graham, George, F.F.A., A.A.S., A.I.A., Vice-President, Central States Life Ins. Co., St. Louis, Mo.

ADMITTED

1915 GREENE, WINFIELD WARDWELL, PH.B., F.C.A.S., Actuary and Comptroller, General Reinsurance Corporation, 80 Maiden Lane, New York, N.Y.

1920 *GUNN, MARCUS, A.B., Actuary, Western States Life Ins. Co., San Francisco, Cal.

† HAIGHT, FRANK J., 812 Hume Mansur Bldg., Indianapolis, Ind.

† HANN, ALFRED GILBERT, Actuary, Pacific Mutual Life Ins. Co., Los Angeles, Cal.

1913 HARVEY, JULIAN CORNEILLE, LL.B., General Manager and Actuary, St. Louis Mutual Life Ins. Co., St. Louis, Mo.

1920 HEMSING, FRANK C., A.A.S., Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1914 *HERFURTH, CARL E., Secretary and Actuary, Montana Life Ins. Co., Helena, Mont.

† HIGDON, JOHN E., A.B., 424 Argyle Bldg., Kansas City, Mo.

1924 *HOHAUS, REINHARD ARTHUR, B.A., F.A.S., Assistant Actuary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

† HOMANS, I. SMITH, A.A.S., Vice-President and Actuary, Commonwealth Life Ins. Co., Louisville, Ky.

1918 HOPE, FRANCIS MOFFAT, F.F.A., A.I.A., A.A.S., Actuary, Occidental Life Ins. Co., Los Angeles, Cal.

1920 HUENERGARD, ARTHUR JACOB CONRAD, B.A., F.A.S., Assistant Actuary, Dominion Life Assurance Co., Waterloo, Ont., Canada.

1914 HUGHES, CHARLES, A.A.S., F.C.A.S., Auditor and Actuary, New York State Insurance Dept., 165 Broadway, New York, N.Y.

1916 HUNTER, ARTHUR, F.A.S., F.F.A., F.C.A.S. Cor. Mem. I.A. Eng. and I.A. Fr., Third Vice-President and Chief Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

1919 HUNTER, ROBERTSON G., F.A.S., A.I.A., Second Vice-President and Actuary, Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1921 HUTCHESON, WILLIAM ANDERSON, F.I.A., F.F.A., F.A.S., Second Vice-President and Actuary, Mutual Life Ins. Co., 34 Nassau St., New York, N.Y.

ADMITTED

† HYDE, EDWARD WYLLYS, B.C.E., C.E. (Retired), Milford, Ohio.

1925 *JACKSON, HENRY HOLLISTER, M.A., F.A.S., Associate Actuary, National Life Ins. Co., Montpelier, Vt.

1925 *JAMISON, (MISS) DOROTHY MAY, A.B., George Washington Life Ins. Co., Charleston, W.Va.

1925 *JOHNSON, WILBUR MOORE, A.M., Vice-President and Actuary, Royal Union Life Ins. Co., Des Moines, Iowa.

1925 *KENNON, RALPH EUGENE, B.A., M.S., Mem. A.M.S., Actuary, Iowa Insurance Dept., Des Moines, Iowa.

1926 *KINEKE, FRANK D., F.A.S., Assistant Mathematician, Prudential Ins. Co., Newark, N.J.

1916 KING, WALTER IRVING, A.B., F.A.S., F.C.A.S., Secretary Group Ins. Dept., Connecticut General Life Ins. Co., Hartford, Conn.

1926 *KOCH, (MISS) ELOISE BENDER, A.B., Central States Life Ins. Co., St. Louis, Mo.

1918 LAIRD, JOHN MELVIN, B.A., A.I.A., F.A.S., F.C.A.S., Secretary, Connecticut General Life Ins. Co., Hartford, Conn.

1923 *LARSEN, ARTHUR WILLIAM, M.A., A.A.S., Actuary, Volunteer State Life Ins. Co., Chattanooga, Tenn.

1926 *LEHMAN, A. THOMAS, A.M., Actuary, Standard Life Ins. Co., Pittsburgh, Pa.

1924 LINTON, MORRIS ALBERT, B.S., M.A., F.I.A., F.A.S., Vice-President, Provident Mutual Life Ins. Co., Philadelphia, Pa.

1924 LITTLE, JAMES FULTON, F.I.A., F.A.S., F.C.A.S., Associate Actuary, Prudential Ins. Co. of America, Newark, N.J.

1919 MACFARLANE, WILLIAM, F.F.A., F.A.S., Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

1921 MACLEAN, ALEXANDER T., F.F.A., F.A.S., Associate Actuary, Massachusetts Mutual Life Ins. Co., Springfield, Mass.

1926 MARSHALL, EDWARD WAYNE, F.A.S., Associate Actuary, Provident Mutual Life Insurance Co., Philadelphia, Pa.

1919 *MARSHALL, ERSTON LELAND, A.B., J.D., 919 Hubbell Bldg., Des Moines, Iowa.

1926 *MARTIN, LESLIE REYNOLDS, F.A.S., Assistant Actuary, Connecticut Mutual Life Ins. Co., Hartford, Conn.

ADMITTED

1924 *Marvin, Harry Cooper, A.M., Consulting Actuary, 2105 N. Meridian St., Indianapolis, Ind.

1926 *McAndless, A.J., B.A., Secretary, Lincoln National Life Ins. Co.,Fort Wayne, Ind.

1923 *McCankie, Reginald Campbell, F.A.S., Associate Actuary, Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1916 McIntosh, James, F.F.A., A.A.S., Associate Actuary, The Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1924 McKechnie, James Baldwin, M.A., F.I.A., F.A.S., General Manager, Manufacturers Life Ins. Co., Toronto, Canada.

1914 Mead, Franklin Bush, A.B., F.A.S., Vice-President, Lincoln National Life Ins. Co., Fort Wayne, Ind.

1919 Moir, Henry, F.F.A., F.I.A., F.A.S., President, United States Life Ins. Co., 105 Fifth Ave., New York, N.Y.

1913 Montgomery, Paul Vaughan, B.A., Vice-President and Actuary, Southland Life Ins. Co., Dallas, Tex.

1920 Morris, Edward Bontecou, Ph.B., F.A.S., F.C.A.S. Actuary, Life Dept., The Travelers Ins. Co., Hartford, Conn.

1913 Morris, William Oscar, Actuary, North American Life Ins. Co., North American Bldg., Chicago, Ill.

1926 *Moyer, Ross Emerson, A.B., A.A.S., Assistant Actuary, Columbian National Life Ins. Co., Boston, Mass.

1913 Napier, William Ewart, Vice-President and Secretary, Scranton Life Ins. Co., Scranton, Pa.

† Nitchie, Joseph Howard, A.B., F.A.S., Consulting Actuary, 19 South La Salle St., Chicago, Ill.

† Nueske, Bernhard Robert, Old Colony Life Ins. Co., Chicago, Ill.

† Orr, Robert Kelly, F.C.A.S., Lansing, Mich.

1920 Parker, John Gowans, B.A., A.I.A., F.A.S., Actuary, Imperial Life Assurance Co., Toronto, Canada.

1914 Pattison, George Benjamin, A.A.S., Secretary and Actuary, Peoria Life Ins. Co., Peoria, Ill.

1918 Perrin, Oliver Winfred, M.A., F.A.S., Mem. A.M.S., Assistant Actuary, Penn Mutual Life Ins. Co., Philadelphia, Pa.

1916 *Phelps, Shaffner Leonard, B.S., Secretary, Volunteer State Life Ins. Co., Chattanooga, Tenn.

ADMITTED

1915 PHILLIPS, THOMAS ASHLEY, B.A., A.I.A., A.A.S., Vice-President, Minnesota Mutual Life Ins. Co., St. Paul, Minn.

1918 PIPE, SIDNEY HERBERT, F.A.S., A.I.A., Actuary, Temple Bldg., Toronto, Canada.

1925 *POORMAN, WILLIAM FRANK, M.A., A.A.S., Actuary, Central Life Assurance Society, Des Moines, Iowa.

1924 *PORTER, ELDER ALEXANDER, A.B., F.A.S., M.A.M.S., Actuary, The Indianapolis Life Ins. Co., Indianapolis, Ind.

† REYNOLDS, JOSEPH B., President, Kansas City Life Ins. Co., Kansas City, Mo.

1920 RHODES, EDWARD EVERETT, F.A.S., Vice-President, Mutual Benefit Life Ins. Co., Newark, N.J.

† RIETZ, PROFESSOR HENRY LEWIS, PH.D., F.A.A.S., F.S.S., University of Iowa, Iowa City, Iowa.

1916 *RIETZ, J. CHARLES, A.B., MEM. A.M.S., Actuary, Midland Mutual Life Ins. Co., Columbus, Ohio.

1910 ROCHE, JOHN FRANCIS, F.A.S., Vice-President, Manhattan Life Ins. Co., 66 Broadway, New York, N.Y.

1912 ROSLINGTON, GEORGE, F.C.A., Vice-President, Occidental Life Ins. Co., Albuquerque, N.M.

1916 SANDERS, GEORGE WHITE, A.B., F.A.S., Actuary, Michigan Mutual Life Ins. Co., Detroit, Mich.

† SEITZ, JACOB CHARLES, A.B., Secretary and Actuary, Security Life Ins. Co. of America, 134 N. La Salle St., Chicago, Ill.

1915 SIGTENHORST, ANDREW, Texas Life Ins. Co., Waco, Tex.

1925 SMITH, VICTOR ROY, M.A., A.I.A., A.A.S., Actuary, Confederation Life Association, Toronto (2), Canada.

1915 SPEERS, ALEXANDER ALBERT, A.B., F.A.S., Actuary, The Toledo Travelers Life Ins. Co., Toledo, Ohio.

1915 STEIN, CALVERT FREDERICK, Secretary-Actuary, Life Dept., Provident Life and Accident Ins. Co., Chattanooga, Tenn.

† STOOKEY, BERT JAY, Secretary, Illinois Life Ins. Co., 1212 Lake Shore Drive, Chicago, Ill.

1921 STRONG, WENDELL MELVILLE, B.A., M.A., PH.D., LL.B., F.A.S., F.C.A.S., Associate Actuary, Mutual Life Ins. Co., 34 Nassau St., New York, N.Y.

1913 THOMSON, GORDON WILLIAM, F.F.A., A.I.A., A.A.S., Vice-President, West Coast Life Ins. Co., 605 Market St., San Francisco, Cal.

ADMITTED

1924 TORREY, MORRIS WHITTEMORE, F.A.S., Assistant Secretary, Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y.

† VAIL, CARLETON M., A.B., Room 1005, 110 South Dearborn St., Chicago, Ill.

1916 WASHBURN, JAMES HERMAN, A.A.S., A.C.A.S., Consulting Actuary, 165 Broadway, New York, N.Y.

1916 WATT, WILLIAM ARTHUR, M.A., A.I.A., F.A.S., Secretary and Actuary, Pilot Life Ins. Co., Greensboro, N.C.

† WEBB, ROBERT MONTAGUE, Kansas City Life Ins. Co., Kansas City, Mo.

1921 WELCH, ARCHIBALD ASHLEY, A.B., A.M., F.A.S., President, Phoenix Mutual Life Ins. Co., Hartford, Conn.

1916 WILLIAMS, FREDERICK ALFRED, F.A.S., A.I.A., F.S.S., Manager and Actuary, "La Nacional" Compañia de Seguros sobre la Vida, S.A., Apartado 1420, Mexico City, Mexico.

1923 *WILLIAMS, (MISS) HELEN JACKSON, A.B., M.S., Bureau of Statistics and Research, 510 Electric Bldg., Cleveland, O.

† WITHINGTON, FREDERIC SCHERER, Actuary, Midland Life Ins. Co., Kansas City, Mo.

1925 WOOD, ARTHUR BARTON, B.A., F.I.A., F.A.S., F.C.A.S., Vice-President and Actuary, Sun Life Assurance Co. of Canada, Montreal, Canada.

1924 WOOD, WILLIAM ARCHIBALD PORTER, B.A., A.I.A., F.A.S., Actuary, Canada Life Assurance Co., Toronto, Canada.

1915 WOODWARD, JOSEPH HOOKER, PH.B., F.A.S., F.C.A.S., Consulting Actuary, Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y.

1925 YOUNG, WILLIAM, F.F.A., F.A.S., F.C.A.S., Actuary, New York Life Ins. Co., 346 Broadway, New York, N.Y.

ASSOCIATES

(Numerals indicate what parts of the Fellowship examinations have been passed.)

ADMITTED

1926 *ANDERSON, T. LOYAL, B.S., Actuary, Illinois Insurance Dept., Springfield, Ill.

1926 *ATRUBIN, HARRY MORTON, B.A., Dominion Life Assurance Co., Waterloo, Ont., Canada.

ADMITTED

1926 *Batho, Elgin Robertson, B.A., A.A.S., A.C.A.S., Bankers Life Co., Des Moines, Iowa.

1915 *Beach, (Miss) Henricka Bryant, M.A., Assistant Secretary and Actuary, Provident Life Ins. Co., Bismarck, N.D.

1913 Beecher, Benjamin Sanford, M.A., Vice-President, National Guardian Life Ins. Co., Madison, Wis.

1925 *Bell, David Wright, B.A., Imperial Life Assurance Co., Toronto, Canada.

1926 *Beveridge, James Robertson, B.A., A.A.S., Manufacturers Life Ins. Co., Toronto, Canada.

1923 Birkenshaw, John Henry, B.A., F.A.S., Assistant Actuary, Confederation Life Association, Toronto, Canada.

† Bishop, George Walter, The Bankers Reserve Life Co., Omaha, Neb.

1923 *Bjorn, Walter, B.S., A.A.S., Connecticut General Life Ins. Co., Hartford, Conn. (*9–11*)

1915 *Bowerman, Walter Gregory, A.M., F.A.S., New York Life Ins. Co., 346 Broadway, New York, N.Y.

1922 *Boyd, Lorne Thomas, B.A., Secretary and Actuary, Commonwealth Life Ins. Co., Hamilton, Ont., Canada. (*9, 10*)

† Brimstin, William Edgar, Secretary, Federal Life Ins. Co., Chicago, Ill.

1919 Brooks, Charles Edward, Ph.D., 613 Beavers Insurance Bldg., Madison, Wis.

1921 Brown, Everett Garrabrant, F.A.S., Actuary, Southwestern Life Ins. Co., Dallas, Tex.

1917 Brown, (Mrs.) Helen Elizabeth, M.A., 702 S. Sixth St., Ironton, Ohio.

1919 Brunnquell, Helmuth G., A.C.A.S., Actuary, Wisconsin Insurance Dept., Madison, Wis.

1911 *Buckman, Harold Hibbs, Actuary, California State Life Ins. Co., Sacramento, Cal.

1926 *Buckner, David Ernest, A.B., A.A.S., Assistant Actuary, Jefferson Standard Life Ins. Co., Greensboro, N.C.

1914 Burkart, Charles J., Kansas City Life Ins. Co., Kansas City, Mo.

1919 *Clark, (Miss) Helen Lucy, A.B., Assistant Actuary, American Central Life Ins. Co., Indianapolis, Ind. (*9, 10*)

ADMITTED

1926 *Conrod, Stuart Fulton, B.A., Great-West Life Assurance Co., Winnipeg, Man., Canada.

† Copeland, John Alexander, A.B., F.C.A.S., Consulting Actuary, 1027 Candler Bldg., Atlanta, Ga.

1924 *Cross, Cecil Frank, A.B., Actuary, American Life Ins. Co., Detroit, Mich. (*9, 10*)

1925 *Dandy, John Percy, B.A., A.A.S., Confederation Life Association, Toronto, Canada. (*9, 10*)

1924 *Dubuar, Charles Carroll, A.B., Assistant Actuary, New York Insurance Dept., Albany, N.Y. (*9*)

1925 *Eastlack, Allen Carlson, A.M., Actuary, Gem City Life Ins. Co., Dayton, Ohio. (*10*)

1926 *Evans, John Henry, Actuary, Ohio National Life Ins. Co., Cincinnati, Ohio.

1912 *Fetsch, Harry Christian, A.B., A.A.S., Actuary and Assistant Secretary-Treasurer, Ohio State Life Ins. Co., Columbus, Ohio.

1924 *Folz, Clifford H., A.B., Actuary, Public Savings Ins. Co., Indianapolis, Ind.

1926 Geddes, George Wilson, F.A.S., Actuary, Ontario Equitable Life and Accident Ins. Co., Waterloo, Ont., Canada.

1926 Gerhard, Frederick Bruce, B.S., F.A.S., Actuary, Department of Banking and Insurance, Trenton, N.J.

† Grimes, Irvin Lorenzo, A.B., Actuary and Assistant Secretary, Mutual Trust Life Ins. Co., 77 W. Washington St., Chicago, Ill.

1916 Griswold, Robert N., B.A., F.A.S., Actuary, Inter-Mountain Life Ins. Co., Salt Lake City, Utah.

1923 Hale, John Stewart, B.A., Actuary, Northwestern National Life Ins. Co., Minneapolis, Minn.

1919 *Hanna, John Vestal, Actuary and Assistant Secretary, United Life and Accident Ins. Co., Concord, N.H.

1915 *Hiller, William John, A.B., Associate Actuary, Bankers Life of Nebraska, Lincoln, Neb.

1914 *Hipp, Grady Hayne, A.B., Actuary, New York Insurance Dept., Albany, N.Y.

1926 Hollenberg, Maximilian Richard, F.A.S., Guardian Life Ins. Co., 50 Union Sq., New York, N.Y.

ADMITTED

1925 HOWELL, VALENTINE, B.S. in Econ., F.A.S., Associate Actuary, Guardian Life Ins. Co., 50 Union Sq., New York, N.Y.

1926 *HUEHL, WALTER HAROLD, B.A., Insurance Department, State of Indiana, Indianapolis, Ind.

† HUSKINSON, GEORGE WILLIAM H., Assistant Superintendent, Illinois Insurance Dept., Springfield, Ill.

1926 *IRWIN, PHILIP CHASE, B.S., Assistant Actuary, Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.

1924 *JENKINS, WILMER ATKINSON, A.A.S., A.M., L. A. Glover & Co., 29 S. La Salle St., Chicago, Ill.

1926 JOHNSON, MURAT LOUIS, A.B., F.A.S., Assitant Actuary, Penn Mutual Life Ins. Co., Philadelphia, Pa.

1918 JORDAN, F. RUSSELL, B.A., A.A.S., Actuary, Franklin Life Ins. Co., Springfield, Ill.

1925 *KALMBACH, LELAND JOHN, A.B., Lincoln National Life Ins. Co., Fort Wayne, Ind. (*10*)

1924 KATTELL, SHERMAN C., B.S., F.A.S., Actuary, Lincoln National Life Ins. Co., Fort Wayne, Ind.

1925 *KENIGSON, GEORGE, A.B., A.A.S., Sun Life Ins. Co. of America, Baltimore, Md.

1924 LITHGOW, JAMES HECTOR, F.A.S., A.I.A., Actuary, Manufacturers Life Ins. Co., Toronto, Canada.

1925 *MACKINNON, WILL DONALD, B.S., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa. (*9*, *10*)

1916 MAYNARD, JOSEPH DURYEA, Assistant Actuary, Northwestern Mutual Life Ins. Co., Milwaukee, Wis.

1923 MCBRIDE, WILLIAM HUGH, Actuary, National Life and Accident Ins. Co., Nashville, Tenn.

1925 *MCCLURG, D. RALPH, PH.B., Secretary and Treasurer, National Equity Life Ins. Co., Little Rock, Ark. (*10*, *12*)

1922 MCCONAGHY, CHARLES ALEXANDER, B.A., A.I.A., F.A.S., Actuary, Continental Life Ins. Co., St. Louis, Mo.

1926 *MENGE, WALTER O., A.M., A.A.S.A., Actuary, Grange Life Ins. Co., Lansing, Mich.

1921 *MYERS, GEORGE ELMER, A.A.S., Sun Life Assurance Co., Montreal, Canada.

1923 *OATES, MERLIN, A.B., Actuary, American Life Reinsurance Co., Dallas, Tex. (*9–11*)

ADMITTED

1922 *Pardieck, (Miss) Gertrude Lenore, A.B., Assistant Actuary, State Life Ins. Co., Indianapolis, Ind.

1925 Pedoe, Arthur, B.Sc. (London), F.I.A., A.A.S., Actuary, Continental Life Ins. Co., Toronto, Canada.

1925 Phillips, James Thompson, B.A., F.A.S., New York Life Ins. Co., 346 Broadway, New York, N. Y.

1918 *Powell, John Mark, A.B., F.A.S., A.C.A.S., Actuary, Columbian National Life Ins. Co., Boston, Mass. (*9, 10*)

1924 *Prentice, George Taylor, B.A., A.A.S., Imperial Life Assurance Co., Toronto, Canada. (*9, 10*)

1920 Reichgott, Henry, Ph.B., A.A.S., Second Vice-President, Missouri State Life Ins. Co., St. Louis, Mo.

1920 *Reilly, Dr. John Franklin, A.M., Ph.D., 212 Physics Bldg., Iowa City, Iowa.

1926 *Richter, Otto Clarence, B.A., A.C.A.S., American Telephone and Telegraph Co., 195 Broadway, New York, N.Y.

1922 *Robbins, Dr. Rainard Benton, A.M., Ph.D., A.A.S., A.C.A.S., Assistant Actuary, New York Insurance Dept., 165 Broadway, New York, N.Y.

1916 *Rothschild, Stanford Z., A.B., A.A.S., Third Vice-President and Actuary, Sun Life Ins. Co. of America, Baltimore, Md.

1914 *Scott, Martin R., Actuary, Reserve Loan Life Ins. Co., Indianapolis, Ind.

1916 Sellman, Henry G., A.B., Consulting Actuary, 700 Times Bldg., St. Louis, Mo. (*10–12*)

1926 *Shannon, Simon, M.A., A.A.S., Great-West Life Assurance Co., Winnipeg, Man., Canada.

1924 *Shepherd, Bruce Edwin, Ph.B., A.A.S., Chief Assistant Actuary, State Department of Banking and Insurance, Trenton, N.J. (*9, 10*)

1916 *Shepherd, Clinton O., F.A.S., Actuary, Missouri State Life Ins. Co., St. Louis, Mo.

1925 *Shudde, Louis Otto, B.A., M.A., Actuary and Assistant Secretary, San Jacinto Life Ins. Co., Beaumont, Tex. (*9, 10*)

ADMITTED

1921 SINCLAIR, COLL CLAUDE, B.A., F.A.S., Assistant Actuary, Great-West Life Assurance Co., Winnipeg, Man., Canada.

† STAYMAN, GUY L., P.O. Box 86, Indianapolis, Ind.

1926 STEVENS, LAWRENCE L., F.A.S., Penn Mutual Life Ins. Co., Philadelphia, Pa.

1925 STOWE, HUDSON J., B.A., F.A.S., Manufacturers Life Ins. Co., Toronto, Canada. (*9–11*)

1913 STRUDELL, FRED DAVID, Consulting Actuary, 722 Chestnut St., St. Louis, Mo.

1915 *STURTEVANT, ROBERT BROWN, A.B., Assistant Secretary, American Central Life Ins. Co., Indianapolis, Ind.

1919 SUZUKI, TOSHIKAZU, M.S., Actuary, First Mutual Life Ins. Co., Kyobashi, Tokio, Japan.

1925 THOMSON, JOHN WALTER, F.F.A., A.I.A., F.S.S., Actuary, North American Reassurance Co., 250 Park Ave., New York, N.Y.

1915 TICKNER, VERNE L., Actuary and Agency Manager, United American Life Ins. Co., Jacksonville, Fla.

† VAIL, ROGER SHERMAN, A.B., 110 South Dearborn St., Chicago, Ill.

1925 *WALKER, (MISS) MARGARET, M.A., Lincoln National Life Ins. Co., Fort Wayne, Ind.

1925 *WALTON, HAROLD GRAHAM, B.S., American Central Life Ins. Co., Indianapolis, Ind.

1923 WARREN, PROFESSOR LLOYD ARTHUR HEBER, M.A., PH.D., F.R.A.S., F.C.A.S., A.A.S., University of Manitoba, Winnipeg, Man., Canada. (*9*)

1926 *WELLMAN, ALEX C., A.B., A.C.A.S., Actuary, Alabama National Life Ins. Co., Birmingham, Ala.

1917 WERKENTHIN, BERNARD, Texas Insurance Dept. Austin, Tex.

† WIGGINS, HORACE SCOVILLE, 514–15 Bankers Life Bldg., Lincoln, Neb.

1926 WRIGHT, BYRON, F.A.S., North American Life Assurance Co., Toronto, Canada.

1915 YORT, J. P., P.O. Box 1369, Washington, D.C.

1926 *YOUNG, FLOYD EUGENE, A.B., A.C.A.S., Associate Actuary, Western Union Life, Spokane, Wash.

1925 *YOUNG, JAY ROLLIN, A.A.S., Franklin Life Ins. Co., Springfield, Ill. (*10*)

CONTRIBUTING MEMBERS

ADMITTED

1921 Abraham Lincoln Life Ins. Co., Springfield Ill.
1921 Aetna Life Ins. Co., Hartford, Conn.
1916 American Central Life Ins. Co., Indianapolis, Ind.
1920 American Life Ins. Co., Denver, Colo.
1923 American Life Ins. Co., Detroit, Mich.
1923 American Life Reinsurance Co., Dallas, Tex.
1917 American National Ins. Co., Galveston, Tex.
1921 Amicable Life Ins. Co., Waco, Tex.
1919 Atlantic Life Ins. Co., Richmond, Va.
1921 Bank Savings Life Ins. Co., Topeka, Kan.
1920 Bankers Life Co., Des Moines, Iowa.
† Bankers Reserve Life Co., Omaha, Neb.
1921 Berkshire Life Ins. Co., Pittsfield, Mass.
1922 Canada Life Assurance Co., Toronto, Canada.
1912 Capitol Life Ins. Co., Denver, Colo.
1919 Cedar Rapids Life Ins. Co., Cedar Rapids, Iowa.
1916 Central Life Assurance Society of the U.S., Des Moines, Iowa.
† Central Life Ins. Co. of Illinois, Chicago, Ill.
1915 Central States Life Ins. Co., St. Louis, Mo.
† Cleveland Life Ins. Co., Cleveland, Ohio.
1920 Columbus Mutual Life Ins. Co., Columbus, Ohio.
1922 Confederation Life Association, Toronto, Canada.
1921 Connecticut General Life Ins. Co., Hartford, Conn.
1926 Connecticut Mutual Life Ins. Co., Hartford, Conn.
1920 Continental American Life Ins. Co., Wilmington, Del.
1923 Continental Assurance Co., Chicago, Ill.
1922 Continental Life Ins. Co., St. Louis, Mo.
1919 Dakota Life Ins. Co., Watertown, S.D.
1922 Detroit Life Ins. Co., Detroit, Mich.
1925 Dominion Life Assurance Co., Waterloo, Ont., Canada.
1918 Elkhorn Life & Accident Ins. Co., Norfolk, Neb.
1920 Equitable Life Ins. Co., Washington, D.C.
1920 Equitable Life Ins. Co. of Iowa, Des Moines, Iowa.
1922 Excelsior Life Ins. Co., Toronto, Canada.
1920 Farmers & Bankers Life Ins. Co., Wichita, Kan.
1923 Farmers National Life Ins. Co., Chicago, Ill.
† Federal Life Ins. Co., Chicago, Ill.

ADMITTED

1921 Fidelity Mutual Life Ins. Co., Philadelphia, Pa.
1916 Franklin Life Ins. Co., Springfield, Ill.
1914 George Washington Life Ins. Co., Charleston, W.Va.
1923 Great Northern Life Ins. Co., Chicago, Ill.
1916 Great Southern Life Ins. Co., Houston, Tex.
1921 Great-West Life Assurance Co., Winnipeg, Man., Canada.
1920 Guaranty Life Ins. Co., Davenport, Iowa.
1921 Guardian Life Ins. Co. of America, New York, N.Y.
† Illinois Life Ins. Co., Chicago, Ill.
1912 Indianapolis Life Ins. Co., Indianapolis, Ind.
† International Life Ins. Co., St. Louis, Mo.
1923 International Life & Trust Co., Moline, Ill.
1922 Imperial Life Assurance Co., Toronto, Canada.
1915 Jefferson Standard Life Ins. Co., Greensboro, N.C.
1921 John Hancock Mutual Life Ins. Co., Boston, Mass.
† Kansas City Life Ins. Co., Kansas City, Mo.
1913 LaFayette Life Ins. Co., LaFayette, Ind.
1925 Liberty Life Ins. Co., Topeka, Kan.
† Lincoln National Life Ins. Co., Fort Wayne, Ind.
1922 London Life Ins. Co., London, Ont., Canada.
1926 Louisiana State Life Ins. Co., Shreveport, La.
1923 Manhattan Life Ins. Co., New York, N.Y.
1922 Manufacturers Life Ins. Co., Toronto, Canada.
1921 Maryland Life Ins. Co., Baltimore, Md.
1921 Massachusetts Mutual Life Ins. Co., Springfield, Mass.
1920 Merchants Life Ins. Co., Des Moines, Iowa.
1921 Metropolitan Life Ins. Co., New York, N.Y.
1915 Michigan Mutual Life Ins. Co., Detroit, Mich.
1920 Midland Life Ins. Co., Kansas City, Mo.
1915 Midland Mutual Life Ins. Co., Columbus, Ohio.
1919 Minnesota Mutual Life Ins. Co., St. Paul, Minn.
1912 Missouri State Life Ins. Co., St. Louis, Mo.
1925 Monarch Life Assurance Co., Winnipeg, Man., Canada.
1920 Montana Life Ins. Co., Helena, Mont.
1919 Mutual Benefit Life Ins. Co., Newark, N.J.
1926 Mutual Life Assurance Co. of Canada, Waterloo, Ont., Canada.
1926 Mutual Life Ins. Co. of New York, New York, N.Y.

ADMITTED

1919 Mutual Trust Life Ins. Co., Chicago, Ill.
1919 National Life & Accident Ins. Co., Nashville, Tenn.
1921 National Life Ins. Co., Montpelier, Vt.
† National Life Ins. Co. of the U.S.A., Chicago, Ill.
1923 National Guardian Life Ins. Co., Madison, Wis.
1921 New England Mutual Life Ins. Co., Boston, Mass.
1925 North American Life Assurance Co., Toronto, Canada.
1915 North American Life Ins. Co., Chicago, Ill.
1920 Northern Life Ins. Co., Seattle, Wash.
1924 Northern States Life Ins. Co., Hammond, Ind.
1915 Northwestern National Life Ins. Co., Minneapolis, Minn.
1921 Occidental Life Ins. Co., Los Angeles, Cal.
1915 Ohio National Life Ins. Co., Cincinnati, Ohio.
1912 Ohio State Life Ins. Co., Columbus, Ohio.
† Old Colony Life Ins. Co., Chicago, Ill.
1915 Old Line Life Ins. Co. of America, Milwaukee, Wis.
1917 Pacific Mutual Life Ins. Co., Los Angeles, Cal.
1916 Pan-American Life Ins. Co., New Orleans, La.
1923 Penn Mutual Life Ins. Co., Philadelphia, Pa.
1920 Peoples Life Ins. Co., Frankfort, Ind.
1915 Peoples Life Ins. Co. of Illinois, Chicago, Ill.
1915 Peoria Life Ins. Co., Peoria, Ill.
1921 Philadelphia Life Ins. Co., Philadelphia, Pa.
1921 Phoenix Mutual Life Ins. Co., Hartford, Conn.
1921 Provident Mutual Life Ins. Co., Philadelphia, Pa.
1921 Prudential Ins. Co. of America, Newark, N.J.
1913 Public Savings Ins. Co. of America, Indianapolis, Ind.
1916 Register Life Ins. Co., Davenport, Iowa.
1920 Reinsurance Life Co. of America, Des Moines, Iowa
1915 Reliance Life Ins. Co., Pittsburgh, Pa.
† Reserve Loan Life Ins. Co., Indianapolis, Ind.
† Royal Union Life Ins. Co., Des Moines, Iowa.
1923 San Jacinto Life Ins. Co., Beaumont, Tex.
1910 Scranton Life Ins. Co., Scranton, Pa.
† Security Life Ins. Co. of America, Chicago, Ill.
1922 Security Mutual Life Ins. Co., Binghamton, N.Y.
1923 Shenandoah Life Ins. Co., Roanoke, Va.
1922 Southern States Life Ins. Co., Atlanta, Ga.

ADMITTED

1924 Southern Union Life Ins. Co., Fort Worth, Tex.
1912 Southland Life Ins. Co., Dallas, Tex.
1916 Southwestern Life Ins. Co., Dallas, Tex.
1926 Springfield Life Ins. Co., Springfield, Ill.
† State Life Ins. Co., Indianapolis, Ind.
1925 Sun Life Assurance Co. of Canada, Montreal, Canada.
1921 Travelers Ins. Co., Hartford, Conn.
1921 Two Republics Life Ins. Co., El Paso, Tex.
1921 Union Central Life Ins. Co., Cincinnati, Ohio.
1920 West Coast Life Ins. Co., San Francisco, Cal.
1912 Western Reserve Life Ins. Co., Muncie, Ind.
1911 Western States Life Ins. Co., San Francisco, Cal.
1917 Western Union Life Ins. Co., Spokane, Wash.
1914 Wisconsin Life Ins. Co., Madison, Wis.

DECEASED FELLOWS

ADMITTED		DIED
1924	Clarence Edmund Cale	1924
†	Carroll Booth Carr	1919
†	Horton W. Cochnower	1913
1914	Isaac Davenport	1916
†	John Maynard Emery	1919
†	Virgil Morrison Kime	1918
1918	Louis Linzmeyer	1918
†	Lucius McAdam	1913
†	Malcolm McNeill	1923
1918	Percy Charles Herbert Papps	1926
1914	Albert Garfield Portch	1918
†	Henry Sherman Vail	1919
†	Geert Andries Van der Sluis	1917
1913	Harris Elias Vineberg	1920
†	Benjamin A. Welstead	1910
†	Paul Livingston Woolston	1924
†	Wilbur S. Wynn	1913

DECEASED ASSOCIATES

ADMITTED		DIED
†	Nels Nelson	1912
1919	Arthur Enoch Clement	1924

STUDENTS

THOSE WHO HAVE PASSED ANY PART OF THE EXAMINATIONS DURING THE LAST THREE YEARS AND HAVE NOT BEEN ADMITTED AS ASSOCIATES

(Italic numerals indicate what parts of the examinations have been passed; bold numerals indicate parts waived.)

ABERNETHY, JOHN R., Jefferson Standard Life Ins. Co., Greensboro, N.C. (*1–3*)
ABRAHAMSON, T. ARNOLD, 699 Valour Road, Winnipeg, Man., Canada. (*2*)
AMBLER, ARTHUR B., St. Louis Mutual Life Ins. Co., St. Louis, Mo. (*1–3*)
AMERMAN, GUY H., 615 Church St., Ann Arbor, Mich. (*1–4*)
AMOSS, G. BRUCE, Great-West Life Assurance Co., Winnipeg, Man., Canada. (*4*)
ANDREWS, (MISS) LUCY R., Missouri State Life Ins. Co., St. Louis, Mo. (*1, 2*)
ANGERER, JOHN W., Michigan Insurance Dept., Lansing, Mich. (1, *2–4, 6*)
ARMANTROUT, EVERETT D., Provident Mutual Life Ins. Co., Philadelphia, Pa. (*1–5*)
ATWATER, ROBERT BURDICK, Bankers Life Co., Des Moines, Iowa. (*1, 2, 4*)
AUDEN, MARCUS F., Confederation Life Association, Toronto, Canada. (*1–4*)
BARNES, ORLIN M., Franklin Life Ins. Co., Springfield, Ill. (*1, 2*)
BARTHEL, (MISS) CATHERINE E., Missouri State Life Ins. Co., St. Louis, Mo. (*1*)
BATHO, M. BRUCE W., 105 Gerard St., Winnipeg, Man., Canada. (*1, 2*)
BELL, WRAY M., Burwash Hall, Victoria College, Toronto, Canada. (*1–4*)
BENNER, (MISS) AMY L., Peoria Life Ins. Co., Peoria, Ill. (*1–5*)
BENSON, CHASE H., Box 1459, Greensboro, N.C. (*1, 2*)
BITTEL, W. HAROLD, Peoria Life Ins. Co., Peoria, Ill. (*1*)
BOISSARD, RICHARD, National Guardian Life Ins. Co., Madison, Wis. (*1–4*)
BOURSCHEIDT, PAUL F., Peoria Life Ins. Co., Peoria, Ill. (*1, 2*)
BOWER, P. S., Great-West Life Assurance Co., Winnipeg, Man., Canada. (*1, 2*)
BOWES, GEORGE H., Great-West Life Assurance Co., Winnipeg, Man., Canada. (*1*)
BRAHANA, HENRY R., 707 S. McCullough Ave., Urbana, Ill. (*1–7*)
BREITHAUPT, WALTER H., 172 Margaret Ave., Kitchener, Ont., Canada. (*1*)
BRONSON, DORRANCE C., 615 Monroe St., Ann Arbor, Mich. (*1–4*)
BROWN, EDWARD D., JR., Jefferson Standard Life Ins. Co., Greensboro, N.C. (*1–4*)
BUCKNELL, EARL F., The Maccabees, Detroit, Mich. (*1–3*)
BURCHELL, JAMES R., Imperial Life Assurance Co., Toronto, Canada. (*2*)
BURGAR, WILLIAM J., 54 Murray St., Toronto, Canada. (*1–4*)
CARSON, (MISS) CHRISTIANA E., 235 Eugene St., Norwood, Man., Canada. (*1, 2, 4*)
CHEN, SSU-TU, 303 W. 100 St., New York, N.Y. (*2–4*)
CLAIR, JOSEPH J., Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y. (*1–4*)
CLARK, GILBERT C., Equitable Life Ins. Co., Washington, D.C. (*1, 2*)
CLEMO, AMBROSE J., Montana Life Ins. Co., Helena, Mont. (*1–4*)
COATES, CLARENCE S., 2019 Los Angeles Ave., Berkeley, Cal. (*1–7*)
COLEMAN, JOHN H., Lafayette Life Ins. Co., Lumberton, N.C. (*1, 2*)
COX, LEWIS C., National Life & Accident Ins. Co., Nashville, Tenn. (*1–4, 6*)
CRAIG, CECIL C., 701 S. Division St., Ann Arbor, Mich. (*1–4*)
CRAIG, DOUGLAS S., Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y. (*2, 4*)
CRIPPEN, LLOYD K., Tuberculosis Hospital, 14th and Upshur Sts., N.W., Washington, D.C. (*1–6, 8*)
DANZINGER, LEO J., 24 Hart Ave., Winnipeg, Man., Canada. (*1–3, 5*)
DAVIDSON, WAID J., Iowa Insurance Dept., Des Moines, Iowa. (*1–4*)
DECKER, MYRON F., 143 East 39th St., New York, N.Y. (*1–4*)
DELAHUNTY, GILBERT T., Peoria Life Ins. Co., Peoria, Ill. (*1*)

Dominguez, Mario, 7a Dr. Velasco No. 195, Mexico City, Mexico. (*1, 2*)
Doran, Alan B., Home Life Ins. Co., 256 Broadway, New York, N.Y. (*1–3*)
Drebin, Harry I., 228 S. Warren Ave., Big Rapids, Mich. (*3*)
Easley, Knight H., Amicable Life Ins. Co., Waco, Tex. (*1, 2*)
Ede, Cecil F., West Coast Life Ins. Co., San Francisco, Cal. (*1–5*)
Elander, Albin E., 2638 Garfield St., Lincoln, Neb. (*2*)
Ellsworth, Leon W., Southwestern Life Ins. Co., Dallas, Tex. (*1, 3*)
Erway, Richard E., 439 S. Division St., Ann Arbor, Mich. (*1*)
Estes, E. Forrest, Bankers Reserve Life Co., Omaha, Neb. (*1–6*)
Faith, Edward L., Missouri State Life Ins. Co., St. Louis, Mo. (*1, 2*)
Feldman, Israel, 769 Magnus Ave., Winnipeg, Man., Canada. (*1, 2*)
Ferguson, Arthur A., Western Union Life Ins. Co., Spokane, Wash. (*1–4*)
Fisher, (Miss) Gladys A., Kansas City Life Ins. Co., Kansas City, Mo. (*1*)
Flesner, (Miss) Irene E., Board of Pensions and Relief, 740 Rush St., Chicago, Ill. (*1–4*)
Fletcher, J. Gordon, Mutual Life Assurance Co., Waterloo, Ont., Canada. (*1–3*)
Fletcher, Robert V., 644 Gertrude Ave., Winnipeg, Man., Canada. (*2*)
Frederick, Russell A., Franklin Life Ins. Co., Springfield, Ill. (*1, 2*)
Glover, Lynn A., 29 S. LaSalle St., Chicago, Ill. (*1–5, 8*)
Graeser, A. Otis, Ohio National Life Ins. Co., Cincinnati, Ohio. (*1, 2, 4*)
Grassby, Gerald M., 160 Yale Ave., Winnipeg, Man., Canada. (*1, 2*)
Green, Walter C., 354 Pine St., San Francisco, Cal. (*1–5*)
Hacking, Frank, 320 Brooklyn St., Winnipeg, Man., Canada. (*2*)
Hall, William D., National Bureau of Casualty and Surety Underwriters, 120 West 42nd St., New York, N.Y. (*1*)
Harper, Rudolph J., Guardian Life Ins. Co., 50 Union Sq., New York, N.Y. (*1*)
Hartwell, Maurice, care of The Maccabees, Detroit, Mich. (*3*)
Higdon, John C., Business Men's Assurance Co., Kansas City, Mo. (*1–3*)
Higginson, George W., Philadelphia Life Ins. Co., Philadelphia, Pa. (*1, 2*)
Hilliard, W. L., Dominion Life Assurance Co., Waterloo, Ont., Canada. (*1–4*)
Hogan, Frank J., Lincoln National Life Ins. Co., Fort Wayne, Ind. (*2–4*)
Hollies, Wm. St. G., Occidental Life Ins. Co., Los Angeles, Calif. (*1–3*)
Holmes, Benjamin T., Confederation Life Association, Toronto, Canada. (*1–4*)
Holtzman, Edward, Pan-American Life Ins. Co., New Orleans, La. (*1–5*)
Hondorp, Peter, 425 S. Division Ave., Ann Arbor, Mich. (*1–4*)
Howard, Wilkie A. M., Confederation Life Association, Toronto, Canada. (*1, 3*)
Huston, Francis E., Northern Life Ins. Co., Seattle, Wash. (*1, 2*)
Huth, George, North American Life Ins. Co., Chicago, Ill. (*1*)
Jekel, Alfred H., 517 S. Kirkwood Road, Kirkwood, Mo. (*1–4*)
Jennings, William C., 226 S. Observatory, Ann Arbor, Mich. (*2–4*)
Johanssen, Tryggve, Jennestad, Norway. (*1*)
Johnson, George F., Whitewood Banking Co., Whitewood, S.D. (*1*)
Johnson, Herbert E., 5118 Huron St., Chicago, Ill. (*1, 2, 3*)
Jones, (Miss) Ollie H., Alamo Life Ins. Co., San Antonio, Texas. (*1, 2*)
Jones, W. Ralph, 502 County Bldg., Chicago, Ill. (*1–4*)
Keene, Earle L., Columbian National Life Ins. Co., Boston, Mass. (*2, 4*)
Keith, (Miss) Marguerite W., 707 Westgate Ave., St. Louis, Mo. (*1, 2*)
Kirkpatrick, Clair C., Iowa Insurance Dept., Des Moines, Iowa. (*1–4*)
Kuhlemeier, Charles A., 1206 Fischer Ave., Detroit, Mich. (*1–4*)
Larsen, Murray N., 140 Ethelbert St., Winnipeg, Man., Canada. (*1–3*)
Lee, Frank H., The Maccabees, Detroit, Mich. (*1–5*)
Lehane, Leo J., Univ. of Manitoba, Winnipeg, Man., Canada. (*1–3*)
Lemon, Herschel B., 111 E. Washington St., Ann Arbor, Mich. (*1, 2, 4*)
Li, Shou-Kun, 1127 Church St., Ann Arbor, Mich. (*2, 3*)
Linder, Joseph, care of Woodward, Fondiller and Ryan, 75 Fulton St., New York, N.Y. (*1–4, 5–7*)

Little, John E., The Maccabees, 5065 Woodward Ave., Detroit, Mich. (*1–7*)
Long, Leon L., Bankers Life Co., Des Moines, Iowa. (*1–4*)
Longenecker, Joshua Van S., Farmers and Bankers Life Ins. Co., Wichita, Kans. (*1–4, 6, 8*)
Louis, Pak Him, 811 Catherine St., Ann Arbor, Mich. (*3*)
Lyle, (Miss) Annie M., Cotton States Life Ins. Co., Nashville, Tenn. (*1–6*)
Maclean, Neil B., Dept. of Mathematics & Astronomy, Univ. of Manitoba, Winnipeg, Man., Canada. (*1–4*)
Mackenzie, (Miss) Ruth S., 408 Wellington Crescent, Winnipeg, Man., Canada. (*1, 2*)
Mansz, Arno A., Dominion Life Assurance Co., Waterloo, Ont., Canada. (*1*)
Martin, Ronald J., 120 Rose St., Winnipeg, Man., Canada. (*1, 4*)
Merkle, John W., care of L. A. Glover & Co., 29 S. LaSalle St., Chicago, Ill. (*1–3*)
Messinger, Leroi W., 109 N. Ingalls St., Ann Arbor, Mich. (*1–3*)
Michener, Samuel M., 580 E. Broad St., Columbus, Ohio. (*1–7*)
Mims, J. L., Southern Union Life Ins. Co., Fort Worth, Tex. (*1–4, 5*)
Mincks, W. Walter, Equitable Life Assurance Society, 393 Seventh Ave., New York, N.Y. (*1–4*)
Morse, John J., Central States Life Ins. Co., St. Louis, Mo. (*1, 2*)
Mott, A. B. C., Midland Life Ins. Co., Kansas City, Mo. (*1–4, 6*)
Mott, Thomas M., The Legal Reserve Ins. Co., Topeka, Kan. (*1–5*)
Murdoch, John M., 293 St. John's Ave., Winnipeg, Man., Canada. (*1, 3*)
Myers, Glen W., 900 S. State St., Ann Arbor, Mich. (*4*)
Myers, Owen J., Occidental Life Ins. Co., Los Angeles, Calif. (*1, 2, 4*)
Nelson, Carroll E., Missouri State Life Ins. Co., St. Louis, Mo. (*1–6, 8*)
Nicholson, Earl H., Lincoln National Life Ins. Co., Fort Wayne, Ind. (*1–3*)
Orloff, Conrad A., Univ. of Manitoba, Winnipeg, Man., Canada. (*1*)
Pakes, Eugene, Guaranty Life Ins. Co., Davenport, Iowa. (*1, 3, 4*)
Pall, Gordon, 133 Hallett St., Winnipeg, Man., Canada. (*1–7*)
Parks, George A., Bankers Life Co., Des Moines, Iowa. (*1–4*)
Pendergast, (Miss) Dell, Missouri State Life Ins. Co., St. Louis, Mo. (*1, 2*)
Perkins, Frederick P., Aetna Life Ins. Co., Hartford, Conn. (*1–4*)
Pierce, Roland W., Philadelphia Life Ins. Co., Philadelphia, Pa. (*1*)
Porter, Randall R., 819 Higley Bldg., Cedar Rapids, Iowa. (*1*)
Prenner, Myron R., North Dakota Insurance Dept., Bismarck, N.D. (*1–4*)
Preston, Robert F., Northwestern Nat. Life Ins. Co., Minneapolis, Minn. (*1–6, 8*)
Pullen, J. Gilbert, Bankers Life Company, Des Moines, Iowa. (*2*)
Raiford, Theodore E., 1512 Granger Ave., Ann Arbor, Mich. (*1–4*)
Reilly, George E., Midland Mutual Life Ins. Co., Columbus, Ohio. (*2–4*)
Roberts, Donald A. H., Mutual Life Assurance Co. of Canada, Waterloo, Ont., Canada. (*2*)
Ruben, Isadore, 1404 Bushwick Ave., Brooklyn, N.Y. (*1–3*)
Ruby, Harlan H., Equitable Life Ins. Co. of Iowa, Des Moines, Iowa. (*1*)
Rust, (Miss) Eleanor, Guardian Life Ins. Co., 50 Union Sq., New York, N.Y. (*1–3*)
Sarason, Harry M., Missouri State Life Ins. Co., St. Louis, Mo. (*1–4*)
Schillig, (Miss) Eva M., North American Reassurance Co., 250 Park Ave., New York, N.Y. (*1–4, 6*)
Schirm, Oscar G., Peoria Life Ins. Co., Peoria, Ill. (*1, 2*)
Schroeder, Gordon D., Confederation Life Assoc., Toronto, Canada. (*1*)
Shepherd, Pearce, North American Reassurance Co., 250 Park Ave., New York, N.Y. (*1–7*)
Skelding, George A., London Life Ins. Co., Bloor and Bathurst Sts., Toronto, Canada. (*1–4*)
Smith, Lewis G., Confederation Life Assoc., Toronto, Canada. (*1–7*)
Stocking, L. Alfred, Equitable Life Ins. Co. of Iowa, Des Moines, Iowa. (*1*)

Sullivan, Chester O., Midland Mutual Life Ins. Co., Columbus, Ohio. (*1, 2*)
Thompson, Emerson W., Travelers Life Ins. Co., Hartford, Conn. (*1*)
Tibbs, (Miss) Esther O., 3507 Grand Boulevard, Chicago, Ill. (*1*)
Tuck, Edward B., Mutual Life Assurance Co., Waterloo, Ont., Canada. (*1–4*)
Turnbull, David McI., 178 Anderson Ave., Winnipeg, Man., Canada. (*1, 2*)
Turner, Paul A., Philadelphia Life Ins. Co., Philadelphia, Pa. (*1–4*)
Twiss, John F., 1020 Union St., Kingston, Ont., Canada. (*1–4*)
Ungerleider, Arthur L., Peoples Life Ins. Co., Frankfort, Ind. (*2–4*)
Unsworth, Norman R., Confederation Life Assoc., Toronto, Canada. (*1, 3*)
Van Fleet, George, 420 Thompson St., Ann Arbor, Mich. (*2, 3*)
Veatch, Virgil V., Merchants Life Ins. Co., Des Moines, Iowa. (*3*)
Vernon, (Miss) Grace H., Illinois Life Ins. Co., 1212 Lake Shore Drive, Chicago, Ill. (*2*)
Vetter, Lester H., Minnesota Mutual Life Ins. Co., St. Paul, Minn. (*1, 2*)
Wadleigh, William L., Metropolitan Life Ins. Co., New York, N.Y. (*1–4*)
Wallace, William, Carolina Life Ins. Co., Columbia, S.C. (*1–5*)
Whipple, (Miss) Ruth, 4919 McPherson St., St. Louis, Mo. (*1–3*)
Whitbread, Robert B. T., 372 Ash St., Winnipeg, Man., Canada. (*1, 2*)
Wilks, Lee, Acacia Mutual Life Assoc., Washington, D.C. (*1, 2*)
Wilson, (Miss) Elizabeth W., 1431 Meridian St., N.W., Washington, D.C. (*1–6*)
Wood, Douglas, Missouri State Life Ins. Co., St. Louis, Mo. (*1–4*)
Woolery, J. Myron, Inter Southern Life Ins. Co., Louisville, Ky. (*1–6, 8*)
Yates, Stanley D., Metropolitan Life Ins. Co., 1 Madison Ave., New York, N.Y. (*1, 3*)
Yudkoff, Harry, 627 Kenilworth Ave., Detroit, Mich. (*2, 3*)

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